Right-Eigenstate-Based Approach to Non-Hermitian Superfluidity with Two-Body Loss

Xuezhu Liu[1,2], Ming Lu[3*], and Haiwen Liu[1,2,4*]

[1]*Center for Advanced Quantum Studies, School of Physics and Astronomy, Beijing Normal University, Beijing 100875, China*
[2]*Key Laboratory of Multiscale Spin Physics (Ministry of Education), Beijing Normal University, Beijing 100875, China*
[3]*Beijing Academy of Quantum Information Sciences, Beijing 100193, China*
[4]*Interdisciplinary Center for Theoretical Physics and Information Sciences, Fudan University, Shanghai 200433, China*

(Received 24 June 2025; accepted manuscript online 25 September 2025)

We theoretically explore a non-Hermitian superfluid model with complex-valued interaction, inspired by two-body loss stemming from inelastic scattering observed in ultracold atomic experiments. Utilizing both the right-eigenstate-based mean-field theory and its biorthogonal counterpart, we study the properties of the system. Notably, the right-eigenstate-based framework produces smooth and continuous solutions, in stark contrast to the absence of nontrivial solutions and the abrupt discontinuities observed in the biorthogonal-eigenstate-based framework under moderate dissipation. In addition, the lower condensation energy obtained in the former framework suggests its superior suitability for describing this system. Furthermore, we explore the impact of backscattering, a crucial factor in realistic systems. Our analysis reveals that, facilitated by two-body loss, even moderate backscattering destabilizes the superfluid state. Sufficiently strong backscattering completely destroys it, highlighting a key mechanism for the fragility of this non-Hermitian quantum phase.

DOI: 10.1088/0256-307X/42/11/110706 CSTR: 32039.14.0256-307X.42.11.110706

1. Introduction. Superconducting systems, characterized by their unique macroscopic quantum phenomena and wide-ranging applications in modern quantum technologies, constitute a core area of condensed matter physics.[1] While numerous unconventional and exotic superconductors have been discovered within the framework of closed Hermitian systems,[2–12] real-world superconductors are inherently open systems, interacting with their environment. This interaction introduces dissipative effects, which are effectively captured by non-Hermitian (NH) Hamiltonians.[13] Despite being relatively underexplored, NH superfluid systems have recently been associated with intriguing phenomena, including $\mathcal{PT}$-symmetric quantum critical effects,[14,15] nonorthogonal Majorana zero modes,[16] exceptional odd-frequency pairing,[17] the paramagnetic Meissner effect,[18] unconventional phase transitions,[19–27] NH-modulated superconducting fluctuation,[28] and exceptional fermionic superfluidity.[29]

Recent experimental advances in ultracold atomic systems have enabled the realization of superfluid states mediated by orbital Feshbach resonance in alkaline-earth atoms.[30–35] These systems exhibit metastable states with strong inelastic collisions, resulting in tunable two-body loss.[36] Unlike elastic scattering or one-body loss, two-body loss introduces complex-valued interaction, offering a novel platform for investigating NH superfluid. A mean-field theory based on the biorthogonal basis has been initially developed to demonstrate how superfluidity is modified under two-body loss.[19,24] However, recent studies have shown that the right-eigenstate-based definition of observables is both experimentally more relevant[37–41] and physically meaningful.[42–46] For instance, studies on normal metal-insulator-NH superconductor junctions have shown that the right-eigenstate-based definition accurately describes the Andreev reflection process. In contrast, using the biorthogonal basis to define current leads to a predicted reduction in conductance due to Andreev reflection, which violates fundamental physical principles.[47] As we know, in the context of mean-field theories, the definition of the order parameter plays a pivotal role. A right-eigenstate-based NH mean-field theory has been developed, successfully describing the first-order $\mathcal{PT}$ phase transitions in NH superconductors.[48] However, within this framework, the effects of two-body loss and impurity scattering on superfluidity remain an open question, warranting further exploration.

In this work, we explore an open superfluid system modeled with complex-valued interaction. By numerically solving the gap equations derived from both the right-eigenstate-based mean-field theory and its biorthogonal counterpart, we evaluate the suitability of these two ap-

*Corresponding authors. Email: luming@baqis.ac.cn; haiwen.liu@bnu.edu.cn
© 2025 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

proaches in describing the NH superfluid systems. We find that the biorthogonal approach fails to yield nontrivial solutions in the regime of moderate dissipation and exhibits discontinuities. In contrast, the right-eigenstate-based approach successfully produces continuous solutions and achieves lower condensation energy, indicating its superior suitability for describing the NH superfluid systems. Furthermore, our analysis reveals that although two-body loss amplifies the magnitude of the order parameter, it concurrently introduces a positive real part to the condensation energy, thereby driving the superfluid state into a metastable regime. In contrast, the intrinsic s-wave pairing interaction mitigates this destabilizing effect, promoting the stabilization of the superfluid state. The interplay between them gives rise to a transition in superfluid stability. Additionally, backscattering further disrupts superfluidity, causing the order parameter to rapidly decay to zero. Notably, moderate backscattering amplifies the destabilizing effect of two-body loss, thereby shrinking the stable superfluid phase in the phase diagram.

2. Model and the NH Gap Equation. We consider an NH superfluid model on a cubic lattice with two-body loss described by the Hamiltonian

$$\begin{aligned} H = & -w\sum_{\langle n,m\rangle} c_n^\dagger c_m - \mu\sum_n c_n^\dagger c_n \\ & - U\sum_n c_{n\uparrow}^\dagger c_{n\downarrow}^\dagger c_{n\downarrow} c_{n\uparrow}. \end{aligned} \tag{1}$$

Here, w represents the nearest-neighbor hopping amplitude, μ is the chemical potential, and $U = U_0 + \mathrm{i}\gamma/2$, where γ represents the two-body loss rate. The two-body loss, induced by the coupling between the environment and the superfluid, results in a complex-valued interaction U, as described by the Lindblad quantum master equation [see the Supplementary Materials (SM) for details],[19,49] which fundamentally accounts for the emergence of non-Hermiticity.

According to the right-eigenstate-based NH mean-field theory,[48] the order parameters are defined as

$$\begin{aligned} \Delta &= -\frac{U}{N}\sum_k \frac{\langle\Psi_0|c_{-k\downarrow}c_{k\uparrow}|\Psi_0\rangle}{\langle\Psi_0\mid\Psi_0\rangle}, \\ \bar{\Delta} &= -\frac{U}{N}\sum_k \frac{\langle\Psi_0|c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger|\Psi_0\rangle}{\langle\Psi_0\mid\Psi_0\rangle}, \end{aligned} \tag{2}$$

where N is the number of unit cells, and $|\Psi_0\rangle$ denotes the NH Bardeen–Cooper–Schrieffer (BCS) ground state as detailed below.[50] By performing the mean-field approximation, we obtain

$$H_{\mathrm{MF}} = \sum_k \begin{pmatrix} c_{k\uparrow}^\dagger & c_{-k\downarrow} \end{pmatrix} \begin{pmatrix} \xi_k & \Delta \\ \bar{\Delta} & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^\dagger \end{pmatrix}, \tag{3}$$

where $\xi_k = \varepsilon_k - \mu$ and $\varepsilon_k = -2w(\cos k_x + \cos k_y + \cos k_z)$ is the energy dispersion of a cubic lattice with nearest-neighbor hopping. Equation (3) can be diagonalized as $H_{\mathrm{MF}} = \sum_k E_k(\bar{\gamma}_{k\uparrow}\gamma_{k\uparrow} + \bar{\gamma}_{-k\downarrow}\gamma_{-k\downarrow})$ by the NH Bogoliubov transformation, where $\bar{\gamma}_{k\sigma}, \gamma_{k\sigma}$ are the quasiparticle creation and annihilation operators with detailed forms in the (SM),* and $E_k = \sqrt{\xi_k^2 + \Delta\bar{\Delta}}$ are the quasiparticle excitation energies. It is noteworthy that, unlike the Hermitian case, $\bar{\gamma}_{k\sigma} \neq \gamma_{k\sigma}^\dagger$; these quasiparticles are neither conventional fermions nor bosons, although $\{\bar{\gamma}_{k\sigma}, \gamma_{k'\sigma'}\} = \delta_{kk'}\delta_{\sigma\sigma'}$. In the ground state, all quasiparticle excitations are absent. Consequently, $|\Psi_0\rangle = \prod_k \gamma_{k\uparrow}\gamma_{-k\downarrow}|0\rangle$, where $|0\rangle$ denotes the fermionic vacuum. Combining this with Eq. (2), we derive the NH gap equation

$$\frac{1}{U} = \frac{1}{2N}\sum_k \frac{E_k^* + \xi_k}{(|u_k|^2 + |v_k|^2)|E_k||E_k + \xi_k|}, \tag{4}$$

where E_k^* denotes the complex conjugate of E_k, and u_k, v_k represent the coefficients of the NH Bogoliubov transformation, with their explicit forms provided in the SM. In the cubic lattice model, both theoretical considerations and numerical calculations indicate that small changes to the Fermi level have little impact on the system's physical properties. For simplicity, we focus on the half-filling case ($\mu = 0$) and normalize the energy scale by setting $w = 1$ in the subsequent discussion.

It is essential to emphasize that, although the interaction is complex-valued, the U(1) symmetry of the original Hamiltonian Eq. (1) is preserved. With the mean-field treatment, Eq. (2), we can generally define

$$\Delta = \Delta_0 \mathrm{e}^{\mathrm{i}\phi}, \quad \bar{\Delta} = \Delta_0 \mathrm{e}^{-\mathrm{i}\phi}, \tag{5}$$

where ϕ represents the U(1) gauge phase and $\Delta_0 \in \mathbb{C}$. Under this definition, $\Delta\bar{\Delta} = \Delta_0^2$ is gauge-invariant. Consequently, E_k and the gap equation are also independent of the gauge phase ϕ. Moreover, Δ_0 serves as the order parameter, with both its real and imaginary components being gauge-invariant and essential to the properties of the system. It can be analytically and numerically demonstrated that $\mathrm{Re}\Delta_0$ corresponds to the real part of the spectral gap, while $\mathrm{Im}\Delta_0$ coincides with the maximum of $\mathrm{Im}E_k$. In the Hermitian limit ($\gamma = 0$), $E_k = \sqrt{\xi_k^2 + \Delta_0^2}$, reducing Eq. (4) to $\frac{1}{U} = \frac{1}{2N}\sum_k \frac{1}{E_k}$.

3. Numerical Solutions of the Gap Equation. By solving the NH gap equation [Eq. (4)] numerically, we obtain the dependence of Δ_0 on the two-body loss rate γ with a fixed $U_0 = 2.8$, as illustrated in Fig. 1(a). With the gauge choice $\phi = 0$, U and Δ_0 are shown to share the same complex angle θ. Meanwhile, $\mathrm{Re}\Delta_0$ exhibits a positive correlation with U_0 and remains nearly invariant with respect to γ, whereas $\mathrm{Im}\Delta_0$ is positively correlated with γ. The inset of Fig. 1(a) illustrates the phase-locking between Δ_0 (dots) and U (line).

In NH systems, energy possesses both real and imaginary components, where the real part is associated with particle occupations, while the imaginary part reflects particle lifetimes. Accordingly, the real part of the condensation energy, $\mathrm{Re}E_{\mathrm{c}}$, indicates whether the NH superfluid

*See the SM for detailed derivations of the right-eigenstate-based mean-field theory and its biorthogonal counterpart, as well as a comprehensive comparison between the two frameworks.

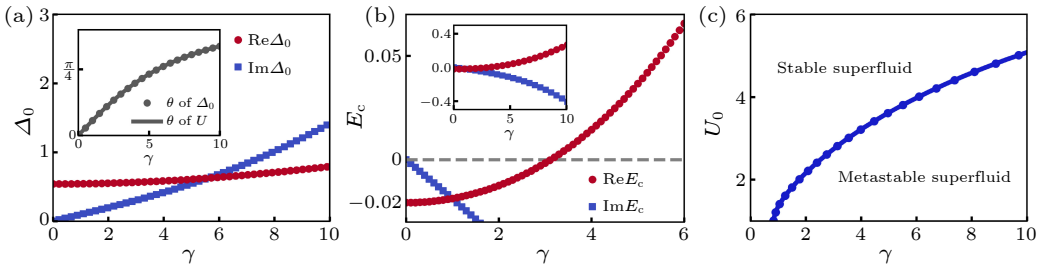

Fig. 1. Numerical solutions derived from the NH gap equation, Eq. (4). (a) Δ_0 plotted against γ for $U_0 = 2.8$. The inset shows the complex angle θ of Δ_0 (grey dot) and U (grey line) as functions of γ for $U_0 = 2.8$, demonstrating exact coincidence. (b) E_c as a function of γ for $U_0 = 2.8$, with the inset presenting results for E_c over a broader range of γ. (c) The phase diagram of superfluid stability at half-filling, where the blue curve indicates $\mathrm{Re}E_c = 0$.

state is energetically favorable.[19] For $\mu = 0$, the particle number is given by $N_e = \sum_{k\sigma}\langle\Psi_0|c^\dagger_{k\sigma}c_{k\sigma}|\Psi_0\rangle/\langle\Psi_0 \mid \Psi_0\rangle = N$, corresponding to the half-filling condition. Under this scenario, the condensation energy E_c is

$$E_c = \frac{1}{N}\sum_k(-E_k + \xi_k + |\xi_k|) + \frac{1}{U}\Delta_0^2. \tag{6}$$

Figure 1(b) illustrates the dependence of E_c on γ with a fixed $U_0 = 2.8$. As γ increases, $\mathrm{Re}E_c$ rises and eventually surpasses zero, indicating that the superfluid state ceases to be the stable ground state of the system. The two-body loss γ, by virtue of the phase-locking property, facilitates the formation of the superfluid order parameter. However, the induced non-Hermiticity ultimately undermines the stability of the system, rendering the superfluid state metastable. The inset of Fig. 1(b) shows E_c over a wider range of γ.

Building on the analysis of $\mathrm{Re}E_c$, we construct the superfluid stability phase diagram in the U_0–γ plane, as shown in Fig. 1(c). Here, we only choose $U_0 > 1$, since it is computationally more demanding to reach the required accuracy for smaller U_0. The transition in superfluid stability, driven by two-body loss, emerges as a universal phenomenon. For a fixed U_0, as γ increases, the system transitions from a stable superfluid state to a metastable superfluid state. Notably, the larger the U_0, the higher the critical value of γ at the transition point. In summary, the pairing interaction preserves superfluid stability, while two-body loss undermines it, and their competition ultimately governs the superfluid stability transitions.

4. The Two Definitions of the Order Parameters. In NH physics, the eigenstates of an NH Hamiltonian are generally non-orthogonal, and their evolution is inherently nonunitary.[51] These characteristics introduce unique challenges and stimulate discussions regarding the proper definition of observables. For an NH matrix O of dimension $n \times n$ with n nondegenerate eigenvalues, or with the degenerate eigenvalues where the algebraic multiplicity equals the geometric multiplicity, its right eigenstates constitute a complete basis (SM). At present, two primary approaches to defining observables exist: one based on the right eigenstate and the other on the biorthogonal basis.[42] While the biorthogonal definition provides a more concise mathematical framework, the right-eigenstate-based definition offers a more intuitive probabilistic interpretation[42–47] and aligns more closely with experimental observations.[37–41] The choice of order parameter definition is particularly crucial in the context of mean-field theory. In this section, we investigate the applicability of these two definitions to the NH superfluid systems, based on numerical simulation results from the complex-valued interaction superfluid model.

The order parameter Δ_0^{LR}, defined within the biorthogonal basis, satisfies the gap equation $\frac{1}{U} = \frac{1}{2N}\sum_k \frac{1}{E_k^{\mathrm{LR}}}$, where $E_k^{\mathrm{LR}} = \sqrt{\xi_k^2 + (\Delta_0^{\mathrm{LR}})^2}$ (see the SM for details). With a fixed $U_0 = 1.8$, we numerically calculate Δ_0 and $\mathrm{Re}E_c$ as functions of γ under both frameworks. Firstly, as illustrated in Fig. 2(a), within the moderate dissipation region, the biorthogonal approach yields no nontrivial solutions (red points), indicating a normal phase.

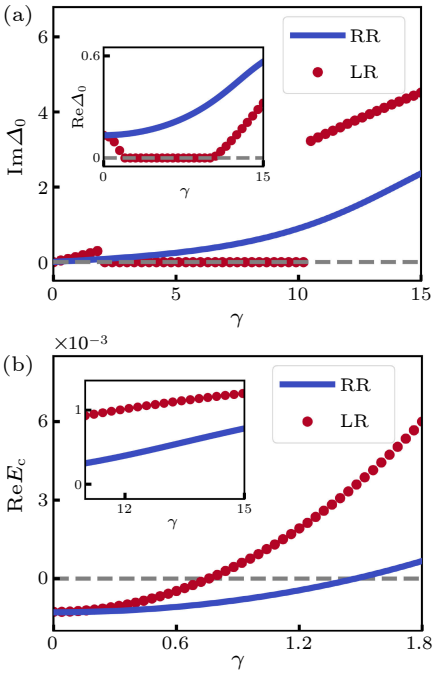

Fig. 2. Comparison of numerical solutions to the NH gap equation under different definitions of the order parameters. (a) Δ_0 and (b) $\mathrm{Re}E_c$ as functions of γ for $U_0 = 1.8$. The blue lines represent solutions based on the right-eigenstate-based approach (RR), while the red points correspond to solutions based on the biorthogonal approach (LR). The inset in (b) highlights the results for the metastable superfluid phase, whereas the main panel depicts the stable superfluid phase.

As γ increases further, $\Delta_0^{\rm LR}$ undergoes an abrupt change, leading to the reemergence of superfluidity. However, these abnormal discontinuities can be readily avoided by using the right-eigenstate-based approach. As shown in Fig. 2(a), blue solid lines are shown as continuous solutions across the entire parameter space, without any nonanalytic behavior. Secondly, as shown in Fig. 2(b), the right-eigenstate-based approach results in lower condensation energy, applicable to both stable and metastable superfluid phases. We thereby conclude that, for complex-valued interaction superfluid, the right-eigenstate-based mean-field theory proves to be more suitable. This finding further supports discussions in related literature.[48]

5. The Backscattering Effect. In this section, we examine the impact of impurity scattering on the NH superfluid using an exactly solvable simplified model—the backscattering model. This model posits that electrons with momentum k, upon encountering an impurity, have their momentum reversed to $-k$. Although this simplified model does not precisely represent real random impurities or phonon scattering, it effectively captures the attenuation of flow and the finite lifetime of quasiparticles induced by impurity scattering in many-body systems.[52]

The NH superfluid model with two-body loss in the presence of backscattering is described by

$$H_{\rm MF}^{\rm BS} = H_{\rm MF} + \sum_{k>0,\sigma} \Gamma_{\rm BS}(c_{k\sigma}^\dagger c_{-k\sigma} + c_{-k\sigma}^\dagger c_{k\sigma}), \quad (7)$$

where $\Gamma_{\rm BS}$ represents the backscattering strength and $H_{\rm MF}$ is detailed in Eq. (3). $H_{\rm MF}^{\rm BS}$ can be diagonalized as $H_{\rm MF}^{\rm BS} = \sum_{k>0} E_{\alpha k}(\bar\gamma_{\alpha k\uparrow}\gamma_{\alpha k\uparrow} + \bar\gamma_{\alpha k\downarrow}\gamma_{\alpha k\downarrow})$ through an NH Bogoliubov transformation (see the SM), where $\alpha = 1, 2$ labels the branches of quasiparticles and $\bar\gamma_{\alpha k\sigma}, \gamma_{\alpha k\sigma}$ are quasiparticle operators. The quasiparticle excitation energy is given by $E_{\alpha k} = \sqrt{[\xi_k - (-1)^\alpha \Gamma_{\rm BS}]^2 + \Delta\bar\Delta}$. Thus, the NH BCS ground state is defined as $|\Psi_0\rangle_{\rm BS} = \prod_{\alpha,k>0} \gamma_{\alpha k\uparrow}\gamma_{\alpha k\downarrow}|0\rangle$. Combining this with Eq. (2), we derive the NH gap equation in the presence of backscattering,

$$\frac{1}{U} = \frac{1}{2N}\sum_{\alpha,k>0}\left\{\frac{E_{\alpha k}^* + [\xi_k - (-1)^\alpha \Gamma_{\rm BS}]}{C|E_{\alpha k}||E_{\alpha k} + [\xi_k - (-1)^\alpha \Gamma_{\rm BS}]|}\right\}, \quad (8)$$

where C is the normalization factor of $|\Psi_0\rangle_{\rm BS}$ (SM). In the weak scattering limit ($\Gamma_{\rm BS} \to 0$), this equation simplifies to Eq. (4). Furthermore, in the Hermitian limit ($\gamma = 0$), it reduces to $\frac{1}{U} = \frac{1}{2N}\sum_{\alpha,k>0}\frac{1}{E_{\alpha k}}$ where $E_{\alpha k} = \sqrt{[\xi_k - (-1)^\alpha \Gamma_{\rm BS}]^2 + \Delta_0^2}$.[52] It is crucial to emphasize that, with the definition of the order parameters [Eq. (2)] being unaffected by the backscattering, the formulation in Eq. (5) remains applicable. As a result, Δ_0, $E_{\alpha k}$, and the gap equation all exhibit gauge invariance.

By the NH gap equation in the presence of backscattering [Eq. (8)], we analyze the dependence of Δ_0 on the backscattering strength $\Gamma_{\rm BS}$ for $U_0 = 6.8$ and $\gamma = 10$, as shown in Fig. 3(a). With increasing $\Gamma_{\rm BS}$, Δ_0 rapidly decays to 0, driving the system into a normal state where stable Cooper pairs can no longer form, similar to the Hermitian case.[52] Upon normalization to the maximum value,

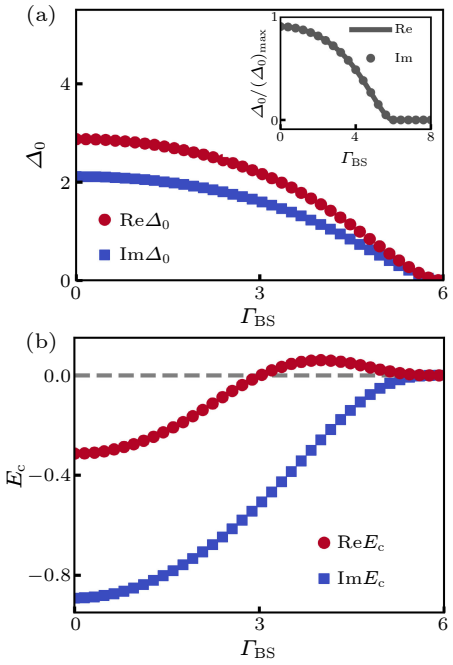

Fig. 3. Numerical solutions derived from the NH gap equation with backscattering [Eq. (8)]. (a) Δ_0 and (b) $E_{\rm c}$ are shown as functions of $\Gamma_{\rm BS}$ for $U_0 = 6.8$ and $\gamma = 10$. The inset in (a) displays ReΔ_0 (grey line) and ImΔ_0 (grey dot) normalized to their maximum values.

both ReΔ_0 and ImΔ_0 exhibit identical dependence on $\Gamma_{\rm BS}$, as shown in the inset of Fig. 3(a). This behavior stems from the phase-locking between Δ_0 and U, which ensures that the phase of Δ_0 remains invariant with respect to $\Gamma_{\rm BS}$. Consequently, for a fixed U, as $\Gamma_{\rm BS}$ varies, the relationship ${\rm Re}\Delta_0/{\rm Re}\Delta_{0\,\rm max} = {\rm Im}\Delta_0/{\rm Im}\Delta_{0\,\rm max} = |\Delta_0|/|\Delta_0|_{\rm max}$ is maintained.

By calculating the particle number for $\mu = 0$, we find $N_{\rm e} = N$, indicating the half-filling. Thus, the condensation energy $E_{\rm c}$ is expressed as

$$E_{\rm c} = \frac{1}{N}\sum_{\alpha,k}\Big(-\frac{1}{2}E_{\alpha k} + \xi_k + |\xi_k - |\Gamma_{\rm BS}||\Big) + \frac{1}{U}\Delta_0^2. \quad (9)$$

Figure 3(b) demonstrates the relationship between $E_{\rm c}$ and $\Gamma_{\rm BS}$ with $U_0 = 6.8$ and $\gamma = 10$. The results reveal that, facilitated by two-body loss, moderate backscattering can destabilize the superfluid state, thereby inducing a superfluid stability transition. Specifically, when backscattering is initially introduced, ${\rm Re}E_{\rm c} < 0$, signifying a stable superfluid phase. As $\Gamma_{\rm BS}$ increases, the system transitions to a metastable superfluid state, characterized by ${\rm Re}E_{\rm c} > 0$. Finally, with further increases in $\Gamma_{\rm BS}$, the superfluid state is entirely destroyed, and the system transitions into the normal state, where ${\rm Re}E_{\rm c} = 0$.

By examining the order parameter and condensation energy, we delineate the phase diagram on the U_0–$\Gamma_{\rm BS}$ plane with $\gamma = 10$, revealing the complex interplay between paring interaction, two-body loss, and backscattering, as shown in Fig. 4(a). In the absence of backscattering ($\Gamma_{\rm BS} = 0$), two-body loss destabilizes the superfluid stability, while the pairing interaction U_0 preserves it. Their competition induces a superfluid stability transition.

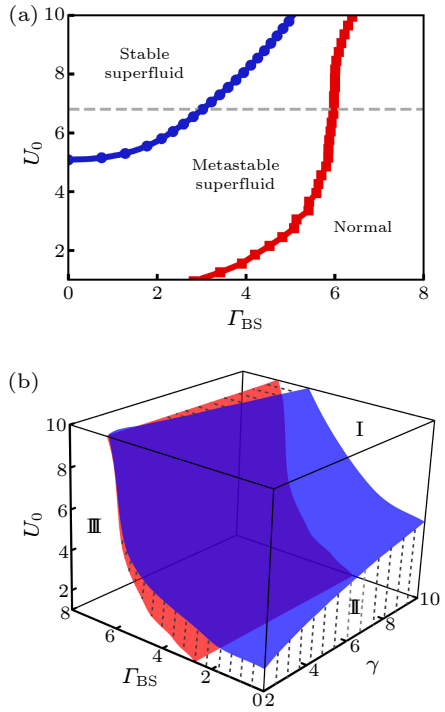

Fig. 4. Phase diagrams. (a) Phase diagram in the U_0–$\Gamma_{\rm BS}$ plane at $\gamma = 10$. The grey dashed line corresponds to the phase transition described in Fig. 3. (b) Three-dimensional phase diagram in the U_0–γ–$\Gamma_{\rm BS}$ parameter space. Region I: the stable superfluid phase; Region Ⅱ: the metastable superfluid phase; Region Ⅲ: the normal phase. The blue curve (or surface) represents the condition $\mathrm{Re}E_{\rm c} = 0$ with $\Delta_0 \neq 0$, while the red curve (or surface) signifies the onset of $\Delta_0 = 0$.

When backscattering is introduced, U_0 required to sustain a stable superfluid phase increases significantly, as indicated by the blue curve in Fig. 4(a). This observation suggests that backscattering amplifies the destabilizing effect of two-body loss on superfluidity. With further increase in $\Gamma_{\rm BS}$, the system transitions into the normal phase with both $\Delta_0 = 0$ and $\mathrm{Re}E_{\rm c} = 0$, and the transition curve is marked red, as shown in Fig. 4(a). Notably, at the critical point, a larger U_0 corresponds to a higher $\Gamma_{\rm BS}$. Figure 4(b) presents the three-dimensional phase diagram within the U_0–γ–$\Gamma_{\rm BS}$ parameter space. The transition to the normal phase, represented by the red surface, is exclusively governed by $\Gamma_{\rm BS}$. As shown in Fig. S1(d) of the SM, the cross-sections of the red surface for various values of γ exhibit complete overlap. In contrast, the superfluid stability transition, depicted by the blue surface, is jointly influenced by both γ and $\Gamma_{\rm BS}$. Within the superfluid phase, maintaining stability becomes increasingly challenging as the effects of two-body loss and backscattering intensify.

In short, the pairing interaction strength U_0 promotes the superfluid phase formation, whereas two-body loss and backscattering act as destabilizing forces. The interplay among these factors leads to the emergence of a distinctive metastable superfluid phase in NH superfluid systems. Currently, fermionic superfluidity under two-body loss has been experimentally achieved in ultracold atomic systems, such as ytterbium atoms.[34,36] In these systems, the interaction strength is tunable via orbital Feshbach resonance,[30–33,35] while the two-body loss rate can be precisely controlled using photoassociation techniques.[53] The impact of two-body loss on the superfluid gap can be experimentally probed through angle-resolved photoemission spectroscopy.[34,54–56] These advancements in ultracold atomic systems establish a versatile and robust platform for investigating both stable and metastable NH superfluid phases, hopefully will provide valuable insights into the unconventional physics in these systems.

6. Summary. We have systematically investigated an NH superfluid model with complex-valued interaction by employing a right-eigenstate-based mean-field theory. Firstly, our work resolves a critical ambiguity in the theoretical description of such systems, demonstrating that the right-eigenstate-based approach provides a more physically consistent and energetically favorable framework than its biorthogonal counterpart. Secondly, we identified a mechanism where two-body loss, while enhancing the pairing amplitude, paradoxically drives the system towards a metastable state. Thirdly, we showed that backscattering acts as a potent destabilizing agent that can ultimately destroy the superfluid phase. These results underscore the delicate interplay between pairing, dissipation, and disorder in NH superfluids. This work not only provides a robust theoretical tool for analyzing open superfluids but also offers direct, testable predictions for ultracold atom experiments with tunable two-body loss. Future investigations could extend this framework to explore nonequilibrium dynamics, the role of thermal fluctuations, and the potential for engineering novel, dissipation-stabilized topological phases in NH systems.

Acknowledgements. This work was financially supported by the National Key Research and Development Program of China (Grant No. 2024YFA1409001), the National Natural Science Foundation of China (Grants Nos. 12374037 and 12204044), and the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDB28000000), and the Fundamental Research Funds for the Central Universities.

References

[1] Acín A, Bloch I, Buhrman H, Calarco T, Eichler C, Eisert J, Esteve D, Gisin N, Glaser S J, Jelezko F, Kuhr S, Lewenstein M, Riedel M F, Schmidt P O, Thew R, Wallraff A, Walmsley I, and Wilhelm F K 2018 *New J. Phys* **20** 080201
[2] Sigrist M and Ueda K 1991 *Rev. Mod. Phys.* **63** 239
[3] Lee P A, Nagaosa N, and Wen X G 2006 *Rev. Mod. Phys.* **78** 17
[4] Stewart G R 1984 *Rev. Mod. Phys.* **56** 755
[5] Stewart G R 2011 *Rev. Mod. Phys.* **83** 1589
[6] Fulde P and Ferrell R A 1964 *Phys. Rev.* **135** A550
[7] Larkin A and Ovchinnikov I 1965 *Sov. Phys. JETP* **20** 762
[8] Kopnin N B 2001 *Theory of nonequilibrium superconductivity* (Oxford University Press)
[9] Fausti D, Tobey R I, Dean N, Kaiser S, Dienst A, Hoffmann M C, Pyon S, Takayama T, Takagi H, and Cavalleri A 2011 *Science* **331** 189
[10] Kundu A and Seradjeh B 2013 *Phys. Rev. Lett.* **111** 136402

[11] Mankowsky R, Subedi A, Först M, Mariager S O, Chollet M, Lemke H T, Robinson J S, Glownia J M, Minitti M P, Frano A, Fechner M, Spaldin N A, Loew T, Keimer B, Georges A, and Cavalleri A 2014 *Nature* **516** 71

[12] Khim S, Landaeta J F, Banda J, Bannor N, Brando M, Brydon P M R, Hafner D, Küchler R, Cardoso-Gil R, Stockert U, Mackenzie A P, Agterberg D F, Geibel C, and Hassinger E 2021 *Science* **373** 1012

[13] Moiseyev N 2011 *Non-Hermitian quantum mechanics* (Cambridge University Press)

[14] Ashida Y, Furukawa S, and Ueda M 2017 *Nat. Commun.* **8** 15791

[15] Lourenço J A S, Eneias R L, and Pereira R G 2018 *Phys. Rev. B* **98** 085126

[16] Kawabata K, Ashida Y, Katsura H, and Ueda M 2018 *Phys. Rev. B* **98** 085116

[17] Cayao J and Black-Schaffer A M 2022 *Phys. Rev. B* **105** 094502

[18] Tamura S, Müller H, Aliani L, and Kornich V 2025 *Phys. Rev. B* **111** l180503

[19] Yamamoto K, Nakagawa M, Adachi K, Takasan K, Ueda M, and Kawakami N 2019 *Phys. Rev. Lett.* **123** 123601

[20] Yamamoto K, Nakagawa M, Tsuji N, Ueda M, and Kawakami N 2021 *Phys. Rev. Lett.* **127** 055301

[21] Iskin M 2021 *Phys. Rev. A* **103** 013724

[22] He P, Ding H T, and Zhu S L 2021 *Phys. Rev. A* **103** 043329

[23] Li H, Yu X H, Nakagawa M, and Ueda M 2023 *Phys. Rev. Lett.* **131** 216001

[24] Shi T, Wang S, Zheng Z, and Zhang W 2024 *Phys. Rev. A* **109** 063306

[25] Li C, Wu Y, and Liu W M 2024 *Phys. Rev. B* **109** 214306

[26] Tajima H, Sekino Y, Inotani D, Dohi A, Nagataki S, and Hayata T 2024 *Phys. Rev. Res.* **6** 023060

[27] Takemori S, Yamamoto K, and Koga A 2024 *Phys. Rev. B* **109** l060501

[28] Chtchelkatchev N M, Golubov A A, Baturina T I, and Vinokur V M 2012 *Phys. Rev. Lett.* **109** 150405

[29] Takemori S, Yamamoto K, and Koga A 2025 arXiv:2504.19525 [cond-mat.quant-gas]

[30] Zhang R, Cheng Y, Zhai H, and Zhang P 2015 *Phys. Rev. Lett.* **115** 135301

[31] He L, Wang J, Peng S G, Liu X J, and Hu H 2016 *Phys. Rev. A* **94** 043624

[32] Höfer M, Riegger L, Scazza F, Hofrichter C, Fernandes D, Parish M, Levinsen J, Bloch I, and Fölling S 2015 *Phys. Rev. Lett.* **115** 265302

[33] Pagano G, Mancini M, Cappellini G, Livi L, Sias C, Catani J, Inguscio M, and Fallani L 2015 *Phys. Rev. Lett.* **115** 265301

[34] Darkwah Oppong N, Riegger L, Bettermann O, Höfer M, Levinsen J, Parish M, Bloch I, and Fölling S 2019 *Phys. Rev. Lett.* **122** 193604

[35] Cappellini G, Livi L, Franchi L, Tusi D, Benedicto Orenes D, Inguscio M, Catani J, and Fallani L 2019 *Phys. Rev. X* **9** 011028

[36] Tomita T, Nakajima S, Takasu Y, and Takahashi Y 2019 *Phys. Rev. A* **99** 031601

[37] Barontini G, Labouvie R, Stubenrauch F, Vogler A, Guarrera V, and Ott H 2013 *Phys. Rev. Lett.* **110** 035302

[38] Li J, Harter A K, Liu J, de Melo L, Joglekar Y N, and Luo L 2019 *Nat. Commun.* **10** 855

[39] Wu Y, Liu W, Geng J, Song X, Ye X, Duan C K, Rong X, and Du J 2019 *Science* **364** 878

[40] Yamamoto K, Nakagawa M, Tezuka M, Ueda M, and Kawakami N 2022 *Phys. Rev. B* **105** 205125

[41] Yamamoto K and Kawakami N 2023 *Phys. Rev. B* **107** 045110

[42] Meden V, Grunwald L, and Kennes D M 2023 *Rep. Prog. Phys.* **86** 124501

[43] Graefe E M, Korsch H J, and Niederle A E 2008 *Phys. Rev. Lett.* **101** 150408

[44] Dóra B, Heyl M, and Moessner R 2019 *Nat. Commun.* **10** 2254

[45] Sticlet D, Dóra B, and Moca C P 2022 *Phys. Rev. Lett.* **128** 016802

[46] Yan Q, Li H, Sun Q F, and Xie X C 2024 *Phys. Rev. B* **110** 045138

[47] Kornich V 2023 *Phys. Rev. Lett.* **131** 116001

[48] Liu X, Lu M, Liu H, and Xie X C 2025 arXiv:2509.03072 [cond-mat.supr-con]

[49] Dalibard J, Castin Y, and Mølmer K 1992 *Phys. Rev. Lett.* **68** 580

[50] Bardeen J, Cooper L N, and Schrieffer J R 1957 *Phys. Rev.* **106** 162

[51] Brody D C 2013 *J. Phys. A: Math. Theor.* **47** 035305

[52] Chen H 1993 *Phys. Rev. Lett.* **71** 2304

[53] Tomita T, Nakajima S, Danshita I, Takasu Y, and Takahashi Y 2017 *Sci. Adv.* **3** e1701513

[54] Damascelli A, Hussain Z, and Shen Z X 2003 *Rev. Mod. Phys.* **75** 473

[55] Vishik I M, Lee W S, He R H, Hashimoto M, Hussain Z, Devereaux T P, and Shen Z X 2010 *N. J. Phys.* **12** 105008

[56] Brown P T, Guardado-Sanchez E, Spar B M, Huang E W, Devereaux T P, and Bakr W S 2019 *Nat. Phys.* **16** 26