Light Hadronic Decays of Spin-0 Partner of $X(3872)$

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The $X(3872)$ is theoretically predicted to have a spin-0 partner state denoted as X_0. Assuming the X_0 as a $D\bar{D}$ molecular bound state, we calculate the decay widths of $X_0 \to VV$ and $X_0 \to PP$ (V and P stand for light vector and pseudoscalar mesons, respectively) via intermediate charmed meson loops. Three different configurations of the X_0, i.e., pure neutral components ($\theta = 0$), isospin singlet ($\theta = \pi/4$), and pure charged components ($\theta = \pi/2$), are investigated. Within a commonly accepted range of the model parameter α, the predicted decay widths of $X_0 \to VV$ are on the order of a few hundred keV, while the decay widths of $X_0 \to PP$ can reach several MeV. The $X_0 \to \rho\rho$ and $\pi\pi$ have larger decay rates. The relative width ratios between the channels are nearly model-independent. Moreover, among those channels only with isovector or isoscalar mesons, the relevant ratios are also independent of the phase angle. The predicted ratios are helpful for searching the X_0 in the future experiments at BESIII and Belle II.

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1. Introduction. In 2003, the Belle Collaboration observed the $X(3872)$ resonance in the $\pi^+\pi^- J/\psi$ invariant mass spectrum from the $B \to K\pi^+\pi^- J/\psi$ process,[1] marking a pivotal breakthrough in exotic hadron spectroscopy. The LHCb Collaboration determined its quantum number $J^{\rm PC} = 1^{++}$.[2,3] At present, the world average mass of the $X(3872)$ is (3871.64 ± 0.06) MeV, with an extremely narrow total width of (1.19 ± 0.21) MeV.[4] The mass of $X(3872)$ is very close to the threshold of $D^0\bar{D}^{*0}$ threshold ($m_{D^0} + m_{D^{*0}} = 3871.69$ MeV), and its dominant decay channels is $D\bar{D}^*$ final states. Therefore, the $X(3872)$ can be naturally interpreted as the $D\bar{D}^*$ molecular state.[5–29] Meanwhile, there also exist other explanations, such as tetraquark state,[30–36] hybrid charmonium state,[37,38] charmonium-molecule mixing state,[39,40] and conventional charmonium state.[41–45]

Heavy quark spin symmetry (HQSS) provides a crucial theoretical framework for understanding the properties and interactions of hadrons containing heavy quarks.[23,46–48] According to the effective field theory (EFT) and HQSS, the $X(3872)$ should have its HQSS partners.[13] If the $X(3872)$ is $D\bar{D}^*$ molecular state with $J^{\rm PC} = 1^{++}$, there would exist three degenerate partner states with the quantum numbers 1^{+-}, 0^{++}, and 2^{++} using an EFT approach with contact interactions in the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ channels in the heavy quark limit.[49]

Extensive investigations have been conducted on the characteristics and structure of the X_2, the isoscalar 2^{++} $D^*\bar{D}^*$ partner of the $X(3872)$.[50–57] The mass of X_2 is theoretically predicted to be around 4012 MeV with a width of approximately the same magnitude as that of the $X(3872)$.[18,58–63] In 2022, Belle collaboration observed a new structure with a mass of $(4014.3 \pm 4.0 \pm 1.5)$ MeV and a width of $(4 \pm 11 \pm 6)$ MeV in the invariant mass distribution of the $\gamma\psi(2S)$.[64] Given that the mass and width of the newly-discovered structure are in accordance with the predicted mass and width of the X_2, it is considered to be a good candidate for the X_2. In Ref. [65], the hidden charmed decays of the $X_2 \to J/\psi V$ and $X_2 \to \eta_c P$ via charmed meson loops were discussed, where $V = \rho^0$, ω, and $P = \pi^0$, η, and η'. The results indicate that the decay widths are significantly influenced by the X_2 mass. In Ref. [66], the charmless decays of the $X_2 \to VV$ and $X_2 \to PP$, where V represents light vector and P stands for pseudoscalar, were investigated. In the cases where the X_2 is a purely neutral $D^{*0}\bar{D}^{*0}$ or a purely charged $D^{*+}D^{*-}$ bound state, the theoretically predicted partial decay widths for the $X_2 \to VV$ and $X_2 \to PP$ are at order of several tens of keV.

The spin-0 partner of $X(3872)$, denoted as X_0 ($J^{\rm PC} = 0^{++}$), is initially predicted using a coupled channel unitary approach[67] involving many PP channels. The only relevant experimental studies are those by the Belle and BaBar Collaborations, which analyzed the $e^+e^- \to J/\psi D\bar{D}$ and $e^+e^- \to D\bar{D}$ reactions.[68–70] However, numerous theoretical studies have been carried out in different processes.[13,18,71–80] For example, in the $D\bar{D}$ mass distribution of $e^+e^- \to J/\psi D\bar{D}$ reaction;[71,72] in the analysis of meson–meson interactions within the heavy meson effective theory;[13,18,73] in the $D^0\bar{D}^0$ mass distribution of the $\psi(3770) \to \gamma D^0\bar{D}^0$ decay.[76] In Refs. [78,79], the authors described the $D\bar{D}$ mass distribution of $\gamma\gamma \to D\bar{D}$ and demonstrated the existence of a bound state near the $D\bar{D}$ mass threshold. Dai *et al.* predicted the $D\bar{D}$ mass distribution in $B^- \to D\bar{D}K^-$ process by considering the state $X(3720)$ mainly coupled to $D\bar{D}$.[80] The width of the $D\bar{D}$ scalar meson $X(3700)$ to the $\eta_c\eta$ channel is 0.85 MeV.[74] According to the HQSS and coupled-channel

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dynamics, the 0^{++} hadronic molecule can not only be a $D\bar{D}$ molecule bound state, but also be regarded as a $D^*\bar{D}^*$ state, or even a mixture of $D\bar{D}$ and $D^*\bar{D}^*$.[13,61] However, this work considers only the case where 0^{++} molecule is composed entirely of $D\bar{D}$. The global analysis of measurements from the Belle, BABAR, BESIII, and LHCb experiments implies that the fraction of unobserved decay modes of the $X(3872)$ is up to 31.9%, which indicates that there is still a lot of room for searching for new decay modes of the $X(3872)$,[81] for example the light hadron decays.[82,83] The X_0, as the spin-0 partner of $X(3872)$, might have some common decay properties with the $X(3872)$. In continuation of our previous work devoted to the light hadron decays of the exotic states,[66,83–85] we shall investigate the light hadronic decays $X_0 \to VV$ and $X_0 \to PP$ via the intermediate charmed meson loops. The calculations were performed under the $D\bar{D}$ molecule ansatz of the X_0 and using an effective Lagrangian approach.

The rest of the paper is organized as follows. In Section 2, we provide the relevant effective Lagrangians and expressions for the decay amplitudes. Then in Section 3, the numerical results and discussions are presented, and a brief summary is given in Section 4.

2. Theoretical Framework.

2.1. Effective Interaction Lagrangians. We assume that X_0 is an S-wave molecular state with the quantum numbers $J^{\rm PC} = 0^{++}$, which can be given by the superposition of the hadronic configurations of $D^0\bar{D}^0$ and D^+D^-,

$$|X_0\rangle = \cos\theta|D^0\bar{D}^0\rangle + \sin\theta|D^+D^-\rangle, \tag{1}$$

where θ represents a phase angle that defines the proportion of the neutral and charged components in the system. The interaction between the X_0 state and the $D\bar{D}$ meson pair can be described by the Lagrangian

$$\mathcal{L}_{X_0} = X_0\left(\chi^0_{\rm nr}D^{0\dagger}\bar{D}^{0\dagger}\cos\theta + \chi^{\rm c}_{\rm nr}D^{+\dagger}D^{-\dagger}\sin\theta\right), \tag{2}$$

where $\chi^0_{\rm nr}$ and $\chi^{\rm c}_{\rm nr}$ represent the coupling constants of the X_0 with the neutral and charged meson pairs, respectively. Their values are related to the X_0 binding energy and estimated by[51,86–88]

$$\chi^2_{\rm nr} = \frac{16\pi}{\mu}\sqrt{\frac{2E_{\rm B}}{\mu}}, \tag{3}$$

where $E_{\rm B} = m_1 + m_2 - M$ and $\mu = m_1m_2/(m_1+m_2)$. Considering that the X_0 is a pure $D\bar{D}$ bound state and the mass of the X_0 is 3721 MeV,[89] we obtain

$$|\chi^0_{\rm nr}| = 2.71\,{\rm GeV}^{-1/2}, \tag{4}$$
$$|\chi^{\rm c}_{\rm nr}| = 3.26\,{\rm GeV}^{-1/2}. \tag{5}$$

Such a distinction is due to the mass difference between the neutral and charged mesons, which would enhance the isospin-breaking effect.

Furthermore, we assume that the X_0 decays to VV and PP (V and P represent light vector and pseudoscalar mesons, respectively) through intermediate charmed meson loops, as shown in Fig. 1. First, the X_0 transforms into a $D\bar{D}$ meson pair. Subsequently, the D and $\bar{D}$ evolve into the final states of VV or PP via exchanging a charmed meson. When the final states are VV, the exchanged charmed meson can be either D or D^*; whereas for the final state of PP, the exchanged meson is exclusively a D^*.

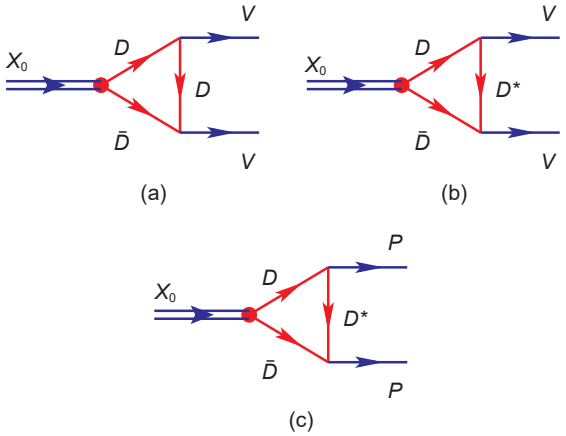

Fig. 1. Feynman diagrams for (a) and (b) the processes $X_0 \to VV$ and (c) $X_0 \to PP$ via intermediate charmed meson loops. The charge conjugated loops are not shown here, but included in the calculations.

Under the heavy quark limit and chiral symmetry, the interactions of the light vector and pseudoscalar mesons with the charmed mesons are described by the following Lagrangian:[46,90–93]

$$\begin{aligned}\mathcal{L} =& -{\rm i}\,g_{DDV}D_i^\dagger\overleftrightarrow{\partial}^\mu D^j(V_\mu^\dagger)^i_j \\ &-2f_{D^*DV}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu V^{\nu\dagger})^i_j(D_i^\dagger\overleftrightarrow{\partial}^\alpha D^{*\beta j} - D_i^{*\beta\dagger}\overleftrightarrow{\partial}^\alpha D^j) \\ &+{\rm i}g_{D^*D^*V}D_i^{*\nu\dagger}\overleftrightarrow{\partial}^\mu D_\nu^{*j}(V_\mu^\dagger)^i_j \\ &+4{\rm i}\,f_{D^*D^*V}D_{i\mu}^{*\dagger}(\partial^\mu V^{\nu\dagger} - \partial^\nu V^{\mu\dagger})^i_j D_\nu^{*j} \\ &-{\rm i}\,g_{D^*DP}\left(D^{i\dagger}\partial^\mu P^\dagger_{ij}D_\mu^{*j} - D_\mu^{*i\dagger}\partial^\mu P^\dagger_{ij}D^j\right) \\ &+\frac{1}{2}g_{D^*D^*P}\epsilon_{\mu\nu\alpha\beta}D_i^{*\mu\dagger}\partial^\nu P^{ij\dagger}\overleftrightarrow{\partial}^\alpha D_j^{*\beta},\end{aligned} \tag{6}$$

where $D^{(*)} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$ and $D^{(*)\dagger} = (\bar{D}^{(*)0}, D^{(*)-}, D_s^{(*)-})$. The V and P are 3×3 matrices representing the light vector and pseudoscalar mesons respectively, and their specific forms are as follows:

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \tag{7a}$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\delta\eta+\gamma\eta'}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\delta\eta+\gamma\eta'}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\gamma\eta+\delta\eta' \end{pmatrix}. \tag{7b}$$

Here the physical states η and η' are superposition of $n\bar{n} = (u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$ flavor eigenstates. Their wave

functions are given by

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \delta & -\gamma \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} |n\bar{n}\rangle \\ |s\bar{s}\rangle \end{pmatrix}, \tag{8}$$

where $\delta = \cos(\theta_{\rm P} + \arctan\sqrt{2})$ and $\gamma = \sin(\theta_{\rm P} + \arctan\sqrt{2})$ with the η–η' mixing angle $\theta_{\rm P}$ varying from -24.6° to -11.5°.[4]

The coupling constants of the charmed mesons to the light vector and pseudoscalar mesons are determined as follows:[92]

$$g_{DDV} = \frac{\beta g_V}{\sqrt{2}}, \tag{9a}$$

$$f_{D^*DV} = \frac{\lambda g_V}{\sqrt{2}}, \tag{9b}$$

$$g_{D^*DP} = \frac{2g}{f_\pi}\sqrt{m_D m_{D^*}}. \tag{9c}$$

Here $g_V = m_\rho/f_\pi$ with the pion decay constant $f_\pi = 132\,\text{MeV}$[92] and the ρ meson mass $m_\rho = 775.26\,\text{MeV}$.[4] Following Ref. [94], $\beta = 0.9$, $\lambda = 0.56\,\text{GeV}^{-1}$, and $g = 0.59$.

2.2. Transition Amplitudes of $X_0 \to VV$ and $X_0 \to PP$. According to the effective Lagrangians above, the amplitudes $\mathcal{M}_V$ for the decay $X_0 \to VV$ through the charmed meson loops in Figs. 1(a) and 1(b) are given as

$$\begin{aligned}\mathcal{M}_V^a =& \int \frac{\mathrm{d}^4 q}{(2\pi)^4}[\chi_{\rm nr}^{0,c}\sqrt{m_{X_0}} m_D \varepsilon^{*\mu}(V_1)\varepsilon^{*\nu}(V_2)] \\ &\times [g_{DDV}(p_1+q)_\mu][g_{DDV}(q-p_2)_\nu] \\ &\times S(p_1,m_D)S(p_2,m_D)S(q,m_D)\mathcal{F}(q^2),\end{aligned} \tag{10}$$

$$\begin{aligned}\mathcal{M}_V^b =& \int \frac{\mathrm{d}^4 q}{(2\pi)^4}[\chi_{\rm nr}^{0,c}\sqrt{m_{X_0}} m_D \varepsilon^{*\mu}(V_1)\varepsilon^{*\nu}(V_2)] \\ &\times [2 f_{D^*DV}\epsilon_{\delta\mu\omega\xi} p_3^\delta (p_1+q)^\omega] \\ &\times [2 f_{D^*DV}\epsilon_{\lambda\nu\gamma\eta} p_4^\lambda (q-p_2)^\gamma] \\ &\times S(p_1,m_D)S(p_2,m_D)S^{\xi\eta}(q,m_{D^*})\mathcal{F}(q^2).\end{aligned} \tag{11}$$

The amplitude of the decay $X_0 \to PP$ shown in Fig. 1(c) is expressed as

$$\begin{aligned}\mathcal{M}_P^c =& \int \frac{\mathrm{d}^4 q}{(2\pi)^4}[\chi_{\rm nr}^{0,c}\sqrt{m_{X_0}} m_D][g_{D^*DP} p_{3\xi}][-g_{D^*DP} p_{4\eta}] \\ &\times S(p_1,m_D)S(p_2,m_D)S^{\xi\eta}(q,m_{D^*})\mathcal{F}(q^2).\end{aligned} \tag{12}$$

In Eqs. (10)–(12) the mass factor $\sqrt{m_{X_0}} m_D$ is responsible for the nonrelativistic normalization of the heavy fields at the $X_0 D\bar{D}$ interaction vertex. The symbols $\varepsilon^{*\mu}(V_1)$ and $\varepsilon^{*\nu}(V_2)$ are the polarization vectors of the final states V_1 and V_2, respectively. The $S(q,m_D)$ represents the propagator for the scalar D, while $S^{\mu\nu}(q,m_{D^*})$ stands for the propagator of the vector D^*, in the following forms:

$$S(q,m_D) = \frac{1}{q^2 - m_D^2 + \mathrm{i}\,\epsilon}, \tag{13}$$

$$S^{\mu\nu}(q,m_{D^*}) = \frac{-g^{\mu\nu} + q^\mu q^\nu/m_{D^*}^2}{q^2 - m_{D^*}^2 + \mathrm{i}\,\epsilon}. \tag{14}$$

To model the off-shell behavior of exchanged mesons and the internal structure of involved mesons, the form factor $\mathcal{F}(q^2)$ is introduced in Eqs. (10)–(12).[88,90–93] The mesons D and $\bar{D}$ interacting with X_0 can be considered as on-shell, since the mass of the X_0 is close to the $D\bar{D}$ threshold. However, the exchanged meson in the triangle loop is off-shell. For the light hadron decay processes studied in this work, we adopt a tripole form factor,

$$\mathcal{F}(q^2) = \left(\frac{m^2 - \Lambda^2}{q^2 - \Lambda^2}\right)^3, \tag{15}$$

where m and q are the mass and momentum of the exchanged meson, respectively. The cutoff Λ is reparameterized as $\Lambda = m + \alpha\Lambda_{\rm QCD}$ with $\Lambda_{\rm QCD} = 0.22\,\text{GeV}$.[27] In our test calculations, the partial decay widths obtained using monopole or dipole form factors are significantly large, even exceeding the full width of the X_0, which is 10 MeV in Ref. [13]. Therefore, we adopt the tripole form factor and set the range of α from 0.5 to 1.5 to obtain reasonable partial decay widths.

The decay width for the two-body processes $X_0 \to VV(PP)$ is expressed as

$$\Gamma = \frac{1}{\mathcal{S}}\frac{|\boldsymbol{p}|}{8\pi m_{X_0}^2}\sum_{\rm spins}\left|\mathcal{M}_{V(P)}\right|^2, \tag{16}$$

where $\mathcal{S}$ is the symmetry factor. This factor is typically assigned a value of 1 unless the final state comprises identical particles, for which $\mathcal{S}$ is set to be 2. The symbol $\sum_{\rm spin}$ means the summation over the spins of final states. $\boldsymbol{p}$ denotes the three momentum of final state.

3. Numerical Results and Discussions. In this section, we first analyze the influence of different α values on the partial decay widths. The parameter α is varied from 0.5 to 1.5. We choose three different phase angles $\theta = 0$, $\pi/4$, and $\pi/2$. The X_0 is a pure neutral state when the phase angle $\theta = 0$. The $\theta = \pi/4$ describes a configuration where the neutral and charged components in the X_0 are of equal proportion. In the case of $\theta = \pi/2$, the X_0 only comprises the charged charmed mesons.

Figure 2 shows the partial decay widths of the $X_0 \to VV$ as functions of the model parameter α. The decay of X_0 into the $K^{*0}\bar{K}^{*0}$ final state can occur only through the intermediate meson loop $[D^+D^-]D_s^{(*)+}$, so that the X_0 must contain charged component for this decay channel. Similarly, the decay into the $K^{*+}K^{*-}$ pair, involving the interaction of $[D^0\bar{D}^0]D_s^{(*)+}$, requires the X_0 to contain neutral component. This indicates that the $K^{*0}\bar{K}^{*0}$ decay channel is completely suppressed at the phase angle of $\theta = 0$ [Fig. 2(a)], while the $K^{*+}K^{*-}$ channel is absent at $\theta = \pi/2$ [Fig. 2(c)]. Moreover, in these two cases, the isospin-violating effect originating from the difference between the u and d quark masses vanishes. Thus, the partial decay width of the $X_0 \to \rho^0\omega$ nearly equals to that of the $X_0 \to \rho^+\rho^-$. Because the couplings of the X_0 to the neutral and charged components are different [see Eqs. (4) and (5)], the absolute widths for these two cases are different accordingly. At the phase angle of $\pi/4$, the neutral and charged components in the X_0 contribute equally. The decay width of the isospin-violating decay $X_0 \to \rho^0\omega$ exhibits a dramatic suppression.

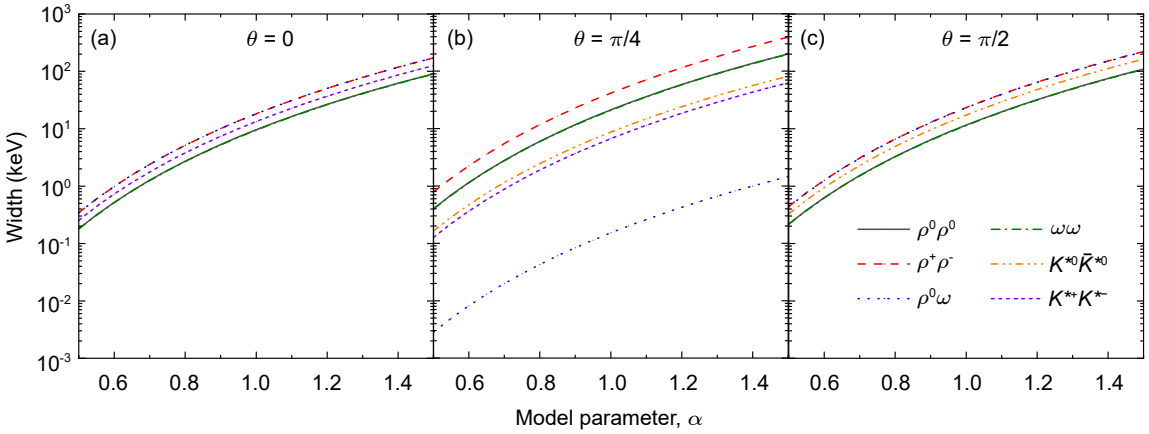

Fig. 2. The dependence of the $X_0 \to VV$ decay widths on the model parameter α at three different phase angles (a) $\theta = 0$, (b) $\pi/4$, and (c) $\pi/2$.

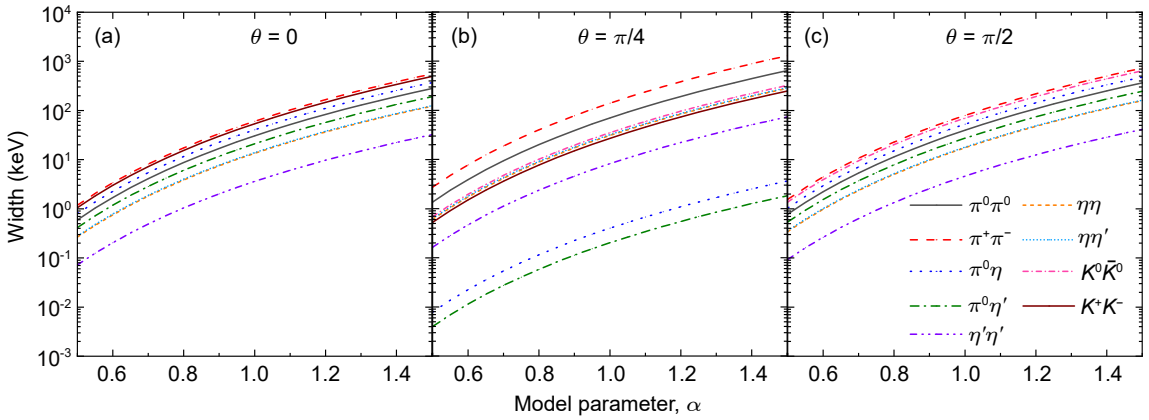

Fig. 3. The dependence of the $X_0 \to PP$ decay widths on the model parameter α at three different phase angles (a) $\theta = 0$, (b) $\pi/4$, and (c) $\pi/2$. The η–η' mixing angle $\theta_{\rm P} = -19.1^\circ$ is taken from Refs. [95,96].

Overall, the partial decay widths for the $X_0 \to VV$ increase as the model parameter α increases; the decays $X_0 \to \rho^0\omega$, $K^{*0}\bar{K}^{*0}$, and $K^{*+}K^{*-}$ exhibit strong dependence on the phase angle θ, while the other three decay channels $X_0 \to \rho^0\rho^0$, $\rho^+\rho^-$, and $\omega\omega$ are weakly phase-angle-dependent. For the later channels, the predicted partial decay widths within the α range from 0.5 to 1.5 are

$$\begin{aligned}\Gamma(X_0 \to \omega\omega) &\approx \Gamma(X_0 \to \rho^0\rho^0) \\ &\approx \frac{1}{2}\Gamma(X_0 \to \rho^+\rho^-) \\ &= (0.2 \sim 200)\ \text{keV}. \end{aligned} \tag{17}$$

In Fig. 3, we plot the partial decay widths of the $X_0 \to PP$ as a function of the model parameter α. As seen, the PP channels exhibit similar behaviors to those of the VV channels when the phase angle θ and model parameter α are varied. For the decay processes $X_0 \to \pi^0\pi^0$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, and $\pi^+\pi^-$, which are nearly independent of the phase angle, the decay widths are

$$\Gamma(X_0 \to \pi^0\pi^0) = (0.6 \sim 600)\ \text{keV}, \tag{18a}$$
$$\Gamma(X_0 \to \eta\eta) = (0.3 \sim 300)\ \text{keV}, \tag{18b}$$
$$\Gamma(X_0 \to \eta\eta') = (0.3 \sim 300)\ \text{keV}, \tag{18c}$$
$$\Gamma(X_0 \to \eta'\eta') = (0.07 \sim 80)\ \text{keV}, \tag{18d}$$
$$\Gamma(X_0 \to \pi^+\pi^-) = (1 \sim 1300)\ \text{keV}. \tag{18e}$$

Table 1. Decay widths (in units of keV) of the $X_0 \to VV$ and PP for different phase angles $\theta = 0$, $\pi/4$, and $\pi/2$. The η–η' mixing angle $\theta_{\rm P} = -19.1^\circ$ is taken from Refs. [95,96] and the model parameter α ranges from 0.5 to 1.5.

Final states	$\theta = 0$	$\theta = \pi/4$	$\theta = \pi/2$
$\rho^0\rho^0$	0.18–90.68	0.40–199.96	0.22–109.73
$\rho^+\rho^-$	0.34–172.08	0.78–392.39	0.44–221.90
$\rho^0\omega$	0.34–171.90	0–1.46	0.44–219.50
$\omega\omega$	0.18–89.67	0.39–197.68	0.22–108.46
$K^{*0}\bar{K}^{*0}$	···	0.16–81.87	0.33–163.75
$K^{*+}K^{*-}$	0.25–125.91	0.13–62.96	···
Total	1.29–650.24	1.86–936.32	1.65–823.34
$\pi^0\pi^0$	0.60–279.18	1.37–639.98	0.78–363.59
$\pi^+\pi^-$	1.19–556.13	2.73–1280.33	1.56–730.11
$\pi^0\eta$	0.79–369.57	0–3.64	1.03–480.51
$\pi^0\eta'$	0.42–190.70	0–1.82	0.54–247.06
$\eta'\eta'$	0.07–32.15	0.16–73.25	0.09–41.39
$\eta\eta$	0.26–122.18	0.61–279.59	0.34–158.60
$\eta\eta'$	0.28–125.76	0.63–287.19	0.36–162.61
$K^0\bar{K}^0$	···	0.69–320.72	1.38–641.43
K^+K^-	1.06–491.76	0.53–245.88	···
Total	4.67–2167.43	6.72–3132.40	6.08–2825.3

In Table 1, we list the partial decay widths for the concerned processes $X_0 \to VV$ and $X_0 \to PP$. In Ref. [97] a

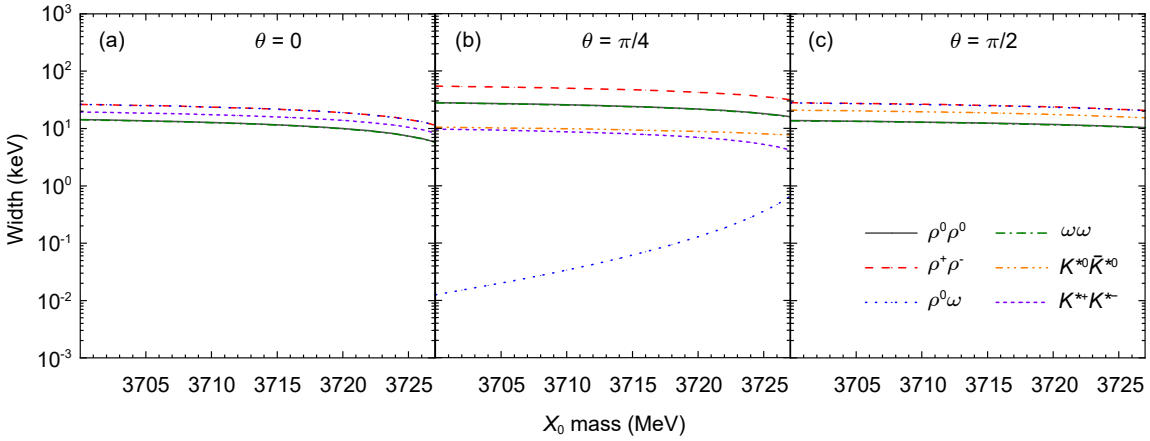

Fig. 4. The X_0 mass dependence of the decay processes $X_0 \to VV$ for different phase angles (a) $\theta = 0$, (b) $\pi/4$, and (c) $\pi/2$. The model parameter α is taken to be 1.0.

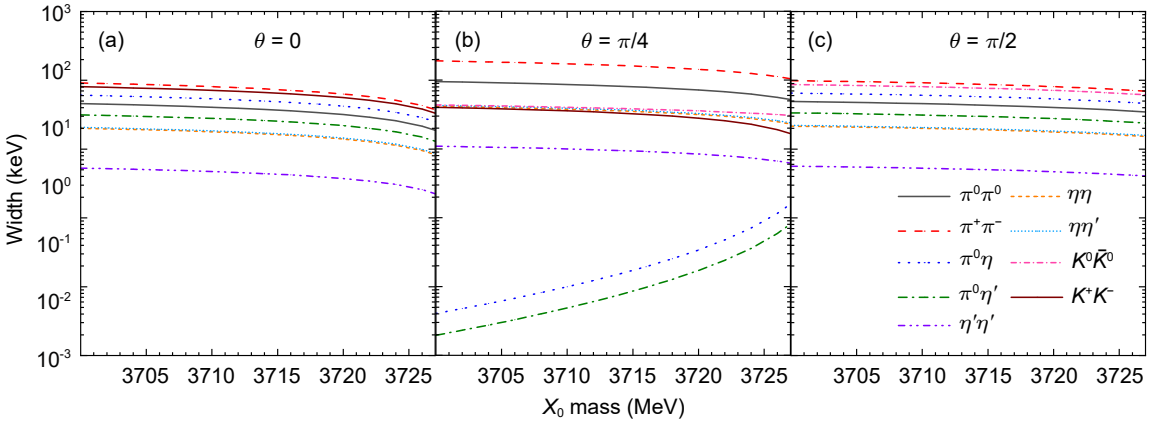

Fig. 5. The X_0 mass dependence of the decay processes $X_0 \to PP$ for different phase angles (a) $\theta = 0$, (b) $\pi/4$, and (c) $\pi/2$. The η–η' mixing angle $\theta_{\rm P} = -19.1^\circ$ is taken from Refs. [95,96] and the model parameter α is taken to be 1.0.

hidden charm resonance with mass 3.722 GeV and width 36 MeV was predicted in a unitarized coupled channel framework. This dynamically generated scalar resonance shows much stronger coupling to the $\eta'\eta'$ than to the $\pi\pi$, in contrast to our present results. The great difference might be useful for future experiments, such as BESⅢ and Belle Ⅱ, to distinguish the possible structures near 3.7 GeV.

In order to investigate the influence of the X_0 mass on the decay widths, we varied the X_0 mass from 3700 MeV to 3727 MeV.[13,18,67,79,98] It is noted that this mass range is below the $D\bar{D}$ thresholds ($m_{D^0} + m_{\bar{D}^0} = 3729.68$ MeV, $m_{D^+} + m_{D^-} = 3739.32$ MeV). The calculated results for the partial decay widths of the $X_0 \to VV$ and PP at $\alpha = 1.0$ are presented in Figs. 4 and 5, respectively. It is seen that except the isospin-breaking decays $X_0 \to \rho^0\omega$ and $X_0 \to \pi^0\eta(\eta')$, the widths of all the other decay processes exhibit slight decrease as the X_0 mass increases. This is because that the coupling strengths $\chi^0_{\rm nr}$ and $\chi^c_{\rm nr}$ governed by Eq. (3) decrease with increasing the X_0 mass. However, the difference between the $\chi^0_{\rm nr}$ and $\chi^c_{\rm nr}$ becomes larger as the mass of X_0 grows. Therefore, the widths of the isospin-breaking decays $X_0 \to \rho^0\omega$ and $X_0 \to \pi^0\eta(\eta')$ that are determined by the difference between neutral and charged meson loops increase clearly with increasing the X_0 mass.

From Figs. 2 and 3, it is seen that the α dependence of the widths is similar for all cases. This model dependence could be canceled for the width ratios. For the $X_0 \to VV$ decay process, we define the following ratios:

$$R_1 = \frac{\Gamma(X_0 \to \rho^0\omega)}{\Gamma(X_0 \to \omega\omega)}, \tag{19a}$$

$$R_2 = \frac{\Gamma(X_0 \to \rho\rho)}{\Gamma(X_0 \to \omega\omega)}, \tag{19b}$$

$$R_3 = \frac{\Gamma(X_0 \to K^{*+}K^{*-})}{\Gamma(X_0 \to \omega\omega)}, \tag{19c}$$

$$R_4 = \frac{\Gamma(X_0 \to K^{*0}\bar{K}^{*0})}{\Gamma(X_0 \to \omega\omega)}, \tag{19d}$$

with $\Gamma(X_0 \to \rho\rho) = \Gamma(X_0 \to \rho^0\rho^0) + \Gamma(X_0 \to \rho^+\rho^-)$.

Similarly, for $X_0 \to PP$, the $\pi^0\pi^0$ and $\pi^+\pi^-$ decay channels are classified as $\pi\pi$, and the relevant ratios are defined as follows:

$$r_1 = \frac{\Gamma(X_0 \to \pi^0\eta)}{\Gamma(X_0 \to \pi\pi)}, \tag{20a}$$

$$r_2 = \frac{\Gamma(X_0 \to \pi^0\eta')}{\Gamma(X_0 \to \pi\pi)}, \tag{20b}$$

$$r_3 = \frac{\Gamma(X_0 \to \eta\eta)}{\Gamma(X_0 \to \pi\pi)}, \tag{20c}$$

$$r_4 = \frac{\Gamma(X_0 \to \eta\eta')}{\Gamma(X_0 \to \pi\pi)}, \tag{20d}$$

$$r_5 = \frac{\Gamma(X_0 \to \eta'\eta')}{\Gamma(X_0 \to \pi\pi)}, \tag{20e}$$

$$r_6 = \frac{\Gamma(X_0 \to K^+K^-)}{\Gamma(X_0 \to \pi\pi)}, \tag{20f}$$

$$r_7 = \frac{\Gamma(X_0 \to K^0\bar{K}^0)}{\Gamma(X_0 \to \pi\pi)}. \tag{20g}$$

The dependence of the ratio R_1 on the model parameter α is shown in Fig. 6. It is evident that R_1 is rather insensitive to α. We have also investigated the dependence of the ratio R_1 on the model parameter α using monopole and dipole form factors, finding that R_1 remains nearly independent of the model parameter α. Moreover, the ratios are approximately equal to those obtained using the tripole form factor. The consistency of the ratio with respect to α suggests that the form factor effectively regulates the cutoff for each channel to a certain extent. However, the ratio exhibits discrepancies under different phase angles, which prompts us to continue discussing its phase-angle dependence.

Since the phase angle characterizes the molecular configuration of the X_0, we investigate the ratios R_i and r_i for different phase angles. In Fig. 7, we plot the ratios R_i defined in Eq. (19) and r_i in Eq. (20) as a function of the phase angle θ. The results were obtained using $\alpha = 1.0$. The trends in the ratios can be broadly categorized into four cases. The ratios R_2 and $r_{3,4,5}$ are nearly independent of the phase angle θ. The ratios R_3 and r_6 decrease with increasing θ, while the ratios R_4 and r_7 increase monotonically as the phase angle θ grows. Within the selected phase angle range, the ratios R_1 and $r_{1,2}$ exhibit a non-monotonic behavior characterized by an initial decrease followed by a subsequent increase. These ratios shown in Fig. 7 may be tested by future experimental measurements at BESⅢ and Belle Ⅱ and can be used to determine the value of the phase angle.

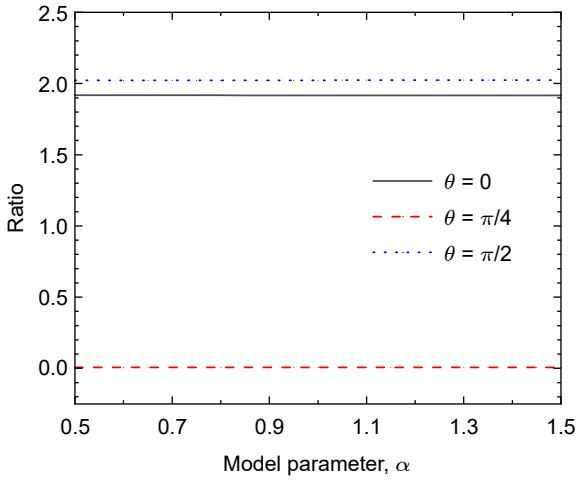

Fig. 6. The α dependence of the ratio R_1 defined in Eq. (19a).

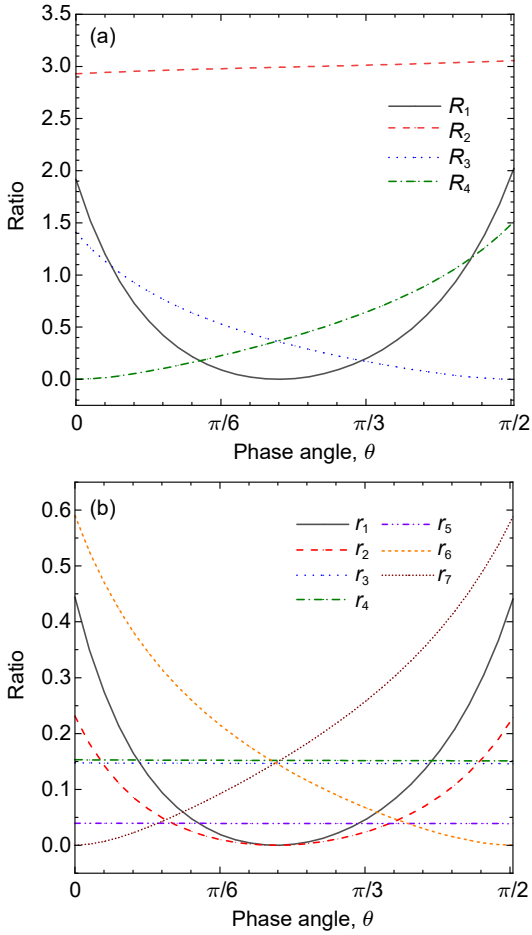

Fig. 7. Ratios (a) R_i defined in Eq. (19) and (b) r_i defined in Eq. (20) are presented as a function of the phase angle θ. The η–η' mixing angle $\theta_P = -19.1°$ is taken from Refs. [95,96], and the model parameter $\alpha = 1.0$.

4. Summary. In this work, we investigated the hadronic decays of the spin-0 partner of $X(3872)$ (called X_0) into two light hadrons. According to the assumption of the X_0 as a $D\bar{D}$ molecular state, we calculated the partial widths of the $X_0 \to VV$ and PP decay processes using an effective Lagrangian approach. We considered three cases: pure neutral state ($\theta = 0$), isospin singlet state ($\theta = \pi/4$), and pure charged state ($\theta = \pi/2$), where the phase angle θ determines the proportion of the neutral and charged components in the X_0. The decay widths depend on the model parameter α. However, the relative width ratios are found to be nearly model-α-independent. As the α increases from 0.5 to 1.5, the total width of the $X_0 \to VV$ is between a few keV and hundreds of keV, while the total width of the $X_0 \to PP$ varies from a few keV to several MeV.

Moreover, we also investigate the influence of the X_0 mass on the decay widths. Based on previous theoretical predictions, we vary the X_0 mass from 3700 MeV to 3727 MeV, which is bellow the $D\bar{D}$ threshold. It was found that the widths for the isospin-breaking decays $X_0 \to \rho^0\omega$ and $X_0 \to \pi^0\eta(\eta')$ increase clearly as the X_0 mass goes up, while the widths for the other decays decrease slightly. Finally, the relative width ratios between different channels

are studied to reflect the influence of the molecular configuration of the X_0. We hope that our calculated results are helpful for searching for the X_0 in the future experiments at BESⅢ and Belle Ⅱ.

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References

[1] Choi S K, Olsen S L, Abe K *et al.* 2003 *Phys. Rev. Lett.* **91** 262001
[2] Aaij R, Abellan Beteta C, Adeva B *et al.* 2013 *Phys. Rev. Lett.* **110** 222001
[3] Aaij R, Adeva B, Adinolfi M *et al.* 2015 *Phys. Rev. D* **92** 011102
[4] Navas S, Amsler C, Gutsche T *et al.* 2024 *Phys. Rev. D* **110** 030001
[5] Close F E and Page P R 2004 *Phys. Lett. B* **578** 119
[6] Swanson E S 2004 *Phys. Lett. B* **588** 189
[7] Wong C Y 2004 *Phys. Rev. C* **69** 055202
[8] Suzuki M 2005 *Phys. Rev. D* **72** 114013
[9] Ebert D, Faustov R N, and Galkin V O 2006 *Phys. Lett. B* **634** 214
[10] Swanson E S 2004 *Phys. Lett. B* **598** 197
[11] Fleming S, Kusunoki M, Mehen T, and van Kolck U 2007 *Phys. Rev. D* **76** 034006
[12] Bignamini C, Grinstein B, Piccinini F, Polosa A D, and Sabelli C 2009 *Phys. Rev. Lett.* **103** 162001
[13] Nieves J and Valderrama M P 2012 *Phys. Rev. D* **86** 056004
[14] AlFiky M T, Gabbiani F, and Petrov A A 2006 *Phys. Lett. B* **640** 238
[15] Hanhart C, Kalashnikova Y S, Kudryavtsev A E, and Nefediev A V 2007 *Phys. Rev. D* **76** 034007
[16] Gamermann D and Oset E 2009 *Phys. Rev. D* **80** 014003
[17] Braaten E and Lu M 2007 *Phys. Rev. D* **76** 094028
[18] Hidalgo-Duque C, Nieves J, and Valderrama M P 2013 *Phys. Rev. D* **87** 076006
[19] Dong Y B, Faessler A, Gutsche T, and Lyubovitskij V E 2008 *Phys. Rev. D* **77** 094013
[20] Braaten E and Lu M 2006 *Phys. Rev. D* **74** 054020
[21] Wang Z G and Huang T 2014 *Eur. Phys. J. C* **74** 2891
[22] Kang X W and Oller J A 2017 *Eur. Phys. J. C* **77** 399
[23] Fleming S and Mehen T 2008 *Phys. Rev. D* **78** 094019
[24] Guo F K, Hidalgo-Duque C, Nieves J, Ozpineci A, and Valderrama M P 2014 *Eur. Phys. J. C* **74** 2885
[25] Ding G J, Liu J F, and Yan M L 2009 *Phys. Rev. D* **79** 054005
[26] Lee I W, Faessler A, Gutsche T, and Lyubovitskij V E 2009 *Phys. Rev. D* **80** 094005
[27] Guo F K, Hanhart C, Kalashnikova Y S, Meißner U G, and Nefediev A V 2015 *Phys. Lett. B* **742** 394
[28] Braaten E and Kusunoki M 2004 *Phys. Rev. D* **69** 074005
[29] Guo F K, Hanhart C, Meißner U G, Wang Q, Zhao Q, and Zou B S 2018 *Rev. Mod. Phys.* **90** 015004
[30] Chen H X 2022 *Commun. Theor. Phys.* **74** 025201
[31] Shi P P, Huang F, and Wang W L 2021 *Phys. Rev. D* **103** 094038
[32] Wang Z G and Huang T 2014 *Phys. Rev. D* **89** 054019
[33] Maiani L, Piccinini F, Polosa A D, and Riquer V 2014 *Phys. Rev. D* **89** 114010
[34] Barnea N, Vijande J, and Valcarce A 2006 *Phys. Rev. D* **73** 054004
[35] Maiani L, Piccinini F, Polosa A D, and Riquer V 2005 *Phys. Rev. D* **71** 014028
[36] Wang Z G 2024 *Phys. Rev. D* **109** 014017
[37] Close F E and Godfrey S 2003 *Phys. Lett. B* **574** 210
[38] Li B A 2005 *Phys. Lett. B* **605** 306
[39] Chen W, Jin H Y, Kleiv R T, Steele T G, Wang M, and Xu Q 2013 *Phys. Rev. D* **88** 045027
[40] Takizawa M and Takeuchi S 2013 *PTEP* **2013** 093D01
[41] Barnes T and Godfrey S 2004 *Phys. Rev. D* **69** 054008
[42] Quigg C 2005 *Nucl. Phys. B Proc. Suppl.* **142** 87
[43] Achasov N N and Shestakov G N 2024 *Phys. Rev. D* **109** 036028
[44] Achasov N N and Shestakov G N 2024 *Phys. Rev. D* **110** 016023
[45] Eichten E J, Lane K, and Quigg C 2004 *Phys. Rev. D* **69** 094019
[46] Yan T M, Cheng H Y, Cheung C Y, Lin G L, Lin Y C, and Yu H L 1992 *Phys. Rev. D* **46** 1148
[47] Neubert M 1994 *Phys. Rept.* **245** 259
[48] Isgur N and Wise M B 1989 *Phys. Lett. B* **232** 113
[49] Baru V, Epelbaum E, Filin A A, Hanhart C, and Nefediev A V 2017 *EPJ Web Conf.* **137** 06002
[50] Albaladejo M, Guo F K, Hidalgo-Duque C, Nieves J, and Valderrama M P 2015 *Eur. Phys. J. C* **75** 547
[51] Shi P P, Dias J M, and Guo F K 2023 *Phys. Lett. B* **843** 137987
[52] Liu S D, Wang F, Jia Z S, Li G, Liu X H, and Xie J J 2024 *Phys. Rev. D* **110** 054048
[53] Ganbold G and Ivanov M A 2025 *Phys. Rev. D* **111** 014007
[54] Shi P P, Baru V, Guo F K, Hanhart C, and Nefediev A 2024 *Chin. Phys. Lett.* **41** 031301
[55] Montana G, Ramos A, Tolos L, and Torres-Rincon J M 2023 *Phys. Rev. D* **107** 054014
[56] Liu M Z, Wu T W, Pavon Valderrama M, Xie J J, and Geng L S 2019 *Phys. Rev. D* **99** 094018
[57] Guo F K, Hidalgo-Duque C, Nieves J, and Valderrama M P 2013 *Phys. Rev. D* **88** 054007
[58] Guo F K, Meißner U G, and Yang Z 2015 *Phys. Lett. B* **740** 42
[59] Baru V, Epelbaum E, Filin A A, Hanhart C, Meißner U G, and Nefediev A V 2016 *Phys. Lett. B* **763** 20
[60] Baru V, Hanhart C, and Nefediev A V 2017 *JHEP* **06** 010
[61] Baru V, Guo F K, Hanhart C, and Nefediev A 2024 *Phys. Rev. D* **109** L111501
[62] Wang Z G 2021 *Int. J. Mod. Phys. A* **36** 2150107
[63] Shi P P, Albaladejo M, Du M L, Guo F K, and Nieves J 2025 *Phys. Rev. D* **111** 074043
[64] Wang X L, Gao B S, Zhu W J *et al.* 2022 *Phys. Rev. D* **105** 112011
[65] Zheng Y, Cai Z, Li G, Liu S, Wu J, and Wu Q 2024 *Phys. Rev. D* **109** 014027
[66] Cai Z X, Jia Z S, Li G, Liu S D, and Xie J J 2024 *Eur. Phys. J. C* **84** 1019
[67] Gamermann D, Oset E, Strottman D, and Vicente Vacas M J 2007 *Phys. Rev. D* **76** 074016
[68] Pakhlov P, Adachi I, Aihara H *et al.* 2008 *Phys. Rev. Lett.* **100** 202001
[69] Chilikin K, Adachi I, Aihara H *et al.* 2017 *Phys. Rev. D* **95** 112003
[70] Aubert B, Karyotakis Y, Lees J P *et al.* 2010 *Phys. Rev. D* **81** 092003
[71] Gamermann D and Oset E 2008 *Eur. Phys. J. A* **36** 189
[72] Wang E, Liang W H, and Oset E 2021 *Eur. Phys. J. A* **57** 38
[73] Ding Z M, Jiang H Y, and He J 2020 *Eur. Phys. J. C* **80** 1179
[74] Xiao C W and Oset E 2013 *Eur. Phys. J. A* **49** 52
[75] Brandão P C S, Song J, Abreu L M, and Oset E 2023 *Phys. Rev. D* **108** 054004
[76] Dai L, Toledo G, and Oset E 2020 *Eur. Phys. J. C* **80** 510

[77] Prelovsek S, Collins S, Mohler D, Padmanath M, and Piemonte S 2021 *JHEP* **06** 035
[78] Wang E, Li H S, Liang W H, and Oset E 2021 *Phys. Rev. D* **103** 054008
[79] Deineka O, Danilkin I, and Vanderhaeghen M 2022 *Phys. Lett. B* **827** 136982
[80] Dai L R, Xie J J, and Oset E 2016 *Eur. Phys. J. C* **76** 121
[81] Li C and Yuan C Z 2019 *Phys. Rev. D* **100** 094003
[82] Ablikim M, Achasov M N, Adlarson P *et al.* 2024 *Phys. Rev. D* **109** L011102
[83] Wang Y, Wu Q, Li G, Qin W H, Liu X H, An C S, and Xie J J 2022 *Phys. Rev. D* **106** 074015
[84] Wu Q, Li G, Shao F, and Wang R 2016 *Phys. Rev. D* **94** 014015
[85] Wu Q, Zheng Y, Liu S, and Li G 2023 *Phys. Rev. D* **107** 034028
[86] Weinberg S 1965 *Phys. Rev.* **137** B672
[87] Baru V, Haidenbauer J, Hanhart C, Kalashnikova Y, and Kudryavtsev A E 2004 *Phys. Lett. B* **586** 53
[88] Guo F K, Hanhart C, Meißner U G, Wang Q, and Zhao Q 2013 *Phys. Lett. B* **725** 127
[89] Wei X, Shen Q H, and Xie J J 2022 *Eur. Phys. J. C* **82** 718
[90] Burdman G and Donoghue J F 1992 *Phys. Lett. B* **280** 287
[91] Wise M B 1992 *Phys. Rev. D* **45** R2188
[92] Casalbuoni R, Deandrea A, Di Bartolomeo N, Gatto R, Feruglio F, and Nardulli G 1997 *Phys. Rept.* **281** 145
[93] Cheng H Y, Chua C K, and Soni A 2005 *Phys. Rev. D* **71** 014030
[94] Isola C, Ladisa M, Nardulli G, and Santorelli P 2003 *Phys. Rev. D* **68** 114001
[95] Coffman D, Dubois G P, Eigen G *et al.* 1988 *Phys. Rev. D* **38** 2695
[96] Jousset J, Ajaltouni Z, Falvard A *et al.* 1990 *Phys. Rev. D* **41** 1389
[97] Gamermann D, Oset E, and Zou B S 2009 *Eur. Phys. J. A* **41** 85
[98] Albaladejo M, Hidalgo-Duque C, Nieves J, and Oset E 2013 *Phys. Rev. D* **88** 014510