An Off-Resonant Quantum Collision Model: Realizing a Qubit Coupled with an Effective Negative-Temperature Bath

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The quantum collision model provides a promising tool for investigating system-bath dynamics. Most studies of quantum collision models, however, are in the resonant regime. In quantum dynamics, the off-resonant interactions often result in exciting effects. Therefore, it is important to investigate the quantum collision models in the off-resonant regime. A bath with a negative absolute temperature is expected to be instrumental in the development of thermal devices. The design of an effective bath with negative absolute temperature coupled to a qubit is a significant component for developing such thermal devices. In this study, we establish an effective negative-absolute-temperature bath coupled to a qubit with a quantum collision model in the far-off-resonant regime. We conduct a detailed systematic investigation of the off-resonant collision model and determine the presence of an additional constraint on the collision duration resulting from the far-off resonant collision. The dynamics of the collision model in the far-off-resonant regime are different from those beyond the far-off-resonant regime. Numerical simulations are used to confirm the validity of the proposed approach.

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The quantum collision model[1] is a powerful and effective tool for investigating the dynamics of the system-bath scheme.[2–15] In the quantum collision model, the bath is represented by an extensive collection of subunits (ancillas). The open system is coupled to the bath by the short system-ancilla collisions, that is, short unitary interactions. The collisions are performed using a pairwise sequence. In most quantum collision models, the system resonantly interacts with the ancillas. The off-resonant interactions between quantum objects often surface new, exciting effects[16] compared to resonant interactions. For example, a two-level system driven far off-resonance by an external weak laser exhibits slight shifts in its levels, known as the *A. C. Stark shift*. An atom with an appropriate structure interacting with light can induce a large nonlinear effect when the light drives the specific level transition off-resonance.[17,18] In quantum technology, a widely used approach to couple the levels of interest is to introduce the intermediate auxiliary levels working in the off-resonant regime, as shown in the examples in Refs. [19–22]. These effects illustrate that it is reasonable to expect that the quantum collision model in the off-resonant regime, i.e., the system colliding off-resonantly with the ancillas, may bring in fresh effects.

In recent years, negative absolute temperatures have garnered significant attention. Onsager originally conceived the physical idea of negative absolute temperature in the statistical investigation of point vortices.[23] Subsequently, Purcell and Pound observed the negative absolute temperatures in the nuclear spin systems using an LiF crystal.[24] Ramsey theoretically investigated the thermodynamics and statistical mechanical implications of such negative absolute temperatures.[25] Although the equilibrium of negative absolute temperature presents uncertainty relating to the definition of entropy,[26,27] negative absolute temperatures are now widely accepted in the scientific community and are consistent with experimental observations.[28–36] A negative absolute-temperature bath is expected to develop thermal devices,[37–47] in which the devices involved in the negative absolute temperature bath would perform better than their traditional counterparts based on positive temperatures. In this context, most proposals assume a negative temperature bath to be already present or be realized through external driving or work. In 2024, the authors of Ref. [48] proposed an approach to develop a synthetic negative-temperature bath coupled with a three-level atom. This proposal has drawn attention to ways in which a negative temperature bath coupled to a qubit, a unit of quantum information widely used in constructing quantum thermal devices, could be realized.

In this Letter, we propose the realization of an effective negative temperature bath coupled to a qubit in the context of a quantum-collision model in the off-resonant regime. The qubit is realized by adiabatically eliminating the highest level of a Λ-type qutrit when ancillas of the effective bath far off-resonance collide with the qutrit. Under certain conditions, based on the dynamics of the proposed collision scheme, a Gorini-Kossakowski-Sudarshan-Lindblad master equation representing the dynamics of a qubit coupled to an effective bath with adjustable temperature was determined by performing appropriate approximations. The validity of the master equation was

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confirmed through numerical simulations, which were necessary to ensure the rigor of this study because we did not find any precedents for handling quantum collision models using the proposed approach. We determined that an additional constraint on the collision duration is impressed since the lower bound of the coarse-graining time stems from rapid oscillation dynamics for far-off-resonant collisions. Beyond the far-off-resonant-collision regime, the effective qubit-bath coupling cannot be realized because it is difficult to neglect the dynamics at the highest level of the qutrit.

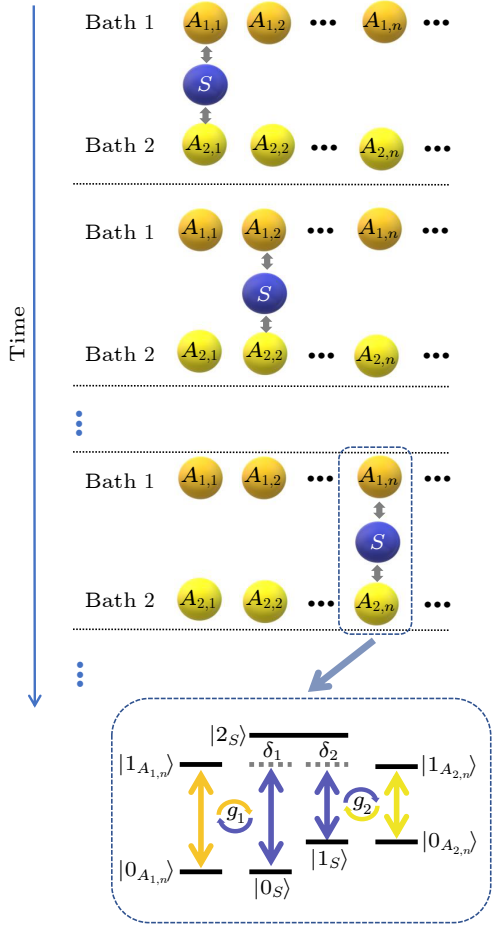

Fig. 1. Collision model: the system S composed of a three-level qutrit undergoes sequence successive collisions with the ancilla qubits in bath 1 and bath 2.

The collision model under consideration is illustrated in Fig. 1. The open system S consists of a Λ-type qutrit. The three levels of S are labeled as $|0_S\rangle$, $|1_S\rangle$, and $|2_S\rangle$, with corresponding level frequencies denoted by $\omega_{S,0}$, $\omega_{S,1}$, and $\omega_{S,2}$, respectively. There are two baths, both of which contain many identical ancillas, that is, noninteracting qubits coupled to the open system through a sequence of system-ancilla collisions. The n-th ($n = 1, 2, 3...$) ancilla of bath m ($m = 1, 2$) is labeled by $A_{m,n}$. The upper and lower levels of $A_{m,n}$ are denoted by $|1_{A_{m,n}}\rangle$ and $|0_{A_{m,n}}\rangle$, respectively. The frequency corresponding to the upper (lower) level of bath m's identical qubits is labeled as $\omega_{A_{m,1}}$ ($\omega_{A_{m,0}}$). Initially, the qubits in bath 1 and 2 are in the Gibbs thermal state with inverse temperatures β_1 and β_2, respectively. We focus on the collisional memoryless model. Each ancilla collides with the system S only once. The n-th collision step is realized by the short interaction of S with $A_{1,n}$ and $A_{2,n}$. The system level transitions $|0_S\rangle \leftrightarrow |2_S\rangle$ and $|1_S\rangle \leftrightarrow |2_S\rangle$ are coupled to the ancilla level transitions $|1_{A_{1,n}}\rangle \leftrightarrow |0_{A_{1,n}}\rangle$ and $|1_{A_{2,n}}\rangle \leftrightarrow |0_{A_{2,n}}\rangle$, respectively. In the interaction picture, the Hamiltonian representing the n-th collision step is given by

$$H_n = g_1\sigma^{10}_{A_{1,n}}\sigma^{02}_S e^{-i\delta_1 t} + g_2\sigma^{10}_{A_{2,n}}\sigma^{12}_S e^{-i\delta_2 t} + \text{h.c.}, \quad (1)$$

where $\sigma^{kk'}_M = |k_M\rangle\langle k'_M|$ is M's level transition and population operators with $M \in \{A_{1,n}, A_{2,n}, S\}$. The interaction strength between $A_{m,n}$ and the system is denoted by g_m, which remains unchanged with the collision step number n. The detunings δ_1 and δ_2 are represented by $\delta_1 = \omega_{S,20} - \omega_{A_1}$ and $\delta_2 = \omega_{S,21} - \omega_{A_2}$, respectively, as shown in Fig. 1. Here, $\omega_{S,kk'} = \omega_{S,k} - \omega_{S,k'}$ denotes the system level transition frequency between the levels $|k_S\rangle$ and $|k'_S\rangle$, and $\omega_{A_m} = \omega_{A_{m,1}} - \omega_{A_{m,0}}$ is the level transition frequency of the qubit in bath m. We have set the reduced Plank constant $\hbar$ as a unit, i.e., $\hbar = 1$. The first (second) term in the Hamiltonian (1) represents the off-resonant interaction between $A_{1,n}$ ($A_{2,n}$) and S when δ_1 (δ_2) is nonzero. In the far-off-resonant case, i.e., the large-detuning case represented as $\delta_m \gg g_m$, the standard adiabatic elimination can be performed[16] to obtain the effective Hamiltonian as

$$\begin{aligned} H_{\text{eff},n} = & -\alpha_{11}\sigma^{11}_{A_{1,n}}\sigma^{00}_S - \alpha_{22}\sigma^{11}_{A_{2,n}}\sigma^{11}_S \\ & - \alpha_{12}\sigma^{01}_{A_{1,n}}\sigma^{10}_{A_{2,n}}\sigma^{10}_S e^{i(\delta_1-\delta_2)t} + \text{h.c.}, \end{aligned} \quad (2)$$

where $\alpha_{mn} = \frac{g_m g_n}{2}(\frac{1}{\delta_m} + \frac{1}{\delta_n})$. The vital condition $|\delta_1 - \delta_2| \ll \alpha_{12}$ should be satisfied so that the dynamics governed by the second line of the effective Hamiltonian (2) play a significant role and are not ignored. For readability, the process of obtaining the effective Hamiltonian is presented in the Supplemental Material. The effective Hamiltonian eliminates the highest level $|2_S\rangle$ of the system, which is referred to as a virtual level in the subsequent analysis. The terms in the first line of the effective Hamiltonian result in slight shifts in the corresponding levels. The second line can be understood from the fact that S jumps between the levels $|0_S\rangle$ and $|1_S\rangle$ via the virtual level; meanwhile, $A_{1,n}$ and $A_{2,n}$ accomplish their corresponding level transitions. It follows that the system S plays the role of an effective qubit with the levels $|0_S\rangle$ and $|1_S\rangle$. The two ancillas cooperatively drive the common effective-qubit-level transition in each collision step. For simplicity, we consider the condition of $\delta_1 = \delta_2 = \delta$, which implies that the energy absorbed (or released) by the effective-qubit-level transition equals the energy released (or absorbed) by the cooperative ancilla-level transitions.

The legitimacy of the adiabatic elimination on the level $|2_S\rangle$ is confirmed in Fig. 2, which shows a numerical simulation of the dynamics governed by the original Hamiltonian (1) and effective Hamiltonian (2) for the far-off-resonant case. In Fig. 2(a), the system-level-population

evolutions governed by the original Hamiltonian agree well with the evolutions governed by the effective Hamiltonian. The population of the level $|2_S\rangle$ governed by the original Hamiltonian is always near zero. As shown in Fig. 2(b), the evolutions of the populations governed by the original Hamiltonian exhibit an extremely small but rapid oscillations with respect to time. A critical approximation used in adiabatic elimination is that the effective Hamiltonian has considered the coarse-grained (or time-averaged) dynamics of the original Hamiltonian. Slight oscillations are not observed in Fig. 2(a) because the evolution time is much longer than the coarse-graining time, and the maximum values of the populations are much larger than the oscillation amplitudes.

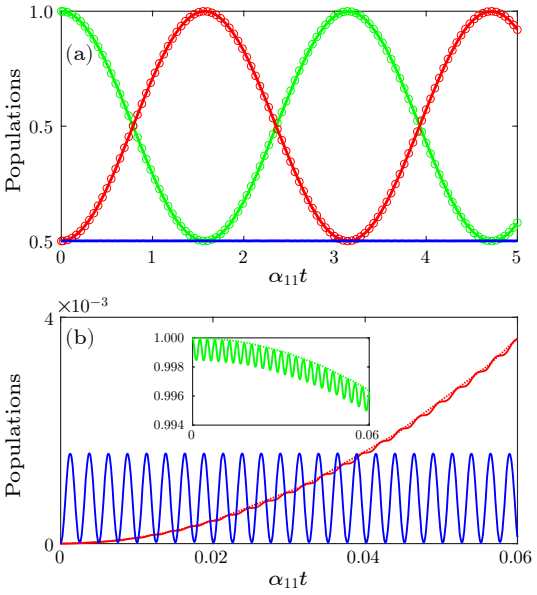

Fig. 2. Numerical simulation of the populations with respect to time governed by the Hamiltonian (1) and (2). The initial state is set as $|1_{A_{1,n}}, 0_{A_{2,n}}, 0_S\rangle$. (a) The evolutions of the populations from $\alpha_{11}t = 0$ to $\alpha_{11}t = 5$, and (b) the evolutions from $\alpha_{11}t = 0$ to $\alpha_{11}t = 0.06$. In (b), to observe the slight oscillations of evolutions governed by the Hamiltonian (1), the populations of the state $|0_S\rangle$ are represented in the insert panel because the difference between the population of $|0_S\rangle$ and the population of $|1_S\rangle$ is much larger than the amplitudes of the oscillations. The original-Hamiltonian-governing populations of $|0_S\rangle$, $|1_S\rangle$, and $|2_S\rangle$ are represented by green, red, and blue solid lines, respectively. In (a), the effective-Hamiltonian-governing populations of $|0_S\rangle$ and $|1_S\rangle$ are represented by green and red hollow circles, respectively. In (b), the effective-Hamiltonian-governing populations of $|0_S\rangle$ and $|1_S\rangle$ are represented by green and red dotted lines, respectively. The parameters are $g_2 = g_1$ and $\delta = 50g_1$.

It is convenient to introduce a rotating frame with respect to the term $-\alpha_{11}\sigma^{11}_{A_{1,n}}\sigma^{00}_S - \alpha_{22}\sigma^{11}_{A_{2,n}}\sigma^{11}_S$. In this rotating frame, the effective Hamiltonian is expressed as

$$V_n = -\alpha\sigma^{01}_{A_{1,n}}\sigma^{10}_{A_{2,n}}\sigma^{10}_S + \text{h.c.}, \tag{3}$$

where $g_1 = g_2 = g$ and $\alpha_{11} = \alpha_{22} = \alpha_{12} = \alpha$. It should be pointed out that although we considered the special case under the condition of $g_1 = g_2$ and $\delta_1 = \delta_2$ for simplicity, the main results will not significantly change when these conditions are violated, as shown in the discussions in the Supplemental Material.

The collision durations in all steps are assumed to be equal and are denoted by τ. The unitary evolution operator $U_n = e^{-iV_n\tau}$ represents the evolution of the n-th collision step.

Before the n-th collision step, the system and n-th qubits of the two baths are considered to be in the state $\Theta_{n-1} = \rho_{n-1} \otimes \eta_{A_{1,n}} \otimes \eta_{A_{2,n}}$, where the density operators ρ_{n-1}, $\eta_{A_{1,n}}$, and $\eta_{A_{2,n}}$ represent the states of S, $A_{1,n}$, and $A_{2,n}$, respectively. When the collision duration τ is considerably smaller than the evolution timescale, we can consider the continuous-time limit; that is, $\frac{\Delta\rho_n}{\tau} \to \frac{d\rho}{dt}$. Then, by performing an approximation up to the second-order of τ (as shown in the Supplemental Material), a continuous-time master equation is obtained as

$$\begin{aligned}\frac{d\rho}{dt} = {} & \Gamma e^{\omega_{S,10}\beta_S}\left(\sigma^{01}_S\rho\sigma^{10}_S - \frac{1}{2}[\rho, \sigma^{11}_S]_+\right) \\ & + \Gamma\left(\sigma^{10}_S\rho\sigma^{01}_S - \frac{1}{2}[\rho, \sigma^{00}_S]_+\right),\end{aligned} \tag{4}$$

in which $\beta_s = \frac{\omega_{A_1}\beta_1 - \omega_{A_2}\beta_2}{\omega_{A_1} - \omega_{A_2}}$ and $\Gamma = \frac{R^2 g^2 \tau}{(1+e^{\omega_{A_1}\beta_1})(1+e^{-\omega_{A_2}\beta_2})}$, where R is the ratio of coupling strength to the detuning, i.e., $R = \frac{g}{\delta}$. The anticommutator satisfies $[A, B]_+ = AB + BA$. The steady-state solution of Eq. (4) can be expressed as

$$\rho_{ss} = \frac{\sigma^{11}_S}{1 + e^{\omega_{S,10}\beta_S}} + \frac{e^{\omega_{S,10}\beta_S}\sigma^{00}_S}{1 + e^{\omega_{S,10}\beta_S}}, \tag{5}$$

indicating that the effective inverse temperature of the effective qubit is β_s. These results represent the coupling between an effective bath synthesized by integrating bath 1 and bath 2 and the effective qubit. The inverse temperature β_s of the effective bath can be tuned using the initial Gibbs states of the ancillas. We now discuss the case where $\omega_{A_1} - \omega_{A_2} = \omega_{S,10} > 0$, i.e., level $|1_S\rangle$ is higher than $|0_S\rangle$. When β_1 decreases and β_2 increases, β_s decreases. This can be understood because a larger population of state $|1_S\rangle$ is associated with a higher temperature of the effective qubit. To stimulate the system to remain in $|1_S\rangle$, we can lower the temperature of bath 2 and increase the temperature of bath 1. When $\frac{\omega_{A_1}}{\omega_{A_2}} > \frac{\beta_2}{\beta_1}$, the the steady-state population of $|1_S\rangle$ is smaller than that of $|0_S\rangle$, corresponding to a positive effective inverse temperature β_s. When $\frac{\omega_{A_1}}{\omega_{A_2}} < \frac{\beta_2}{\beta_1}$, the steady-state population of $|1_S\rangle$ is greater than that of $|0_S\rangle$, which corresponds to a negative effective inverse temperature β_s. This result implies an approach for realizing an effective negative temperature coupled to a qubit based on conventional thermal baths. For example, a qutrit can be coupled to two conventional thermal baths, where the frequencies of the baths are filtered so that they are significantly different from the corresponding level transition frequencies of the qutrit. In practice, the effective negative temperature of the two-level system in the heating stroke in a quantum Otto heat engine can be simulated by adjusting the population of the excited states using radio frequency and gradient fields.[44] The steady-state effective negative temperature of the qubit via the

synthesized bath exhibits the advantage of far fewer stringent operational requirements in the temporal dimension. It is therefore expected to significantly reduce errors stemming from operations in the temporal dimension.

The short but finite collision duration is a coarse-graining time, and the collision step is a coarse-grained procedure. We recall that the rapid slight oscillation dynamics governed by the Hamiltonian (1) have been coarse-grained using the effective Hamiltonian (2) under a large-detuning condition, which naturally imposes a constraint on the collision duration. The constraint states that the collision duration should be significantly longer than the time scale of the rapid-slight-oscillation period. We proceed to demonstrate the validity of the continuous-time master equation (4) numerically under this constraint.

We return to the original Hamiltonian (1) to numerically simulate the accurate dynamics of the collided system. For convenience, we introduce a new rotating frame in which the Hamiltonian is time-independent. In the rotating frame with respect to $-\delta\sigma_S^{22}$, the time-independent Hamiltonian is expressed as

$$H_n' = \delta\sigma_S^{22} + g\sigma_{A_{1,n}}^{10}\sigma_S^{02} + g\sigma_{A_{2,n}}^{10}\sigma_S^{12} + \text{h.c.} \tag{6}$$

The subscript $'$ in the operator O' implies that the operator O is represented in the new rotating frame. After the n-th collision step, the reduced-density operator for the system is given by

$$\rho_n' = \mathrm{Tr}_{A_1,A_2} \sum_{m,m'} e^{-i(\epsilon_m - \epsilon_{m'})\tau} \langle m|\Theta_{n-1}'|m'\rangle |m\rangle\langle m'|, \tag{7}$$

where $|m\rangle$ and $|m'\rangle$ are the eigenvectors of the time-independent Hamiltonian H_n', where ϵ_m and $\epsilon_{m'}$ are the corresponding eigenvalues. Alternatively, ρ_n' can also be determined using the following equation:

$$\rho_n' = \mathrm{Tr}_{A_1,A_2}\Theta_n'(\tau), \tag{8}$$

where $\Theta_n'(\tau)$ is the solution to the Liouville equation $\dot{\Theta}'(t) = -i[H_n', \Theta'(t)]$ at time $t = \tau$ under the initial condition $\Theta'(0) = \Theta_{n-1}'$. Subsequently, a numerical simulation of the system state after each collision step can be obtained using numerical iterations expressed in Eq. (7) or (8). The numerical simulation would fit the accurate dynamics of the system with high precision because no approximation has been performed to obtain the analytical form of Eqs. (7) and (8).

The numerical simulations presented in Fig. 3 compare the accurate dynamics of the collided system with the dynamics of the continuous-time master equation (4) for different collision durations. The accurate dynamics of the system are numerically simulated using Eq. (8), in which the numerical solution of the Liouville equation is derived by the 4-order Runger–Kutta method. Next, we discuss two scenarios.

In *situation (a)*, the collision duration is too small to satisfy the constraint. The evolution described in Eq. (4) significantly differs from the accurate evolution of the system, as confirmed in Fig. 3(a) when $\alpha\tau = 0.01$. In the accurate evolution, the collided system can jump to the level $|2_S\rangle$ with a nonnegligible probability. This can be understood from the fact that, although the coarse-grained dynamics of the level $|2_S\rangle$ is negligible in the large detuning case, the level transition between $|0_S\rangle$ and $|2_S\rangle$ plays the predominant role when the evolution duration is in (or smaller than) the time scale of the rapid-slight-oscillation period, as shown in Fig. 2(b). In this scenario, the coarse-graining on the dynamics does not work well, and the level $|2_S\rangle$ is not negligible. The qutrit cannot be considered an effective qubit, thus making the continuous-time master equation (4) invalid.

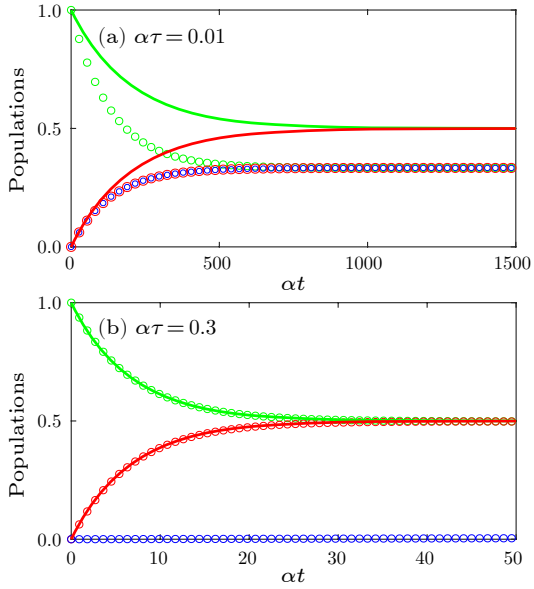

Fig. 3. Numerical simulations of the collided-system-level-population evolutions governed by the original Hamiltonian (1) and the master equation (4). The initial state of the system is $|0_S\rangle$, and the parameters are $\Delta = 200g, \omega_{A_1}\beta_1 = \omega_{A_2}\beta_2 = 10^{-4}$. We take $\alpha\tau = 0.01$ in (a) and $\alpha\tau = 0.3$ in (b). The original-Hamiltonian-governing-collided populations of levels $|0_S\rangle$, $|1_S\rangle$, and $|2_S\rangle$ are plotted using green, red, and blue hollow circles, respectively. The master-equation-governing populations of levels $|0_S\rangle$ and $|1_S\rangle$ are plotted using green and red solid lines, respectively. The blue and red circles show different sizes in (a) because of the overlapping.

In *situation (b)*, the collision duration is significantly longer than the time scale of the rapid-slight-oscillation period and much shorter than the evolution time scale. As shown in Fig. 3(b), the population evolution governed by Eq. (4) agrees reasonably well with the accurate evolution of the system. Intuitively, a high bath temperature is associated with an increasing trend of the system jumping to the high energy level. Therefore, both in Figs. 3(a) and 3(b), the temperatures of the two baths are set near positive infinity. In Fig. 3(b), it is difficult to obtain a noticeable population of level $|2_S\rangle$. The probabilities of the two levels of the effective qubit are equal, implying that the effective temperature is near positive infinity. The continuous-time master equation (4) provides a good representation of the system dynamics.

A continuous-time master equation that is different from the master equation (4) can be obtained based on

the dynamics governed by the Hamiltonian (6). According to the unitary evolution operator $U_n' = e^{-iH_n'\tau}$, the approximation up to the second-order of τ can be made to express the continuous-time master equation as

$$\begin{aligned}\frac{d\rho'}{dt} &= i\delta[\rho', \sigma_S^{22}] + \tau\delta^2\left(\sigma_S^{22}\rho'\sigma_S^{22} - \frac{1}{2}[\rho', \sigma_S^{22}]_+\right) \\ &+ \gamma_1 e^{\omega_{A_1}\beta_1}\left(\sigma_S^{02}\rho'\sigma_S^{20} - \frac{1}{2}[\rho', \sigma_S^{22}]_+\right) \\ &+ \gamma_1\left(\sigma_S^{20}\rho'\sigma_S^{02} - \frac{1}{2}[\rho', \sigma_S^{00}]_+\right) \\ &+ \gamma_2 e^{\omega_{A_2}\beta_2}\left(\sigma_S^{12}\rho'\sigma_S^{21} - \frac{1}{2}[\rho', \sigma_S^{22}]_+\right) \\ &+ \gamma_2\left(\sigma_S^{21}\rho'\sigma_S^{12} - \frac{1}{2}[\rho', \sigma_S^{11}]_+\right), \end{aligned} \tag{9}$$

where $\gamma_m = \frac{g^2\tau}{1+e^{\omega_{A_m}\beta_m}}$. The master equation indicates that the qutrit is coupled to bath 1 and 2; that is, the level transition $|0_S\rangle \leftrightarrow |2_S\rangle$ is coupled to bath 1, and the transition $|1_S\rangle \leftrightarrow |2_S\rangle$ is coupled to bath 2. When $\delta = 0$, the case represented in Ref. [48] is valid.

Although continuous-time master equations (4) and (9) represent different dynamics, they do not conflict because they operate under different conditions. When the detuning is small, Eq. (9) provides a good presentation of the dynamics of the system. However, when the detuning is large, Eq. (9) does not provide a good representation. This can be understood based on the approximations up to the second order of τ. The approximation $U_n' \sim I - iH_n'\tau - \frac{(H_n'\tau)^2}{2}$ requires the high-order terms on $\delta\tau$ and $g\tau$ (in H_n') to be negligible. When the detuning is small, the high-order terms can be safely ignored. However, a large detuning gives rise to the rapid oscillation and the coarse-graining collision duration τ should be significantly larger than the oscillation period. This restriction on the lower bound of τ results in the illegality of the elimination of the high-order term of $\delta\tau$. The master equation (4), which originates from the effective Hamiltonian (2), works well under the case of large detunings because an effective Hamilton is obtained under the large-detuning case. Evidently, Eq. (4) would not perform well when the detuning is small.

In conclusion, we proposed a quantum collision model in which an open system composed of a qutrit collides off-resonance with the ancillas of the two baths. The qutrit interacts with two ancillas from the two baths during each collision step. In the far-off-resonant case, according to the legal adiabatic elimination at the highest level of the qutrit, we show that qutrit-ancilla interactions are effectively considered two ancillas commonly driving the level transition of a qubit. A master equation, which implies that our scheme is considered as a qubit coupled to an effective bath, was derived in the continuous-time limit via coarse-graining. The master equation shows that the temperature of the effective bath can be an arbitrary negative real number. The validity of the approach is confirmed by numerically comparing the accurate dynamics of the system to the qubit dynamics represented by the master equation. Because the far-off-resonant interaction results in rapid oscillation dynamics, the collision duration should be significantly longer than the rapid oscillation period to ensure that each collision process is coarse-grained. This differs from the quantum collision model working in a resonant regime because there are no rapid oscillation dynamics. Beyond the far-off-resonant case, the system dynamics can be represented by a master equation, thereby implying a qutrit coupled to two independent baths, where the highest level of the qutrit cannot be legally adiabatically eliminated, and the qutrit is not regarded as an effective qubit. The work outlines an approach for coupling a qubit with an effective negative temperature based on the quantum collision model. This study systematically investigated the quantum collision model in the off-resonant regime. We expect the proposed scheme to provide meaningful contributions to the study of quantum thermal devices.

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