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Bound and Resonating Multiquark Configurations

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We review the predictions of quark models for multiquark configurations that are bound or resonant states, and compare different methods for estimating the properties of resonances.

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1. History. Exotic hadrons are almost as old as ordinary hadrons, with, however, a regrettable tendency to disappear after some years, or to give rise to endless debates about their interpretation.

In the 1950s and 1960s, there were claims about Z baryons with strangeness $S = +1$, in contrast with the negative strangeness of the recently discovered hyperons. The claims for Z baryons were based on somewhat risky amplitude-analyses of K^+-nucleon scattering, but the data were not sufficient, particularly for the spin-dependent observables, to provide with unambiguous solutions. The Z baryon was removed from the particle data tables. It reappeared later under the name of $\Theta(1540)^+$ thanks to an experiment in Japan, but again, the Θ^+ was not confirmed by experiments using better detectors and statistics. See, e.g., the review in Ref. [1].

The richest and most debated sector is that of scalar light mesons. The simplest constituent models describe them as the $^{2\mathsf{S}+1}L_J = {}^3P_0$ states of $q\bar{q}$, where $q = u, d$ and S is the total quark spin, suggesting a somewhat higher mass, of the order of 1 GeV or more. On the other hand, the meson-exchange model of the nucleon-nucleon interaction requires a scalar meson of mass about 0.5 GeV, to explain the attraction that binds the nuclei. There is, indeed, a strong correlation of two pions when produced in a state of spin and isospin $J = I = 0$. The situation is even more intricate in the mass region around 1.5 GeV–2 GeV, with a rich interplay of $q\bar{q}$ and $s\bar{s}$ (s is the strange quark), as well as multiquarks, glueballs and hybrids. See, e.g., the reviews in Ref. [2].

The baryonium episode contributed to assess some new theoretical concepts and, unfortunately, to develop a kind of mistrust and suspicion about this physics. In short, peaks have been found in some antinucleon-induced cross sections, and interpreted as new kind of mesonic resonances preferentially coupled to the baryon-antibaryon channels, dubbed *baryonium*. A low-energy antiproton ring was built at CERN in the 1980s to study these states, but none of the candidates were confirmed. On the other hand, the associated phenomenology was quite rich and promising: duality which, schematically, dictates the existence of resonances in reactions with abundant Yukawa-type of exchanges in the crossed channel; possibility of "quasi-nuclear" states (Nowadays, this approach is referred to as "molecular dynamics"), the speculations about ad-hoc clustering in multiquark compounds, elaborate string dynamics, etc.

There has been a continuous interest in the possibility of dibaryons with various flavor content, besides the deuteron. We shall discuss the doubly-strange $H(uuddss)$ dibaryon in the next section. There has been various claims for non-strange dibaryons, the most serious one being found at COSY.[3]

With the advent of the heavy quarks c and b in the 1970s, a very clean spectroscopy has emerged in the charmonium and bottomonium sectors. However, with the discovery of the $X(3872)$ in 2003 by the Belle experiment, and further similar states dubbed Y and Z, it was realized that the spectrum above (sometimes just above) the $Q\bar{q} + \bar{Q}q$ threshold becomes more rich and complex. For a review, see Ref. [2]. As for the interpretation of the XYZ states, there are essentially two approaches. The first one, started before the discovery of the $X(3872)$, is based on the interaction between a charmed meson and an anticharmed one (or beauty analogs). The second one describes these new mesons as $Q\bar{Q}q\bar{q}$ configurations, with, however, very often some assumptions about internal the clustering that simplifies the cumbersome four-body problem in a more tractable two-body one. See, e.g., Ref. [4].

As one can see, the history is rather shaky with sometimes adventurous claims on the experimental side, and on the theory side, a flurry of innovative ideas with, however, some lack of rigor in solving models, and, in some cases, redundant claims that do not take into account the existing literature.

2. Bound States. The Holy Grail of the search for exotic hadrons is a stable configuration, lying below any threshold for dissociation, and thus decaying weakly. There are already some speculations about such decays, with, for instance, the prediction that the lifetime of the $bb\bar{u}\bar{d}$ tetraquark is significantly longer than that of the stable b-mesons and b-baryons.[5] Indeed, the energy release

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of the b-quark decay in a tetraquark is partly used to give momentum to the spectators, and the rearrangement of the final-state quarks into hadrons is somewhat suppressed by their larger separation and their color configuration.

2.1. When Does Binding Occurs? Let us consider the class of few-body systems corresponding to the Hamiltonian

$$H = \frac{1}{2m}\sum_i \boldsymbol{p}_i^2 + \sum_{i<j} g_{ij}\, v(r_{ij}), \tag{1}$$

with $r_{ij} = |\boldsymbol{r}_j - \boldsymbol{r}_i|$, where one is provisionally restricted to equal masses and the same functional form $v(r)$ for all pairs.

In the case of N identical bosons with an attractive $v(r)$ (i.e., $g_{ij} = 1$ for any pair), the total strength $\sum g_{ij} = N(N-1)/2$ is larger than its cumulated value for any threshold. Hence the binding increases with N for any reasonable attractive potential.

In the case of atomic physics with $N = 4$, i.e., two electrons and two positrons interactions with $v(r) = -1/r$ (annihilation is disregarded), one gets $\sum g_{ij} = 2$ for both the positronium molecule $e^+e^+e^-e^-$ and its threshold made of two positronium atoms. Hence, binding is not guaranteed at first sight, and, indeed, requires a non-trivial variational function to be established.[6,7]

In the chromoelectric limit of the simple quark models with color operators

$$g_{ij} = -\frac{3}{16}\tilde{\lambda}_i \cdot \tilde{\lambda}_j, \tag{2}$$

where $v(r)$ is the quarkonium potential, one gets again $\sum g_{ij} = 2$ for both a four-quark state of any given color wave function, and the sum of values in the threshold made of two mesons, so that binding is not obvious, and, indeed, not observed in careful estimates. See, e.g., the discussion in Ref. [8]. Hence one should rely on unequal masses, or spin effects, or multibody forces to enforce binding.

2.2. Chromomagnetic Binding. The role of the chromomagnetic interaction in hadron spectroscopy has been stressed by several authors, following Ref. [9], and its extension to multiquarks pioneered in several papers by Jaffe. In Ref. [10], he realized that a chromomagnetic model

$$\sum_{i<j} \tilde{\lambda}_i \cdot \lambda_j\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\, \frac{v_{ss}(r_{ij})}{m_i\, m_j} \tag{3}$$

exhibits dramatic coherences. For $H = uuddss$ with spin 0, in the $SU(3)$ limit where all quark masses m_i are equal, if one adopts the same expectation value $\langle v_{ss}(r_{ij})\rangle$ of the smeared delta function for all pairs in the Λ baryon and the dibaryon H, one can obtain H deeply bound (about 150 MeV) below the $\Lambda\Lambda$ threshold. Unfortunately, all the searches fail to find the H, and a more careful treatment of the 6-body dynamics does not lead to a bound H. For a recent review including the estimates based on lattice QCD, see, e.g., Refs. [11,12].

Note that 10 years after Jaffe's paper on the H, Gignoux *et al.*,[13] and, independently, Lipkin[14] pointed out that the same chromomagnetic mechanism predicts a heavy pentaquark $\bar{Q}qqqq$ bound below the $\bar{Q}q+qqq$ threshold, if one assumes an infinitely heavy flavored antiquark, and, again, $SU(3)$ symmetry in the $qqqq$ sector and the same $\langle v_{ss}(r_{ij})\rangle$ as for ordinary baryons. Once more, however, relaxing these hypotheses works against the stability of such pentaquark.

2.3. Chromoelectric Binding. Just a few years after the first speculations on the H dibaryon, another mechanism was proposed that relies on the spin-independent part of the interaction.[15]

Consider, indeed, a $QQ\bar{q}\bar{q}$ configuration, with masses M and m. If one increases the mass ratio M/m starting from $M/m = 1$ and keeping $M^{-1} + m^{-1}$ fixed, the threshold energy will remain the same in a flavor-independent potential, while the four-body energy decreases, as its Hamiltonian is more and more asymmetric. See, e.g., Ref. [8]. It has been also stressed that it is exactly the same mechanism that makes the hydrogen molecule more stable than the positronium molecule, if one measures the stability by the fraction of extra binding with respect to the energy of the two-atom threshold.

The critical mass ratio M/m at which binding occurs in a purely chromoelectric interaction depends on the details of the interaction and is very large. In practice, as demonstrated first by Janc and Rosina,[16] the isoscalar $cc\bar{u}\bar{d}$ with $J^P = 1^+$ is bound in some realistic potentials, below the DD^* threshold (the T_{cc} found by LHCb[17,18] has however a small decay width into $DD\pi$, thank to the width of the D^* meson), as the chromoelectic effect is supplemented by a favorable chromomagnetic contribution of the $\bar{u}\bar{d}$ pair that has no analog in the threshold. See, also, Ref. [19].

Interestingly, the stability of T_{QQ} for heavy enough Q has been confirmed in studies based on QCD sum rules[20] and on lattice QCD, see, e.g., Refs. [21,22], and references therein.

Note that, while in the case of hydrogen-like molecules and the case of chromoelectric $QQ\bar{q}\bar{q}$, increasing the mass ratio M/m improves the binding, there is a noticeable difference: in the limit of equal masses, the Ps_2 molecule is bound and the $QQ\bar{Q}\bar{Q}$ is not, if one adopts the prescription (2) of a pairwise color-octet-exchange interaction. This is due to a larger asymmetry of the strength coefficients $\{g_{ij}\}$ in the case of Ps_2.[8]

2.4. String Dynamics. A debatable aspect of the simple chromoelectric models is the assumption of a pairwise interaction with color factors, as expressed by Eq. (2), which corresponds to color-octet exchange. The linear part of the potential in quarkonia, $v(r) = \sigma r$, is interpreted as a flux tube of gluons of constant cross-section linking the quark to the antiquark.[23] Its natural generalization to baryons is

$$V = \sigma \min_J (r_{12} + r_{23} + r_{31}), \tag{4}$$

as illustrated in Fig. 1. It corresponds to the well-known problem by Fermat and Torricelli of minimizing the cumulated distance to the three vertices of a triangle, see, e.g., Ref. [24].

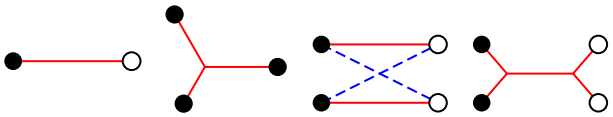

Fig. 1. From left to right: String picture of quark-antiquark, three quarks, and flip–flop and the connected strings for the tetraquark.

In an extension to tetraquarks, the confining potential is the minimum of the connected string diagram with two junctions, and the two possible disconnected pairs of quark-antiquark strings, sometimes referred to as the "flip–flop" model, say

$$V = \sigma \, \min \Big[\min_{a,\,b} \left(r_{1a} + r_{2a} + r_{ab} + r_{b3} + r_{b4}\right), \\ r_{13} + r_{24}, \; r_{14} + r_{23} \Big]. \tag{5}$$

This potential is more attractive than the pairwise model, but, as it is dominated by the flip–flop term, it requires changes of the internal color state when the quark and antiquark move. Hence, the attraction fully operates for non-identical quarks, but is damped by the Pauli principle for identical quarks or antiquarks. See, e.g., Refs. [25,26].

3. Resonances. Bound states probe the short-range dynamics and give access to novel weak decays. On the other hand, the resonances, which are more delicate to study, give valuable information in other spin-flavor channels and on the long-range part of the interaction.

3.1. Calculating Resonance Properties. When one first encounters the task of estimating resonances, one is tempted to build a simple variational expansion

$$\Psi = \sum_k \gamma_k \, \exp\Big[- \sum_{ij} a_{ij}^{(k)} \, r_{ij}^n / 2 \Big], \tag{6}$$

and diagonalize the Hamiltonian in this basis of Gaussians ($n = 2$) or exponentials ($n = 1$) and associate the first level above the threshold with an approximation to the first resonance.

This is unfortunately not that simple, but there are ways to use bound-state type of wave functions (6) to look for resonances.

Consider for instance the well-studied benchmark interaction [27]

$$V(r) = 7.5\, r^2 \, \exp(-r) \tag{7}$$

the so-called Bardsley potential, in units of $\hbar^2/2m = 1/2$. A direct search in the complex-energy plane indicates a resonance at $E = 3.42639 - i\,0.0127745$. If one uses a small basis of five exponentials, i.e., $n = 1$ in Eq. (6), with the range parameters running from $a^{(1)} = 1.5$ to $a^{(5)} = 2.5$ in a geometric series, and then apply an overall rescaling, namely (this α corresponds to $1/\lambda^2$ of some papers in the literature)

$$a_{ij}^{(k)} \;\to\; \alpha \, a_{ij}^{(k)}, \tag{8}$$

one gets the spectrum shown in Fig. 2, with a dramatic succession of avoided crossings (or, say, level rearrangements). The minimal separation at the avoided crossings gives a rough estimate of the width by the Simons formula. [28] This is the method of *stabilization* or *real scaling*.

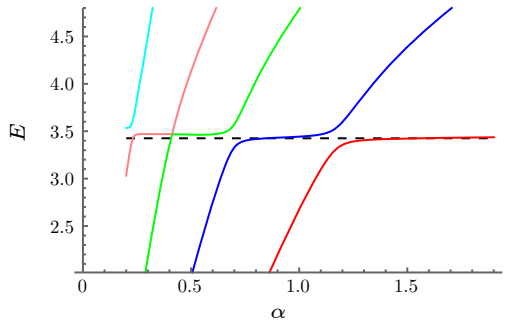

Fig. 2. Stabilization method applied to the Bardsley potential. The dotted line corresponds to the exact value of the resonances. The colored energy levels are given by an expansion with five exponentials, whose ranges are multiplied by a scaling factor α.

The method of *complex scaling* consists of applying a rotation $r \to r\,\exp(i\,t)$ so that the wavefunction of a resonance becomes exponentially decreasing at large distances. [29,30] In its variant based on Eq. (6), the range parameters undergo a rotation $a_{ij} \to a_{ij}\,\exp(-n\,i\,t)$. An illustration is given in Fig. 3.

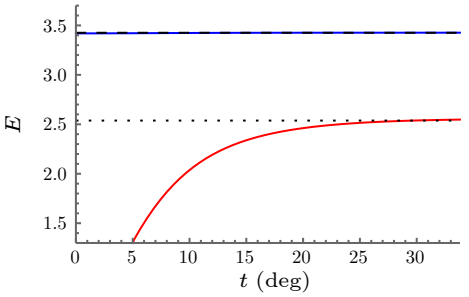

Fig. 3. Complex scaling applied to the Bardsley potential. Blue: real part of the energy, red: imaginary part times −200, as a function of the rotation angle t. The dotted lines correspond to the exact values of the resonances.

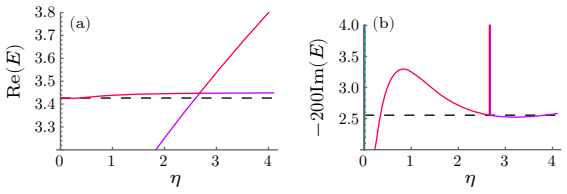

Fig. 4. Real and imaginary part near the first resonance in the Bradsley potential, supplemented with an absorptive confinement starting at $r_c = 2.05$, of variable strength η. The dotted lines correspond to the exact value.

Another trick consists of adding an *external absorption*, for instance a quadratic potential starting at $r = r_c$. Figure 4 shows the energy region of interest, estimated with seven exponentials with range parameters from $a = 2$ to 6 in a geometric series, with the potential (7) supplemented with

$$W = -i\,\eta\,(r - r_c)^2\,\Theta(r - r_c), \tag{9}$$

as a function of η. As already pointed out in Ref. [31], when one deals with a finite basis, the best results correspond to stationary energies, unlike to the $\eta \to 0$ limit with a very large basis. One notices again a kind of level rearrangement: as η is varied, two variational levels take it in turn to approximate the resonance energy.

Remarkably, the different methods give very similar wavefunction, as illustrated in Fig. 5. The wavefunction from real-plus-complex scaling and from external absorption are very similar, and would probably be even closer if the computation were pushed further. Of course, with the latter method, the wavefunction is damped at large r, with complex scaling, it oscillates.

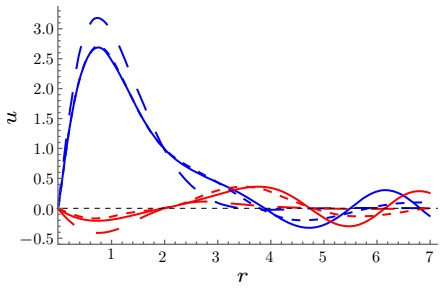

Fig. 5. Comparison of the reduced radial wavefunction $u(r)$ of the first resonance in the Bradsley potential. Solid line: direct numerical search of a state decaying as $\exp(i\,k\,r)$ (Siegert method); short dashing: complex scaling using the parametrization of Blain *et al.* with seven terms; long dashing: complex absorption. Blue: real part; red: imaginary part, with the normalization $u(2) = 1$.

3.2. Application to the Quark Model. As stressed in the previous section, the quark model predicts just a few bound states, in channels where the patterns of the symmetry breaking favor the collective configurations as compared to the threshold. For a survey of bound states in the quark model, see, e.g., Ref. [32].

On the other hand, there are many channels for which the amount of attraction is similar in the collective multiquark and in its threshold, and thus a resonant behavior can reasonably be expected. There is also the possibility of resonances as excited states above some deeply bound state such as $bb\bar{u}\bar{d}$.[33]

This program implies considerable efforts, as dozens of channels are concerned, and in each case, one should performs many tests by changing the size of the bases such as Eq. (6), varying the parameters, using different potentials, etc.

There is already rich literature: in Refs. [34,35], Hiyama *et al.* have applied the method of stabilization to the light and hidden-charm pentaquarks, with an estimate of the width by the Simons formula. These pioneering studies clearly demonstrate the possibility of separating the genuine resonances from the positive-energy states paving the continuum.

In Ref. [33], Meng *et al.* demonstrated the basis of a specific model (that may be too attractive as it slightly overbinds T_{cc}) the possibility of having both bound radial excitations and resonances for $bb\bar{q}\bar{q}$, which are identified by the method of stabilization. Note that a very large basis was used to cover the compact bound states and the more extended excitations.

A special effort has been devoted to "fully charmed" tetraquarks $cc\bar{c}\bar{c}$, since there are some candidates seen by various experiments at the LHC. See, e.g., the compilation by the Quarkonium Working Group.[36] For a recent review of stabilization and complex scaling applied to doubly-heavy and fully heavy states, see, e.g., Ref. [37].

Recently a very small sample of the results of the China–Japan collaboration have been checked.[38] Let us concentrate on the simplest case, namely $cc\bar{c}\bar{c}$ with equal masses, and $J^P = 2^+$ that involves only two spin-color channels. The potential is the one by Barnes, Godfrey and Swanson (BGS),[39] as in Ref. [33], to ease the comparison. First, one can see that if the range parameters of the expansion (6) are multiplied by a factor α, a series of avoided crossings is observed, indicating the location of candidates for resonances. The scale is of course arbitrary: $\alpha = 1$ corresponds to an empirical tuning, and the stability with respect to α signals the vicinity of a resonance. (Note the somewhat oscillating behavior of the energy levels as a function of α in the case of a small number of Gaussians, unlike what is observed when one uses a larger basis, as in Refs. [34,35]. In the discrete part of the spectrum, the ground-state energy, as well as the *sum* of n first levels should exhibit a minimum for some value of α, but here one deals with the continuum.)

Then for a given α, one can apply complex scaling, in its variational variant, i.e., for Gaussians,

$$a_{ij} \to a_{ij}\,\alpha\,\exp(-2\,i\,t). \tag{10}$$

The results are shown in Figs. 6 and 7. A level is clearly stationary near $E = 6.9 - i\,0.04$ GeV, and another one near $E = 7.1 - i\,0.012$ GeV, in rough agreement with the more detailed study in Refs. [40,41]

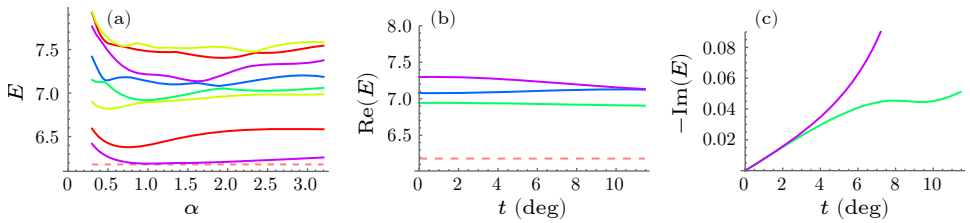

Fig. 6. Combination of stabilization and complex scaling for the $J^P = 2^+$ states of $cc\bar{c}\bar{c}$ resonances with the BGS potential,[39] starting from a wavefunction of type (6) with $n = 2$. Left: stabilization, center and right: real part and imaginary part of the energy for $\alpha = 1.55$ in Eq. (10). The dashed line indicates the lowest threshold.

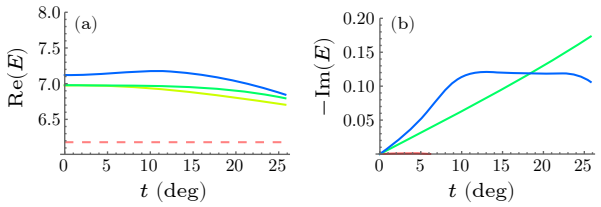

Fig. 7. The same as Fig. 6, but for $\alpha = 2.55$.

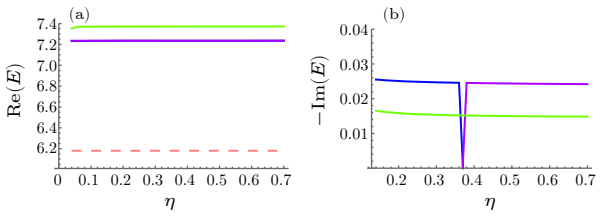

Fig. 8. Levels obtained near 7.2 GeV for $cc\bar{c}\bar{c}$ with spin 2 in the BGS potential by the method of complex confinement.

The above results are confirmed by the method of absorptive confinement. Namely the potential in each pair of the color $\bar{3}3$ channel is supplemented with $-i\,\eta\,(r_{ij} - r_c)^2$ for $r \geq r_c$. In Fig. 8, two levels clearly emerge with a stable energy, near 7 GeV. The plots are drawn with a few Gaussians in each color channel and a radius $r_c = 4\,\text{GeV}^{-1}$ for the absorption. One notices a level rearrangement for the imaginary part of the level near 7 GeV. Clearly, the computing should be repeated with a larger basis, but the pattern already demonstrates a clear separation of the genuine resonances from the background.

Note that neither Fig. 7 nor Fig. 8 is exhaustive. The former focuses on the 7.0 GeV region and the latter on the 7.2 GeV one. A thorough comparison of the methods using various quark models is currently studied by several groups.

4. Outlook. There is now a consensus about the stability of $QQ\bar{q}\bar{q}$ tetraquarks, if the mass ratio is large enough and the $\bar{q}\bar{q}$ pair in a spin 0 state, as the chromoelectric and chromomagnetic mechanisms are combined coherently. Such tetraquarks were first predicted on the basis of potential models and confirmed by elaborate lattice calculations. The techniques for calculating multiquark bound states are now rather mature, but one should remain careful, as some approximations such as ad-hoc clustering tend to overbind the multiquarks and thus could predict the stability of configurations that are unbound in a given model.

Calculating resonances is even more delicate, and usually requires very large bases. It is hoped that more handy methods will be elaborated. Some recent studies indicate the possibility of resonances in the fully heavy configurations such a $cc\bar{c}\bar{c}$ and in channels with both heavy and light quarks and antiquarks. The experimental search is very difficult and restricted to some golden modes such as $J/\psi\,J/\psi$. The most remarkable pattern is the scarcity of multiquark bound states and narrow resonances.

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