Higher-Order Nonlinear Effects on Optical Soliton Propagation and Their Interactions

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(Received 30 May 2024; accepted manuscript online 13 June 2024)

When pursuing femtosecond-scale ultrashort pulse optical communication, one cannot overlook higher-order nonlinear effects. Based on the fundamental theoretical model of the variable coefficient coupled high-order nonlinear Schrödinger equation, we analytically explore the evolution of optical solitons in the presence of high-order nonlinear effects. Moreover, the interactions between two nearby optical solitons and their transmission in a nonuniform fiber are investigated. The stability of optical soliton transmission and interactions are found to be destroyed to varying degrees due to higher-order nonlinear effects. The outcomes may offer some theoretical references for achieving ultra-high energy optical solitons in the future.

DOI: 10.1088/0256-307X/41/7/074204

Traditional fiber optic communication systems should avoid dispersion, loss and nonlinear effects to the greatest extent for the purpose of enhancing the quality of signal transmission in optical fibers. However, due to special requirements on fiber velocity, transmission capacity, interference immunity, security, and other aspects in the fields of communication and national defense, researchers have gradually shifted the focus of research to the nonlinear effects in optical fibers.[1–7] In addition, as the transmission distance increases, a larger channel transmission power is required to overcome fiber loss, which means that the channel sending power will lead to higher-order nonlinear effects in a system. At this time, the impact of higher-order nonlinear effects on the pulse transmission becomes assignable.[8–14] Therefore, nonlinear optics has gradually become an important sub-discipline in optical research, and study of higher-order nonlinear effects on pulse transmission, mode-locked lasers and other applications is of great scientific significance.[15–23]

On the other hand, optical solitons, known as localized electromagnetic waves, have the ability to propagate steadily through nonlinear materials with dispersion effects. Research of optical solitons is a significant subject in nonlinear optics, which is great potential in quantum information processing, optical communication, and optical switching, etc.[24–27] Transmission of optical solitons in an optical fiber is mathematically modeled with the standard nonlinear Schrödinger equation (NLSE), only containing group velocity dispersion (GVD) and self-phase modulation (SPM) effects. As ultrashort pulse technology advances, the standard NLSE is no longer able to describe transmission of ultrashort optical solitons when the transmitted optical soliton is shortened to the femtosecond order. In addition, the dispersion and nonlinear effect in a higher order form of optical fibers must be taken into consideration. Therefore, rather than the standard form equation, the high-order NLSE (HNLSE) is required to express femtosecond optical solitons propagation in optical fibers mathematically.[28–31]

In this Letter, taking the higher-order nonlinear effects into consideration, we analytically investigate the variable coefficient coupled high-order nonlinear Schrödinger equation (VCHNLSE) that can be used to describe the transmission of femtosecond optical solitons in optical fibers. The specific form of the equation is [32]

$$\begin{aligned} &iq_{1,t}+q_{1,xx}+2\sigma(|q_1|^2+2|q_2|^2)q_1+2\sigma q_1^*q_2^2+i\varepsilon(t)q_{1,xxx}\\ &+6i\varepsilon(t)\sigma(|q_1|^2+|q_2|^2)q_{1,x}+6i\varepsilon(t)\sigma(q_1q_2^*+q_1^*q_2)q_{2,x}=0,\\ &iq_{2,t}+q_{2,xx}+2\sigma(2|q_1|^2+|q_2|^2)q_2+2\sigma q_2^*q_1^2+i\varepsilon(t)q_{2,xxx}\\ &+6i\varepsilon(t)\sigma(|q_1|^2+|q_2|^2)q_{2,x}+6i\varepsilon(t)\sigma(q_1q_2^*+q_1^*q_2)q_{1,x}=0, \end{aligned} \tag{1}$$

where $\varepsilon(t)$ is a real function, which is related to high-order nonlinear effects. For Eq. (1), the breather solutions have been obtained, and the modulation instability has been analyzed.[32] However, the influence of high-order nonlinear effects on the transmission and interaction of optical solitons, which is a key issue that urgently needs to be studied in high-speed and long-distance fiber optic transmission systems, was not discussed in Ref. [32]. Here, based on Eq. (1), the bilinear method will be adopted to analyze the evolution properties of femtosecond optical soliton and to investigate how higher-order nonlinear effects affect optical soliton transmission and interactions.

The primary content of this essay are summarized in the following. First, the solutions for both one- and two-soliton solutions are derived through the solution process of Eq. (1). Then we analyze how the higher-order nonlin-

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ear effects influence optical soliton transmission as well as interactions between solitons based on the existing analytical solutions. Finally we provide conclusions.

Analytic Solutions. For Eq. (1), we set

$$q_1 = \frac{G}{F}, \quad q_2 = \frac{H}{F}. \tag{2}$$

Then substituting Eq. (2) into Eq. (1), we can reach the Hirota bilinear forms

$$\begin{aligned}
&\Big[i\mathrm{D}_t + \mathrm{D}_x^2 + i\varepsilon(t)\mathrm{D}_x^3\Big]G\cdot F = \lambda_1 GF,\\
&\Big[i\mathrm{D}_t + \mathrm{D}_x^2 + i\varepsilon(t)\mathrm{D}_x^3\Big]H\cdot F = \lambda_2 HF,\\
&\mathrm{D}_x^2 F\cdot F = 2\sigma\Big(|G|^2+|H|^2\Big),\\
&-2\sigma(GH^*+G^*H)\Big[3i\varepsilon(t)\mathrm{D}_x H\cdot F + HF\Big] = \lambda_1 GF^3,\\
&-2\sigma(GH^*+G^*H)\Big[3i\varepsilon(t)\mathrm{D}_x G\cdot F + GF\Big] = \lambda_2 HF^3.
\end{aligned} \tag{3}$$

In Eq. (3), we set $\lambda_1 = \lambda_2 = 1$, and make the corresponding assumptions to obtain the analytical one- and two-soliton solutions to Eq. (1).

One-Soliton Solution. We can expand the G, H, and F as $G = \eta g_1$, $H = \eta h_1$, and $F = 1+\eta^2 f_2$, and set $g_1 = \alpha_1 e^{\theta}$, $h_1 = \beta_1 e^{\theta}$, and $f_2 = \zeta_1 e^{\theta+\theta^*}$, where $\theta = Ax + B(t) + C$, $\alpha_1 = \alpha_1^r + i\alpha_1^i$, and $\beta_1 = \beta_1^r + i\beta_1^i$. Let all the coefficients of η be equal to zero, the above-mentioned recursive relation among G, H, and F can be reduced. The one-soliton solution is

$$q_1 = \frac{g_1}{1+f_2}, \quad q_2 = \frac{h_1}{1+f_2}, \tag{4}$$

where

$$\begin{aligned}
&\beta_1^r = -\frac{\alpha_1^i\beta_1^i}{\alpha_1^r}, \quad B(t) = \int[-A^2(\varepsilon(t)A - i)]dt,\\
&\zeta_1 = \frac{\sigma(|\alpha_1|^2+|\beta_1|^2)}{(A+A^*)^2},
\end{aligned} \tag{5}$$

with α_1^r, α_1^i, β_1^i being real numbers, A and C the complex numbers.

Two-Soliton Solution. We can expand the G, H and F as $G = \eta g_1 + \eta^3 g_3$, $H = \eta h_1 + \eta^3 h_3$, and $F = 1 + \eta^2 f_2 + \eta^4 f_4$, and set

$$\begin{aligned}
g_1 &= \alpha_1 e^{\theta_1} + \alpha_2 e^{\theta_2},\\
g_3 &= m_1(t)e^{\theta_1+\theta_2+\theta_1^*} + m_2(t)e^{\theta_1+\theta_2+\theta_2^*},\\
h_1 &= \beta_1 e^{\theta_1} + \beta_2 e^{\theta_2},\\
h_3 &= n_1(t)e^{\theta_1+\theta_2+\theta_1^*} + n_2(t)e^{\theta_1+\theta_2+\theta_2^*},\\
f_2 &= \zeta_1(t)e^{\theta_1+\theta_1^*} + \zeta_2(t)e^{\theta_2+\theta_2^*} + \zeta_3(t)e^{\theta_1+\theta_2^*} + \zeta_4(t)e^{\theta_1^*+\theta_2},\\
f_4 &= l_1(t)e^{\theta_1+\theta_1^*+\theta_2+\theta_2^*},
\end{aligned} \tag{6}$$

where $\theta_k = A_k x + B_k(t) + C_k$, $\alpha_k = \alpha_k^r + i\alpha_k^i$, and $\beta_k = \beta_k^r + i\beta_k^i$, $k = 1, 2$, α_k^r, α_k^i, β_k^r, β_k^i are all real numbers, A_k and C_k are complex numbers. The two-soliton solutions are

$$q_1 = \frac{g_1+g_3}{1+f_2+f_4}, \quad q_2 = \frac{h_1+h_3}{1+f_2+f_4}, \tag{7}$$

where

$$\begin{aligned}
&\beta_1^r = -\frac{\alpha_1^i\beta_1^i}{\alpha_1^r}, \quad \beta_2^r = -\frac{\alpha_2^i\beta_2^i}{\alpha_2^r}, \quad \beta_1^i = \frac{\alpha_1^r\beta_2^i}{\alpha_2^r},\\
&B_1(t) = \int[-A_1^2(\varepsilon(t)A_1 - i)]dt,\\
&B_2(t) = \int[-A_2^2(\varepsilon(t)A_2 - i)]dt,\\
&\zeta_1(t) = \frac{\sigma\Big(|\alpha_1|^2+|\beta_1|^2\Big)}{(A_1+A_1^*)^2}, \quad \zeta_2(t) = \frac{\sigma\Big(|\alpha_2|^2+|\beta_2|^2\Big)}{(A_2+A_2^*)^2},\\
&\zeta_3(t) = \frac{\sigma\Big(\alpha_1\alpha_2^* + \beta_1\beta_2^*\Big)}{(A_1+A_2^*)^2}, \quad \zeta_4(t) = \frac{\sigma\Big(\alpha_2\alpha_1^* + \beta_2\beta_1^*\Big)}{(A_2+A_1^*)^2},
\end{aligned}$$

$$\begin{aligned}
m_1(t) = e^{-Q_1}\int\Big\{&\Big[(\zeta_1(t)\alpha_2 + \zeta_4(t)\alpha_1)\frac{d}{dt}B_1^*(t)\\
&+ (\zeta_1(t)\alpha_2 - \zeta_4(t)\alpha_1)\frac{d}{dt}B_1(t)\\
&+ (-\zeta_1(t)\alpha_2 + \zeta_4(t)\alpha_1)\frac{d}{dt}B_2(t)\\
&+ \alpha_2\frac{d}{dt}\zeta_1(t) + \alpha_1\frac{d}{dt}\zeta_4(t)\Big]\\
&+ \alpha_2[i + (A_1 - A_2 + A_1^*)\varepsilon(t)]\\
&\times (A_1 - A_2 + A_1^*)^2\zeta_1(t)\\
&+ \alpha_1[i + (-A_1 + A_2 + A_1^*)\varepsilon(t)]\\
&\times (A_1 - A_2 - A_1^*)^2\zeta_4(t)\Big\}e^{Q_1}dt,
\end{aligned}$$

$$\begin{aligned}
m_2(t) = e^{-Q_2}\int\Big\{&\Big[(\zeta_2(t)\alpha_1 + \zeta_3(t)\alpha_2)\frac{d}{dt}B_2^*(t)\\
&+ (\zeta_3(t)\alpha_2 - \zeta_2(t)\alpha_1)\frac{d}{dt}B_1(t)\\
&+ (-\zeta_3(t)\alpha_2 + \zeta_2(t)\alpha_1)\frac{d}{dt}B_2(t)\\
&+ \alpha_1\frac{d}{dt}\zeta_2(t) + \alpha_2\frac{d}{dt}\zeta_3(t)\Big]\\
&+ \alpha_1[i + (-A_1 + A_2 + A_2^*)\varepsilon(t)]\\
&\times (A_1 - A_2 - A_2^*)^2\zeta_2(t)\\
&+ \alpha_2[i + (A_1 - A_2 + A_2^*)\varepsilon(t)]\\
&\times (A_1 - A_2 + A_2^*)^2\zeta_3(t)\Big\}e^{Q_2}dt,
\end{aligned}$$

$$\begin{aligned}
n_1(t) = e^{-Q_1}\int\Big\{&\Big[(\zeta_1(t)\beta_2 + \zeta_4(t)\beta_1)\frac{d}{dt}B_1^*(t)\\
&+ (\zeta_1(t)\beta_2 - \zeta_4(t)\beta_1)\frac{d}{dt}B_1(t)\\
&+ (-\zeta_1(t)\beta_2 + \zeta_4(t)\beta_1)\frac{d}{dt}B_2(t)\\
&+ \beta_2\frac{d}{dt}\zeta_1(t) + \beta_1\frac{d}{dt}\zeta_4(t)\Big]\\
&+ \beta_2[i + (A_1 - A_2 + A_1^*)\varepsilon(t)]\\
&\times (A_1 - A_2 + A_1^*)^2\zeta_1(t)\\
&+ \beta_1[i + (-A_1 + A_2 + A_1^*)\varepsilon(t)]\\
&\times (A_1 - A_2 - A_1^*)^2\zeta_4(t)\Big\}e^{Q_1}dt,
\end{aligned}$$

$$\begin{aligned}
n_2(t) = e^{-Q_2}\int\Big\{&\Big[(\zeta_2(t)\beta_1 + \zeta_3(t)\beta_2)\frac{d}{dt}B_2^*(t)\\
&+ (\zeta_3(t)\beta_2 - \zeta_2(t)\beta_1)\frac{d}{dt}B_1(t)
\end{aligned}$$

$$
\begin{aligned}
&+(-\zeta_3(t)\beta_2+\zeta_2(t)\beta_1)\frac{d}{dt}B_2(t)\\
&+\beta_1\frac{d}{dt}\zeta_2(t)+\beta_2\frac{d}{dt}\zeta_3(t)\Big]\\
&+\beta_1[i+(-A_1+A_2+A_2^*)\varepsilon(t)]\\
&\times(A_1-A_2-A_2^*)^2\zeta_2(t)\\
&+\beta_2[i+(A_1-A_2+A_2^*)\varepsilon(t)]\\
&\times(A_1-A_2+A_2^*)^2\zeta_3(t)\Big\}e^{Q_2}dt,
\end{aligned}
$$

$$
\begin{aligned}
l_1(t)=(A_1+A_1^*+A_2+A_2^*)^{-2}\Big\{&-\zeta_2(t)\zeta_1(t)\\
&\times(A_1-A_2+A_1^*-A_2^*)^2\\
&-\zeta_4(t)\zeta_3(t)(A_1-A_2-A_1^*+A_2^*)^2\\
&+\sigma[m_1(t)\alpha_2^*+m_2(t)\alpha_1^*+n_1(t)\beta_2^*+n_2(t)\beta_1^*\\
&+m_1^*(t)\alpha_2+m_2^*(t)\alpha_1+n_1^*(t)\beta_2+n_2^*(t)\beta_1]\Big\},
\end{aligned}
$$

$$
\begin{aligned}
Q_1=&(A_1+A_2+A_1^*)^3\int\varepsilon(t)dt+B_1(t)+B_2(t)+B_1^*(t)\\
&-[iA_1^2+(2iA_2+2iA_1^*)A_1+iA_2^2+2iA_1^*A_2+iA_1^{*2}]t,\\
Q_2=&(A_1+A_2+A_2^*)^3\int\varepsilon(t)dt+B_1(t)+B_2(t)+B_2^*(t)\\
&-[iA_1^2+(2iA_2+2iA_2^*)A_1+iA_2^2+2iA_2^*A_2+iA_2^{*2}]t.
\end{aligned}
$$

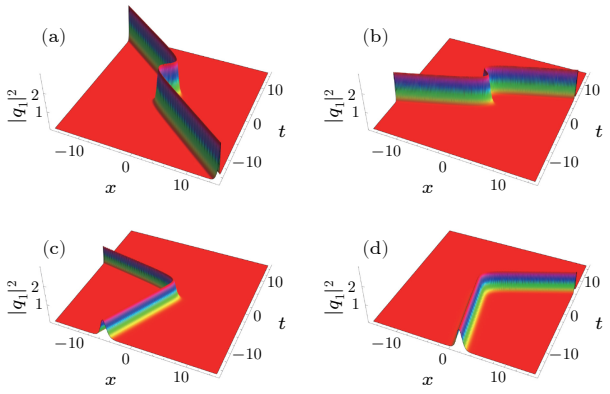

Fig. 1. Influence of changes in A on the transmission of optical solitons with different high-order nonlinear effects. Parameters in solution (5) are $\sigma=1$, $C=0$, $\beta_1^i=1$, $\alpha_1^r=1$, and $\alpha_1^i=1$ with (a) $\varepsilon(t)=\mathrm{sech}(t)$, $A=-2-0.66i$; (b) $\varepsilon(t)=\mathrm{sech}(t)$, $A=-1.8+1.6i$; (c) $\varepsilon(t)=\tanh(t)$, $A=-1.4-1.3i$; (d) $\varepsilon(t)=\tanh(t)$, $A=1.5+0.56i$.

Discussions. For solutions (5) and (7), the properties of q_1 and q_2 are the same, but there is a slight difference in the amplitude of optical solitons. Therefore, we only discuss the optical transmission and interactions of q_1. For the coefficient $\varepsilon(t)$, which is related to high-order effects, we mainly discuss the influence of $\varepsilon(t)$ on the transmission and interaction of optical solitons when it is the hyperbolic tangent and hyperbolic tangent function, respectively. First, we assume that $\sigma=1$, $C=0$, $\beta_1^i=1$, $\alpha_1^r=1$, and $\alpha_1^i=1$ in solution (5). From Figs. 1(a) and 1(b), $\varepsilon(t)=\mathrm{sech}(t)$, we can find that the transmission path of optical solitons exhibits diversity with the different values of A. By selecting different values of A, we can achieve control over the direction and speed of optical soliton transmission. When $\varepsilon(t)=\tanh(t)$, we can see that by changing the values of A, the transmission path and propagation speed of optical solitons have indeed undergone significant changes in Figs. 1(c) and 1(d).

In Fig. 2, we mainly discuss the influence of high-order nonlinear coefficient $\varepsilon(t)$ on the transmission of optical solitons. In Figs. 2(a) and 2(b), we find that they exhibit different transmission characteristics from Figs. 1(a) and 1(b). At the phase transition point, the amplitude of the optical soliton becomes very large, and we can also control the specific amplification position. In Figs. 2(c) and 2(d), we can achieve free switching of optical soliton transmission paths by changing the correlation coefficient of $\varepsilon(t)$.

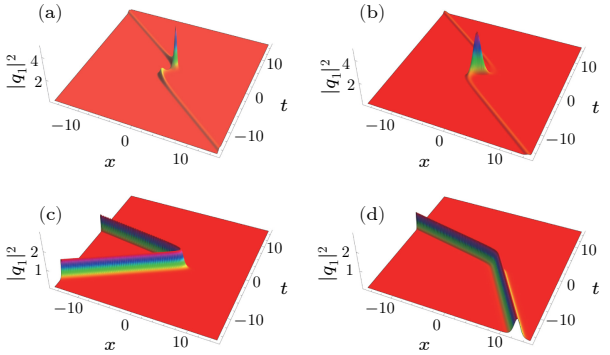

Fig. 2. Influence of changes in $\varepsilon(t)$ on the transmission of optical solitons with the different values of A. Parameters in solution (5) are $\sigma=1$, $C=0$, $\beta_1^i=1$, $\alpha_1^r=1$, and $\alpha_1^i=1$ with (a) $\varepsilon(t)=\mathrm{sech}(-1.5t-0.75)$, $A=-2-0.66i$, (b) $\varepsilon(t)=1.6\mathrm{sech}(1.2t-0.2)$, $A=-2-0.66i$, (c) $\varepsilon(t)=1.4\tanh(1.3t+1.1)$, $A=-1.4-1.3i$, (d) $\varepsilon(t)=0.63\tanh(1.8t+1.9)$, $A=-1.4-1.3i$.

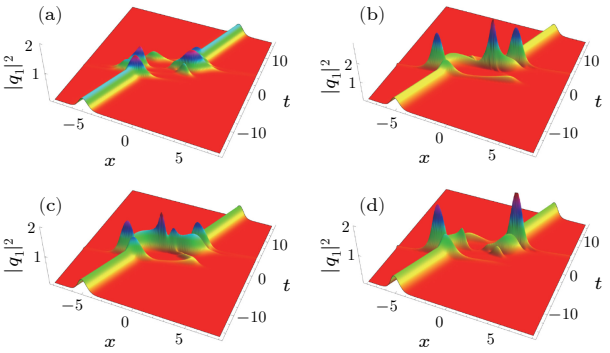

Fig. 3. Influence of changes in $\varepsilon(t)$ on the interactions of optical solitons. Parameters in solution (7) are $\sigma=1$, $C_k=0$, $\beta_k^i=1$, $\alpha_k^r=1$, $\alpha_k^i=1$, $A_1=-1.1+1.5i$, and $A_2=1.3+0.1i$ with (a) $\varepsilon(t)=0.94\mathrm{sech}(1.4t+1.5)$, (b) $\varepsilon(t)=1.2\mathrm{sech}(1.4t-0.4)$, (c) $\varepsilon(t)=1.3\mathrm{sech}(1.8t+0.56)$, (d) $\varepsilon(t)=1.1\mathrm{sech}(1.4t+1.1)$.

In Fig. 3, $A_1=-1.1+1.5i$ and $A_2=1.3+0.1i$, we mainly discuss the influence of $\varepsilon(t)$ on the interaction of optical solitons. In the interaction between two optical solitons, the amplitude of the optical soliton is relatively random. By changing the coefficient of $\varepsilon(t)$, the amplitude of the optical soliton interaction can be effectively amplified. When $\varepsilon(t)$ is a hyperbolic secant function, our

effective control over the interaction process of optical solitons becomes exceptionally evident in Fig. 4. In Fig. 4(a), we achieve amplification in the interaction of optical solitons. In Fig. 4(c), we obtain parallel transmission after soliton interaction by measuring the incident phase of the soliton. In Figs. 4(b) and 4(d), we can effectively control the period of soliton interactions.

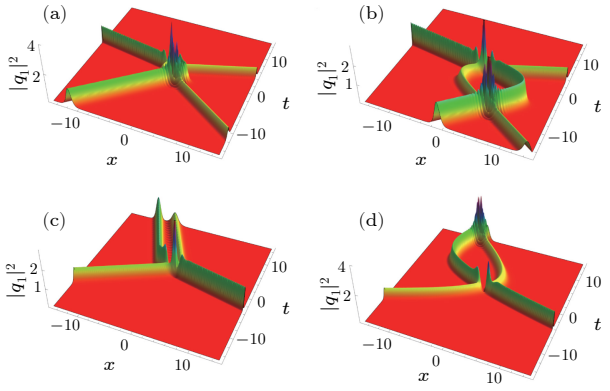

Fig. 4. Influence of changes in $\varepsilon(t)$ on the interactions of optical solitons. Parameters in solution (7) are $\sigma = 1$, $C_k = 0$, $\beta_k^i = 1$, $\alpha_k^r = 1$, $\alpha_k^i = 1$, $A_1 = 1.2 + 0.34i$, and $A_2 = 1.6 - 1.2i$ with (a) $\varepsilon(t) = -2\tanh(-1.2t - 0.9)$, (b) $\varepsilon(t) = -2\tanh(-0.41t - 1.1)$, (c) $\varepsilon(t) = -1.1\tanh(1.9t - 1.4)$, (d) $\varepsilon(t) = 1.7\tanh(-0.16t + 0.13)$.

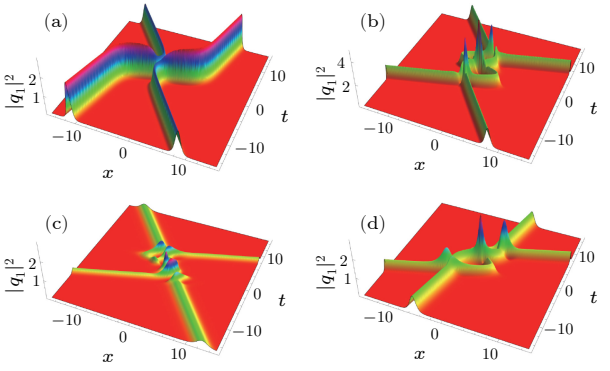

Fig. 5. Influence of changes in A_1 and A_2 on the interactions of optical solitons with the high-order nonlinear coefficient $\varepsilon(t) = \mathrm{sech}(t)$. Parameters in solution (7) are $\sigma = 1$, $C_k = 0$, $\beta_k^i = 1$, $\alpha_k^r = 1$, and $\alpha_k^i = 1$ with (a) $A_1 = -2 + 0.1i$, $A_2 = -1.6 - 0.4i$, (b) $A_1 = -1.6 + 1.9i$, $A_2 = 1.8 - 0.4i$, (c) $A_1 = 0.7 + 1.6i$, $A_2 = -0.8 - 0.4i$, (d) $A_1 = -1.1 + 1.5i$, $A_2 = 1.3 + 0.1i$.

In Fig. 5, $\varepsilon(t) = \mathrm{sech}(t)$, we can see that the interaction characteristics of optical solitons have also changed by adjusting the values of A_1 and A_2. This interaction characteristic is different from Fig. 3. This indicates that by adjusting the values of A_1 and A_2, we have more flexibility in controlling the period and amplitude of soliton interactions. In Fig. 6, we also obtain effective control of soliton interactions. By changing the values of A_1 and A_2, we can amplify optical solitons and specify which specific optical soliton is amplified.

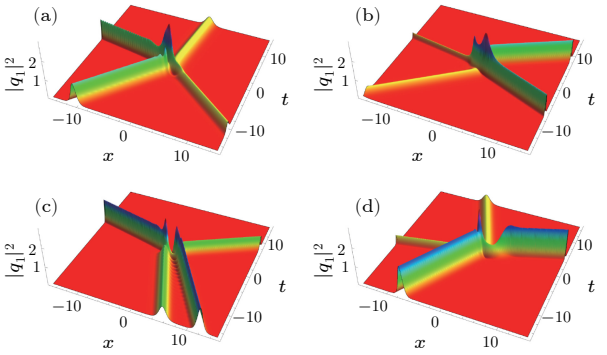

Fig. 6. Influence of changes in A_1 and A_2 on the interactions of optical solitons with the high-order nonlinear coefficient $\varepsilon(t) = \tanh(t)$. Parameters in solution (7) are $\sigma = 1$, $C_k = 0$, $\beta_k^i = 1$, $\alpha_k^r = 1$, and $\alpha_k^i = 1$ with (a) $A_1 = -1 - 0.31i$, $A_2 = 1.5 - 1.4i$, (b) $A_1 = 1.5 - 0.16i$, $A_2 = -0.78 - 1.2i$, (c) $A_1 = 1.2 + 0.34i$, $A_2 = 1.6 - 1.2i$, (d) $A_1 = 1.8 + 0.84i$, $A_2 = 1 + 1.1i$.

In summary, we have investigated the variable coefficient coupled high-order nonlinear Schrödinger equation (1), from which the one soliton solution is obtained and two-soliton solutions under different conditions are derived. We also discuss the nonlinear dynamical behaviors of optical solitons in an optical transmission system. In addition, how the higher-order nonlinear effects affect the transport and interactions of optical soliton is investigated. By selecting different values of A_k and $\varepsilon(t)$, we achieve the control over the direction and speed of optical soliton transmission. Further, we control the specific amplification position and achieve the free switching of optical soliton transmission paths by changing the correlation coefficient. Finally, we amplify the amplitude of the optical soliton in their interactions effectively. The results show that high-power femtosecond pulses can be stably transmitted in optical fibers. This provides a theoretical basis for the stable transmission of femtosecond optical solitons, which will promote the development of the theory of optical soliton dynamics in optical communication systems, and has important reference value for development of future optical devices.

Acknowledgement. This work was supported by the Scientific Research Foundation of Weifang University of Science and Technology (Grant Nos. KJRC2022002 and KJRC2023035).

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