Nondegenerate Soliton Solutions of (2+1)-Dimensional Multi-Component Maccari System

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For a multi-component Maccari system with two spatial dimensions, nondegenerate one-soliton and two-soliton solutions are obtained with the bilinear method. It can be seen by drawing the spatial graphs of nondegenerate solitons that the real component of the system shows a cross-shaped structure, while the two solitons of the complex component show a multi-solitoff structure. At the same time, the asymptotic analysis of the interaction behavior of the two solitons is conducted, and it is found that under partially nondegenerate conditions, the real and complex components of the system experience elastic collision and inelastic collision, respectively.

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As one of the three major theories of nonlinear science, solitons often appear in various fields of physics.[1–11] Among them, nondegenerate soliton is one of the research hotspots in the field of nonlinear physics in recent years, and some meaningful works have been carried out. For instance, researchers used the bilinear method to study a Manakov system, a coupled nonlinear Schrödinger system, and a long-wave-short-wave resonance interaction system, and then obtained the nondegenerate soliton solutions of these systems. The spatial structure and dynamic properties of nondegenerate solitons were analyzed in detail.[12–16] Qin *et al.* obtained nondegenerate bound state solitons solutions of multi-component Bose-Einstein condensates through the developed Darboux transform method, and revealed that the interaction between these bound state solitons is usually inelastic collision phenomenon.[17] Zhang and Zhang used the Riemann–Hilbert method to obtain the high-order nondegenerate soliton solution of the coupled nonlinear Schrödinger equation.[18] Yu *et al.* studied the nondegenerate soliton solution of the coupled Manakov equation with variable coefficients, and obtained different types of nondegenerate solitons with different types of spatial structures by selecting different parameter variables.[19] Hanif and Saleem explored the parity-time symmetric non-local discrete nonlinear Schrödinger equation and analyzed the dynamic properties of its degenerate and nondegenerate solutions.[20] Mo *et al.* used a deep learning algorithm to study the coupled nonlinear Schrödinger equation, and simulated nondegenerate and degenerate soliton solutions by using a pre-fixed multi-stage training algorithm.[21]

For (1+1)-dimensional equations, there are many studies on solving nondegenerate soliton solutions based on the form of bilinear equations, while for (2+1)-dimensional systems, related studies are relatively rare. Therefore, we intend to study such issues with bilinear method for (2+1)-dimensional coupled equations. In this study, we consider the following multi-component Maccari system:[22]

$$iU_t + U_{xx} + UW = 0, \tag{1}$$

$$iV_t + V_{xx} + VW = 0, \tag{2}$$

$$W_t + W_y + (|U|^2 + |V|^2)_x = 0, \tag{3}$$

where U and V are complex functions, and W is a real function. The equations are obtained by the asymptotic reduction of Fourier expansion and spatiotemporal rescaling of the Kadomtsev–Petviashvili equation,[23] and are important equations used to describe two-dimensional nonlinear waves in the fields of fluid mechanics, plasma physics, optics, and other fields.[24–26]

Above all, the system is bilinearized.[27–29] Let

$$U = \frac{g^{(1)}}{f}, \tag{4}$$

$$V = \frac{g^{(2)}}{f}, \tag{5}$$

$$W = 2(\ln f)_{xx}. \tag{6}$$

Then Eqs. (1)–(3) can be transformed into the following bilinear form:

$$(D_x^2 + iD_t)g^{(s)} \cdot f = 0, \quad s = 1, 2, \tag{7}$$

$$(D_xD_y + D_xD_t + \lambda)f \cdot f + \sum_{s=1}^{2} g^{(s)} \cdot g^{(s)^*} = 0, \tag{8}$$

where D is a bilinear symbol, the symbol $*$ represents complex conjugate, and λ is an arbitrary constant. For convenience of discussion, let $\lambda = 0$. Then, for Eqs. (7)–(8), perform power series expansions of $g^{(s)}$ and f as follows:

$$\begin{aligned} g^{(s)} &= \varepsilon g_1^{(s)} + \varepsilon^3 g_3^{(s)} + \cdots, \quad s = 1, 2, \\ f &= 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \cdots. \end{aligned} \tag{9}$$

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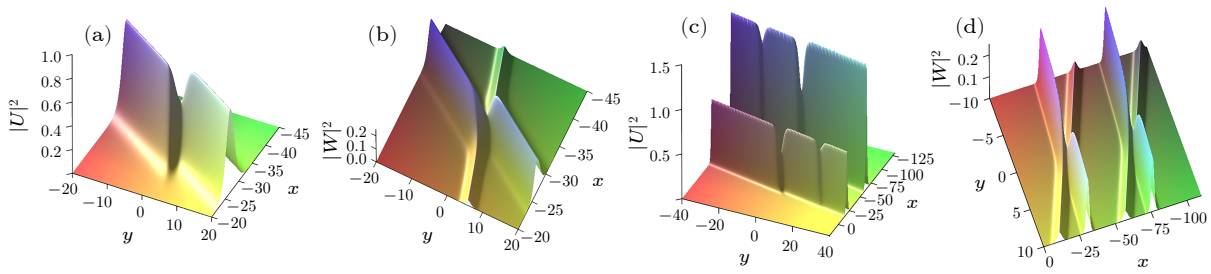

Fig. 1. Various types of nondegenerate solitons defined by Eqs. (12)–(14) and equations in the Supplementary Material for $t=-30$, $\alpha_1=0.45+0.5i$, $\alpha_2=0.45+0.55i$, $\beta_1=0.45+0.45i$, $\beta_2=0.45+0.515i$, $k_1=-0.5+0.5i$, $k_2=-0.5+1.2i$, $p_1=0.2-0.5i$, $p_2=0.2-1.2i$, $m_1=-0.32+0.5i$, $m_2=-0.32+1.2i$, $q_1=0.7-0.5i$, and $q_2=0.7-1.2i$.

In order to get the nondegenerate one-soliton solution, truncate Eq. (9) as follows:

$$\begin{aligned} &g^{(s)}=\varepsilon g_1^{(s)}+\varepsilon^3 g_3^{(s)},\quad s=1,2,\\ &f=1+\varepsilon^2 f_2+\varepsilon^4 f_4, \end{aligned} \tag{10}$$

where

$$\begin{aligned} &g_1^{(1)}=\alpha_1 e^{\eta_1},\quad g_3^{(1)}=e^{\eta_1+2\,\mathrm{Re}(\xi_1)+\Delta_1},\\ &g_1^{(2)}=\beta_1 e^{\xi_1},\quad g_3^{(2)}=e^{\xi_1+2\,\mathrm{Re}(\eta_1)+\Delta_2},\\ &f_2=e^{2\,\mathrm{Re}(\eta_1)+R_1}+e^{2\,\mathrm{Re}(\xi_1)+R_2},\\ &f_4=e^{2\,\mathrm{Re}(\eta_1+\xi_1)+R_3}, \end{aligned} \tag{11}$$

$$\begin{aligned} &\eta_1=k_1x+p_1y+w_1t,\\ &\xi_1=m_1x+q_1y+n_1t. \end{aligned} \tag{12}$$

Here, α_j, β_j, k_j, p_j, m_j, and q_j are complex constants. Then put Eqs. (10)–(12) into bilinear equations (7)–(8), collect the coefficients of $\varepsilon^i (i=1,2,\ldots)$, and make them 0, thus getting

$$\begin{aligned} &w_1=ik_1^2,\quad n_1=im_1^2,\\ &e^{\Delta_1}=\frac{\alpha_1|\beta_1|^2(k_1-m_1)}{4m_{1R}(k_1+m_1^*)(i(m_1^{*2}-m_1^2)-2q_{1R})},\\ &e^{\Delta_2}=\frac{|\alpha_1|^2\beta_1(m_1-k_1)}{4k_{1R}(m_1+k_1^*)(i(k_1^{*2}-k_1^2)-2p_{1R})},\\ &e^{R_1}=\frac{|\alpha_1|^2}{4k_{1R}(i(k_1^{*2}-k_1^2)-2p_{1R})},\\ &e^{R_2}=\frac{|\beta_1|^2}{4m_{1R}(i(m_1^{*2}-m_1^2)-2q_{1R})},\\ &e^{R_3}=\Big[|\alpha_1|^2|\beta_1|^2|k_1-m_1|^2\Big]\cdot\Big[16k_{1R}m_{1R}|k_1+m_1^*|^2\\ &\quad\times(i(k_1^{*2}-k_1^2)-2p_{1R})(i(m_1^{*2}-m_1^2)-2q_{1R})\Big]^{-1}. \end{aligned} \tag{13}$$

Finally, by letting $\varepsilon=1$ in Eq. (10) under the condition of $k_j\neq m_j$ $(j=1,2)$, the nondegenerate one-soliton solution of the system is obtained.

$$\begin{aligned} U&=\frac{\alpha_1 e^{\eta_1}+e^{\eta_1+2\,\mathrm{Re}(\xi_1)+\Delta_1}}{1+e^{2\,\mathrm{Re}(\eta_1)+R_1}+e^{2\,\mathrm{Re}(\xi_1)+R_2}+e^{2\,\mathrm{Re}(\eta_1+\xi_1)+R_3}},\\ V&=\frac{\beta_1 e^{\xi_1}+e^{\xi_1+2\,\mathrm{Re}(\eta_1)+\Delta_2}}{1+e^{2\,\mathrm{Re}(\eta_1)+R_1}+e^{2\,\mathrm{Re}(\xi_1)+R_2}+e^{2\,\mathrm{Re}(\eta_1+\xi_1)+R_3}},\\ W&=2\Big[\ln(1+e^{2\,\mathrm{Re}(\eta_1)+R_1}+e^{2\,\mathrm{Re}(\xi_1)+R_2}\\ &\quad+e^{2\,\mathrm{Re}(\eta_1+\xi_1)+R_3})\Big]_{xx}. \end{aligned} \tag{14}$$

In the same way, in order to get the two-soliton solution of this system, Eq. (9) is truncated as

$$\begin{aligned} &g^{(s)}=\varepsilon g_1^{(s)}+\varepsilon^3 g_3^{(s)}+\varepsilon^5 g_5^{(s)}+\varepsilon^7 g_7^{(s)},\quad s=1,2,\\ &f=1+\varepsilon^2 f_2+\varepsilon^4 f_4+\varepsilon^6 f_6+\varepsilon^8 f_8. \end{aligned} \tag{15}$$

Then follow the process of solving one-soliton solution mentioned above to get the nondegenerate two-soliton solution of the system (see the Supplementary Material for specific expressions). The graphs of the nondegenerate one-soliton and two-soliton of this system are drawn below.

Figures 1(a) and 1(b) and Figs. 1(c) and 1(d) represent nondegenerate one-soliton and nondegenerate two-soliton behaviors, respectively. In Fig. 1(a), the complex component U consists of a pair of solitoffs [30,31] arranged in parallel. In Fig. 1(b), the real component V presents two pairs of solitoff structure with different slopes, which we call X-type nondegenerate soliton. In Fig. 1(c), it is observed that the nondegenerate two soliton of the component U are composed of two sets of parallel three solitoffs. In Fig. 1(d), the component V presents two X-type nondegenerate solitons.

In order to study the interaction behavior between two solitons in this system under partially nondegenerate conditions, we will perform an asymptotic analysis. Firstly, some nondegenerate conditions ($k_2=m_2$, $p_2=q_2$) are introduced. Then select parameters

$$\begin{aligned} &k_{1I}=m_{1I},\quad p_{1I}=q_{1I},\quad k_{iR}<0,\quad m_{iR}<0, k_{iI}>0,\\ &m_{iI}>0,\quad p_{iR}>0,\quad q_{iR}>0,\quad p_{iI}<0,\quad q_{iI}<0,\\ &k_{1I}<k_{2I},\quad p_{1I}>p_{2I},\quad i=1,2. \end{aligned} \tag{16}$$

As a result, the wave variables η_i and ξ_i have the following asymptotic behavior. In the case of $t=\mp\infty$, for the nondegenerate soliton 1: η_{1R}, $\xi_{1R}\simeq 0$, $\eta_{2R}\to\mp\infty$, and for the degenerate soliton 2: $\eta_{2R}\simeq 0$, η_{1R}, $\xi_{1R}\to\pm\infty$. Therefore, we have the following asymptotic expression.

(i) Before collision: $t\to-\infty$

Soliton S^{1-}:

$$U\simeq\frac{e^{i\eta_{1I}}A_1^{1-}\cosh(\xi_{1R}+\Phi_1)}{a_1\cosh(\eta_{1R}+\xi_{1R}+\Phi_2)+(a_1^*)^{-1}\cosh(\eta_{1R}-\xi_{1R}+\Phi_3)}, \tag{17}$$

$$V\simeq\frac{e^{i\xi_{1I}}A_2^{1-}\cosh(\eta_{1R}+\Phi_4)}{a_2\cosh(\eta_{1R}+\xi_{1R}+\Phi_2)+(a_2^*)^{-1}\cosh(\eta_{1R}-\xi_{1R}+\Phi_3)}, \tag{18}$$

$$W \simeq \Big\{4|a_3|^2[k_{3R}^2\cosh(2\eta_{1R}+\Phi_2+\Phi_3) + k_{1R}^2\cosh(2\xi_{1R}+\Phi_2-\Phi_3)] + 2(k_{1R}+k_{3R})^2|a_3|^4 + 2(k_{1R}-k_{3R})^2\Big\}\cdot\Big\{[|a_3|^2\cosh(\eta_{1R}+\xi_{1R}+\Phi_2) + \cosh(\eta_{1R}-\xi_{1R}+\Phi_3)]^2\Big\}^{-1}, \tag{19}$$

where

$$A_1^{1-} = 2\sqrt{\frac{2\alpha_1 k_{1R}(2k_{1R}k_{1I}-p_{1R})}{\alpha_1^*}},$$
$$A_2^{1-} = 2\sqrt{\frac{2\beta_1 m_{1R}(q_{1R}-2m_{1R}m_{1I})}{\beta_1^*}},$$
$$a_1 = \sqrt{\frac{k_1^*-m_1^*}{k_1^*+m_1}},\quad a_2 = \sqrt{\frac{k_1^*-m_1^*}{k_1+m_1^*}},$$
$$a_3 = \sqrt{\frac{k_1-m_1}{k_1+m_1^*}},\quad \Phi_1 = \frac{\phi_{11}-\varrho_{11}}{2},$$
$$\Phi_2 = \frac{\rho_{11}}{2},\quad \Phi_3 = \frac{\theta_{11}-\vartheta_{11}}{2},$$
$$\Phi_4 = \frac{\gamma_{11}-\varrho_{21}}{2},\quad \varrho_{11} = \ln\alpha_1,\quad \varrho_{21} = \ln\beta_1.$$

Soliton S^{2-}:

$$U \simeq e^{i\eta_{2I}}A_1^{2-}\operatorname{sech}(\eta_{2R}+\Phi_5), \tag{20}$$
$$V \simeq e^{i\eta_{2I}}A_2^{2-}\operatorname{sech}(\eta_{2R}+\Phi_5), \tag{21}$$
$$W \simeq A_3^{2-}\operatorname{sech}(\eta_{2R}+\Phi_5)^2, \tag{22}$$

where

$$A_1^{2-} = \frac{1}{2}\exp\Big(\lambda_{11}-\frac{\ln(e^{\chi_{11}}+e^{\iota_{11}})}{2}-\frac{\rho_{11}}{2}\Big),$$
$$A_2^{2-} = \frac{1}{2}\exp\Big(v_{11}-\frac{\ln(e^{\chi_{11}}+e^{\iota_{11}})}{2}-\frac{\rho_{11}}{2}\Big),$$
$$A_3^{2-} = 2k_{2R}^2,\quad \Phi_5 = \frac{\ln(e^{\chi_{11}}+e^{\iota_{11}})}{2}-\frac{\rho_{11}}{2}.$$

(ii) After collision: $t\to\infty$

Soliton S^{1+}:

$$U \simeq \Big[a_4e^{i\eta_{1I}}\cosh(\xi_{1R}+\Phi_6)+a_5e^{i\xi_{1I}}\cosh(\eta_{1R}+\Phi_7)\Big]\cdot\Big[a_6\cosh(\eta_{1R}+\xi_{1R}+\Phi_8)+a_7\cosh(\eta_{1R}-\xi_{1R}+\Phi_9) + a_8(\cosh(\eta_{1I}-\xi_{1I})\cos\Phi_{10} - i\sinh(\eta_{1I}-\xi_{1I})\sin\Phi_{10})\Big]^{-1}, \tag{23}$$

$$V \simeq \Big[a_9e^{i\eta_{1I}}\cosh(\xi_{1R}+\Phi_{11})+a_{10}e^{i\xi_{1I}}\cosh(\eta_{1R}+\Phi_{12})\Big]\cdot\Big[a_6\cosh(\eta_{1R}+\xi_{1R}+\Phi_8)+a_7\cosh(\eta_{1R}-\xi_{1R}+\Phi_9) + a_8(\cosh(\eta_{1I}-\xi_{1I})\cos\Phi_{10} - i\sinh(\eta_{1I}-\xi_{1I})\sin\Phi_{10})\Big]^{-1}, \tag{24}$$

$$W \simeq 2\Big[\ln((e^{\chi_{11}}+e^{\iota_{11}})e^{2\,\mathrm{Re}(\eta_1+\xi_1)} + (e^{\rho_{22}}+e^{\delta_5})e^{2\,\mathrm{Re}(\eta_1)}+(e^{\sigma_{11}}+e^{\delta_6})e^{2\,\mathrm{Re}(\xi_1)} + e^{\eta_1+\xi_1^*+\xi_{21}}+e^{\eta_1^*+\xi_1+\psi_{12}}+e^{\theta_{22}}+e^{9_{22}})\Big]_{xx}, \tag{25}$$

where

$$a_4 = \sqrt{(e^{\phi_{22}}+e^{\delta_1})(e^{\kappa_{11}}+e^{\delta_7})},$$
$$a_5 = \exp(\frac{\lambda_{12}+\varphi_{12}}{2}),$$
$$a_6 = \sqrt{(e^{\chi_{11}}+e^{\iota_{11}})(e^{\theta_{22}}+e^{\vartheta_{22}})},$$
$$a_7 = \sqrt{(e^{\rho_{22}}+e^{\delta_5})(e^{\sigma_{11}}+e^{\delta_6})},$$
$$a_8 = \exp(\frac{\psi_{12}+\zeta_{21}}{2}),\quad a_9 = \exp(\frac{\upsilon_{12}+\mu_{12}}{2}),$$
$$a_{10} = \sqrt{(e^{\nu_{11}}+e^{\delta_9})(e^{\gamma_{22}}+e^{\delta_3})},$$
$$\Phi_6 = \frac{1}{2}\ln(\frac{e^{\kappa_{11}}+e^{\delta_7}}{e^{\phi_{22}}+e^{\delta_1}}),\quad \Phi_7 = \frac{\lambda_{12}-\varphi_{12}}{2},$$
$$\Phi_8 = \frac{1}{2}\ln(\frac{e^{\chi_{11}}+e^{\iota_{11}}}{e^{\theta_{22}}+e^{\vartheta_{22}}}),\quad \Phi_9 = \frac{1}{2}\ln(\frac{e^{\rho_{22}}+e^{\delta_5}}{e^{\sigma_{11}}+e^{\delta_6}}),$$
$$\Phi_{10} = \frac{\varsigma_{21}-\psi_{12}}{2},\quad \Phi_{11} = \frac{\upsilon_{12}-\mu_{12}}{2},$$
$$\Phi_{12} = \frac{1}{2}\ln(\frac{e^{\nu_{11}}+e^{\delta_9}}{e^{\delta_3}+e^{\gamma_{22}}}).$$

Soliton S^{2+}:

$$U \simeq e^{i\eta_{2I}}A_1^{2+}\operatorname{sech}(\eta_{2R}+\Phi_{13}), \tag{26}$$
$$V \simeq e^{i\eta_{2I}}A_2^{2+}\operatorname{sech}(\eta_{2R}+\Phi_{13}), \tag{27}$$
$$W \simeq A_3^{2+}\operatorname{sech}(\eta_{2R}+\Phi_{13})^2, \tag{28}$$

where

$$A_1^{2+} = \frac{1}{2}\exp(\varrho_{12}-\frac{\ln(e^{\theta_{22}}+e^{\vartheta_{22}})}{2}),$$
$$A_2^{2+} = \frac{1}{2}\exp(\varrho_{22}-\frac{\ln(e^{\theta_{22}}+e^{\vartheta_{22}})}{2}),$$
$$A_3^{2+} = 2k_{2R}^2,\quad \Phi_{13} = \frac{1}{2}\ln(e^{\theta_{22}}+e^{\vartheta_{22}}),$$
$$\varrho_{12} = \ln\alpha_2,\quad \varrho_{22} = \ln\beta_2.$$

From the above asymptotic analysis, it can be seen that in the case of $t\to+\infty$, for the asymptotic expressions (23)–(25) of soliton 1, its amplitude cannot be clearly determined. As a result, it is impossible to determine the collision type of soliton 1. Therefore, we can indirectly determine whether energy transformation occurs when soliton 1 and soliton 2 interact by analyzing the amplitude and velocity changes of soliton 2, and then confirm the collision type of the two soliton solutions when they interact.

First of all, it can be found from Eqs. (20)–(22) and (26)–(28) that the traveling velocity of soliton 2 is determined by the variable η_{2R}, which shows that the traveling velocity of soliton 2 remains unchanged before and after the interaction. Then, it is calculated that $|A_1^{2-}|^2 \neq |A_1^{2+}|^2$ and $|A_2^{2-}|^2 \neq |A_2^{2+}|^2$, which shows that the amplitudes of the components U and V for solitons 2 change before and after the collision. Furthermore, it indirectly reflects that the components U and V of solitons 1 and 2 have energy transfer respectively, so the complex components U and V of the two solitons have inelastic collisions. In the same way, for the component W, it can be found that $A_3^{2-} = A_3^{2+}$, so soliton 2 has an elastic collision, which further explains that the real component W of the two solitons has an elastic collision. Next, the interaction phenomenon of the two solitons is explored by drawing the timing diagram of the components U and W.

From Figs. 2(a)–2(c), it can be clearly found that the structure of nondegenerate soliton 1 has changed significantly before and after the interaction, that is, it changes from a parallel two-solitoff structure to an X-shaped structure. On the contrary, according to Fig. 2(d)–2(f), it can be seen that for the nondegenerate soliton 1 and degenerate soliton 2 of the component W, their shapes do not change before and after the interaction. Therefore, the component W occurs as an elastic collision. In addition, by observing Fig. 2(a), it can be seen that in partially nondegenerate conditions, the nondegenerate soliton degenerates from the three-solitoff structure to the two-solitoff structure.

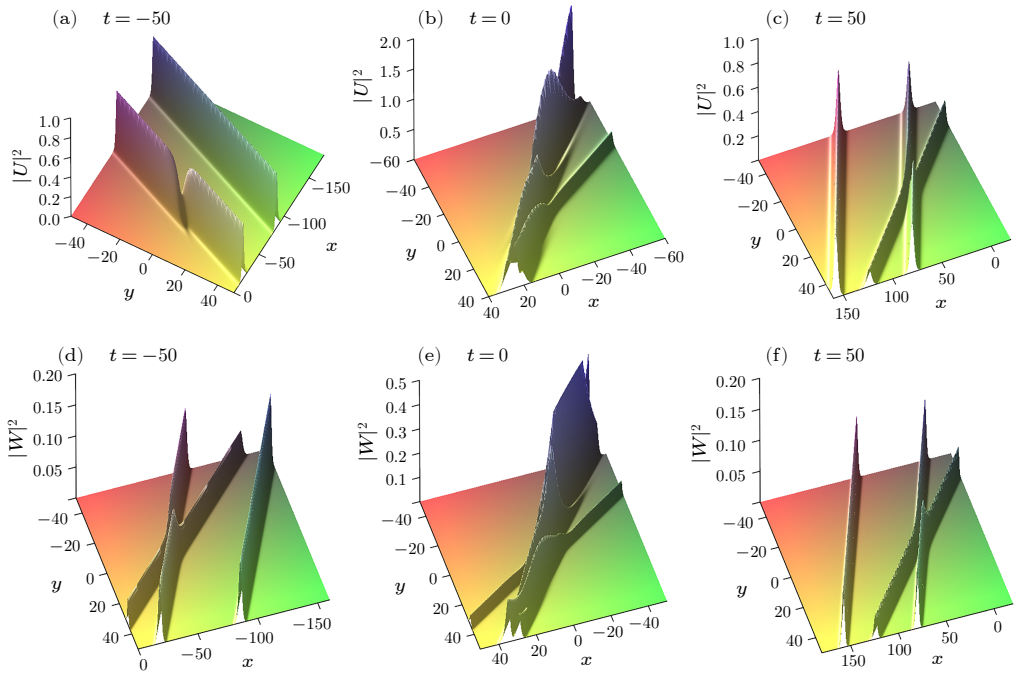

Fig. 2. Interaction between nondegenerate soliton and degenerate soliton defined by in the Supplementary Material for $\alpha_1 = 0.45 + 0.5i$, $\alpha_2 = 0.45 + 0.55i$, $\beta_1 = 0.45 + 0.45i$, $\beta_2 = 0.45 + 0.515i$, $k_1 = -0.4 + 0.5i$, $k_2 = -0.4 + 1.2i$, $p_1 = 0.315 - 0.5i$, $p_2 = 0.315 - 1.2i$, $m_1 = -0.32 + 0.5i$, and $q_1 = 0.5 - 0.5i$.

In summary, in comparison with the previous research on nondegenerate solitons of (1+1)-dimensional systems, we explore the nondegenerate soliton solutions of (2+1)-dimensional multi-component Maccari system, and then use more multiple spatial dimensions to more comprehensively describe the spatial structure of nondegenerate solitons. The nondegenerate one soliton of this system exhibits a parallel arrangement of double solitoff structure or crossed X-shaped structure in two-dimensional space. For the nondegenerate two-soliton solution with complex components, the space structure consists of two sets of three solitoffs. In addition, regarding the interaction phenomenon of two solitons in this system under partially nondegenerate conditions, although the collision types of the complex components are the same as those in literature, they are all inelastic collisions. However, elastic collisions occur for the real component.

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