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Fermionized Dual Vortex Theory for Magnetized Kagomé Spin Liquid

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Inspired by the recent quantum oscillation measurement on the kagomé lattice antiferromagnet in finite magnetic fields, we raise the question about the physical contents of the emergent fermions and the gauge fields if the $U(1)$ spin liquid is relevant for the finite-field kagomé lattice antiferromagnet. Clearly, the magnetic field is non-perturbative in this regime, and the finite-field state has no direct relation with the $U(1)$ Dirac spin liquid proposal at zero field. We here consider the fermionized dual vortex liquid state as one possible candidate theory to understand the magnetized kagomé spin liquid. Within the dual vortex theory, the S^z magnetization is the emergent $U(1)$ gauge flux, and the fermionized dual vortex is the emergent fermion. The magnetic field polarizes the spin component that modulates the $U(1)$ gauge flux for the fermionized vortices and generates the quantum oscillation. Within the mean-field theory, we discuss the gauge field correlation, the vortex–antivortex continuum and the vortex thermal Hall effect.

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Establishing the connection between the microscopic degrees of freedom and the emergent variables in the underlying framework is the key step to build up theories and understand the experimental phenomena for quantum many-body systems. This is particularly so for exotic quantum phases of matter such as quantum spin liquids. In quantum spin liquids, for example, one ought to identify the fractionalized quasiparticles and the gauge fields in the physical spin variables, as the quantum spin liquids are described by the emergent gauge theories in their deconfined phases that support the fractionalized spinon-like quasiparticles. These connections could provide useful mutual feedback between theories and experiments.

Quite recently, there is some experimental progress on the kagomé lattice spin liquid candidate material YCu_3Br.[1] A 1/9 magnetization plateau was observed in the magnetic field, and sets of oscillation emerges in the vicinity of this plateau. This was understood from the response of the fermionic spinons to the emergent $U(1)$ gauge field. In fact, quantum oscillation of spin liquids has been proposed for a long time by Lesik Motrunich.[2] The system in Motrunich's analysis is in the weak Mott regime and has little resemblance with the strong Mott insulating kagomé lattice antiferromagnet. Crudely speaking, the emergent fermionic spinon in the weak Mott regime is not very far from the physical electron. Microscopically, the strong charge fluctuation generates the ring exchange that traps the external magnetic flux and then induces the internal $U(1)$ gauge flux via the scalar spin chirality $(\boldsymbol{S}_i \times \boldsymbol{S}_j) \cdot \boldsymbol{S}_k$. This induction mechanism is the physical origin of quantum oscillation for the weak Mott insulating spin liquid. In the kagomé lattice spin liquid materials that are in the strong Mott regime, Motrunich's mechanism does not apply. In the strong Mott regime, if one still adopts the usual fermionic parton/spinon construction and relates the scalar spin chirality to the $U(1)$ gauge flux for the spinon, the external field does not directly induce the internal gauge flux via the simple Zeeman coupling. The brute-force attack is to examine the spinon-gauge-coupled theory and to analyze the energetics for the spin model by varying the magnetic fields, and then to check if the background $U(1)$ gauge flux is modified in such a way to generate the quantum oscillation.

Instead of invoking the involved energetics with the fermionic parton construction for the spontaneous internal flux generation where the actual spin model is *not* quite clear for the material, we return to the origin. From the experiments, the quantum oscillation was not observed near the zero field where the most attention was drawn and the spin liquid was proposed.[3–5] Thus, it is reasonable to expect that the zero-field spin liquid and the spin liquid near the 1/9 magnetization plateau[6] are different states. The magnetic fields are non-perturbative near the plateau, and the spin symmetry of the Heisenberg model breaks down to $U(1)$. It is tempting to think that a different theoretical framework and description from the slave-fermion construction may be needed for the spin liquid near the 1/9 plateau. The difference would bring a different relation between the physical spin variables with the fractionalized quasiparticles and gauge fields.

Around the 1/9 plateau, the spin component S^z is magnetized. From the internal perspective of the emergent spinon-gauge theory, a finite and varying S^z should be responsible for the generation of the internal $U(1)$ gauge flux. Thus, it is tempting for us to regard S^z to be directly related to the internal gauge flux for the emergent

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fermionic matter. Due to the $U(1)$ symmetry here, it is convenient to think the spin variables in terms of the hardcore bosons where S^z is related to the boson density. If S^z is interpreted as the $U(1)$ gauge flux, from the traditional formulation of the boson-vortex duality, the boson density, i.e., S^z directly serves as the dual $U(1)$ gauge flux for the vortices.[7] Due to the gauge fluctuations, the vortices are interacting with the logarithmic repulsion. With the frustrated spin interaction in the transverse components, the vortices are at half-filling.[8] Following the existing arguments by Fisher and his collaborators,[8–12] one can infer that the strong vortex interaction and the finite vortex density suppress the density fluctuations of the vortices such that the vortex exchange statistics becomes less important. Given the fermionic quantum oscillatory phenomena in experiments, one naturally thinks the vortices as fermions. Formally, one can perform an exact statistical transmission by attach the 2π flux tubes to the bosonic vortex.[8] This exact formulated theory is the starting point for the mean-field theory that describes the low-energy fermionic vortices coupled to the dual $U(1)$ gauge field. In this Letter, we build up a fermionized dual vortex theory as a possible understanding for the magnetized kagomé spin liquid. In this theoretical description, the emergent fermions are the fermionized vortices, and the emergent gauge field is the dual $U(1)$ gauge field. The purpose of this work is to make the specific connection between the recent puzzling quantum oscillation experiments and the fermionized dual vortex theory, and further identify the physical properties within the dual vortex theory.

Duality Formulation for the Spin Model. To formulate the theoretical description, we start from the antiferromagnetic spin model on the kagomé lattice in the external magnetic field,

$$H = \sum_{ij}\Big[J_{z,ij}S_i^zS_j^z + \frac{1}{2}J_{\perp,ij}(S_i^+S_j^- + S_i^-S_j^+)\Big] - \sum_i h\,S_i^z. \tag{1}$$

To keep the model a bit more general, we set the the spin model as the generic spin-1/2 XXZ model in the magnetic field for which the nearest-neighbor Heisenberg model is included as the limited case. As the theory in this work is not a study of the energetics, we simply make sure the model retains the required symmetries, especially the $U(1)$ symmetry, for the construction purpose. In the hardcore boson representation for the spin variable, one sets

$$S_i^z = n_i - \frac{1}{2},\quad S_i^+ = b_i^\dagger,\quad S_i^- = b_i, \tag{2}$$

where n_i is the boson density and $b_i^\dagger$ (b_i) is the creation (annihilation) operator for the boson. Due to the hardcore constraint, $n_i = 0, 1$ for the spin-1/2 variables. The spin model now becomes

$$H = \sum_{ij}\Big[J_{z,ij}\Big(n_i - \frac{1}{2}\Big)\Big(n_j - \frac{1}{2}\Big) + \frac{J_{\perp,ij}}{2}(b_i^\dagger b_j + \text{h.c.})\Big] - \sum h\Big(n_i - \frac{1}{2}\Big). \tag{3}$$

It is more convenient to replace the hardcore bosons with the bosonic rotors by relaxing the hardcore constraint and introducing the onsite Hubbard-U interaction to suppress the boson density. In the bosonic rotor representation, one has $b_i \sim e^{-i\phi_i}$, and the above model is converted to

$$H = \sum_{ij} J_{\perp,ij}\cos(\phi_i - \phi_j) + \sum_i U(n_i - \bar{n})^2 + \sum_{ij} J_{z,ij}(n_i - \bar{n})(n_j - \bar{n}) + \cdots, \tag{4}$$

where the phase variable ϕ_i is conjugate to the boson density n_i with $[\phi_i, n_j] = +i\delta_{ij}$. The onsite interaction U is a strong interaction that is introduced to implement the Hilbert space constraint for the hardcore bosons to enforce the selection of $n_i = 0, 1$. For the zero-magnetization state with $\langle S^z\rangle = 0$, the boson is at half-filling $\bar{n} = 1/2$. For the 1/9-magnetization plateau state that is of interest in this work, we have $\bar{n} = 1/2 + \langle S^z\rangle = 5/9$.

The procedure of the boson-vortex duality is quite standard. To manifest the vortex degrees of freedom explicitly, one performs the duality transformation. The resulting model describes the mobile vortices hop on the lattice sites ($\boldsymbol{r}$, $\boldsymbol{r}'$) of the dual lattice in the background of the fluctuating $U(1)$ gauge field $a_{\boldsymbol{r}\boldsymbol{r}'}$. The background $U(1)$ gauge flux, which is experienced by the vortices, arises from the boson density and/or the magnetization, i.e.,

$$S_i^z \sim n_i \sim \frac{1}{2\pi}(\Delta \times a)_i. \tag{5}$$

The above linear relation between the magnetization and the internal gauge flux encodes the underlying reason for the quantum oscillation within this framework that has been argued in the introduction and will be further discussed in the following. A similar linear gauge-flux induction and the resulting flux-matter coupling[13,14] have been mentioned in the context of the pyrochlore quantum ice $U(1)$ spin liquid except that the matter fields over there are bosonic.[15] No quantum oscillation, however, is expected for the pyrochlore $U(1)$ spin liquid with the bosonic matters.

The dual lattice of the kagomé lattice is a dice lattice and contains three sublattices with different coordination numbers. From the previous experiences,[11] the dual vortex theory on the dice lattice is in a bit of difficulty to deal with when the fermionization procedure is introduced. Instead, a useful trick that circumvents the difficulty is to introduce the integer spin moments with $\langle S^z\rangle = 0$ in the centers of the hexagonal plaquettes.[11] As shown in Fig. 2, these auxiliary integer moments interact with the nearby spins with the weak antiferromagnetic interactions, $J'_\perp$ and J'_z, on the dashed bonds. The hardcore boson mapping on these auxiliary sites yields $n_i = S_i^z$ and $\bar{n}_i = \langle S_i^z\rangle = 0$. This boson-vortex duality then yields the dual vortex model on the dual honeycomb lattice. The dual Hamiltonian on the dual honeycomb lattice now has the following form:

$$H_{\text{dual}} = -\sum_{\langle \boldsymbol{r}\boldsymbol{r}'\rangle} t_{\boldsymbol{r}\boldsymbol{r}'}\cos(\theta_{\boldsymbol{r}} - \theta_{\boldsymbol{r}'} - \bar{a}_{\boldsymbol{r}\boldsymbol{r}'} - a_{\boldsymbol{r}\boldsymbol{r}'})$$

$$+\sum_{\boldsymbol{rr}'}\left(N_{\boldsymbol{r}}-\frac{1}{2}\right)V_{\boldsymbol{rr}'}\left(N_{\boldsymbol{r}'}-\frac{1}{2}\right)$$
$$+\sum_{\boldsymbol{rr}'}2\pi^2 J_{\boldsymbol{rr}'}e^2_{\boldsymbol{rr}'}+\sum_{\boldsymbol{r}}\frac{U}{(2\pi)^2}(\Delta\times a)^2_{\boldsymbol{r}}. \quad (6)$$

Here $\boldsymbol{r}$ and $\boldsymbol{r}'$ refer to the sites of the dual honeycomb lattice in Fig. 1. The phase variable $\theta_{\boldsymbol{r}}$ is conjugate with the vortex density $N_{\boldsymbol{r}}$ for

$$[\theta_{\boldsymbol{r}},\, N_{\boldsymbol{r}'}]=i\delta_{\boldsymbol{rr}'}, \quad (7)$$

such that $e^{\pm i\theta_{\boldsymbol{r}}}$ creates/annihilates the vortex at the lattice $\boldsymbol{r}$. When the vortices hop on the dual lattice, they are coupled to the underlying $U(1)$ gauge field $a_{\boldsymbol{rr}'}$ that is defined on the link of the dual lattice. In the first line of Eq. 6, $a_{\boldsymbol{rr}'}$ is the fluctuating piece of the gauge field, while $\bar{a}_{\boldsymbol{rr}'}$ is the background gauge field to take care of the finite boson density. The second line of Eq. 6 describes the interaction between the vortices from the gauge fluctuations. The third line of Eq. 6 describes the standard Maxwell's terms for the $U(1)$ lattice gauge theory, where the variable $e_{\boldsymbol{rr}'}$ defines the "electric field" for the gauge field on the link. The coupling $J_{\boldsymbol{rr}'}$ is given by $J_{\boldsymbol{rr}'}=J_\perp(J'_\perp)$ such that the link $\boldsymbol{rr}'$ on the dual honeycomb lattice crosses the original triangular lattice link $\langle ij\rangle$ with $J_{\perp,ij}=J_\perp$ $(J'_\perp)$.

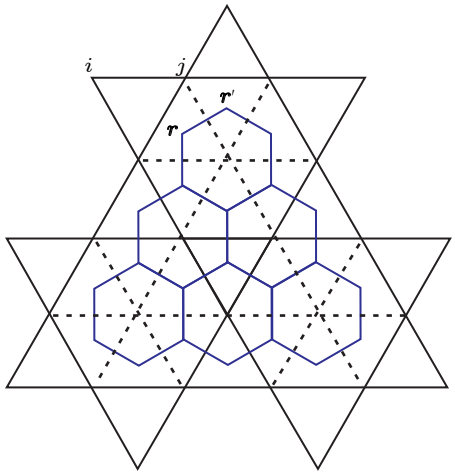

Fig. 1. The interpolated lattice between the kagomé lattice and the triangular lattice. When the interactions on the dashed bonds are tuned to zero, the system becomes a kagomé lattice. Otherwise, it is a triangular lattice. The introduced sites are referred as the auxiliary sites. To distinguish the interactions on the solid bonds, the exchange couplings on the dashed bonds are referred as $J'_\perp$ and J'_z. The dual lattice of the triangular lattice is a honeycomb lattice (in blue).

Due to the inhomogeneous boson distribution on the triangular lattice in Fig. 1, the background $U(1)$ gauge flux also receives an inhomogeneous distribution with

$$(\Delta\times\bar{a})_i\equiv\sum_{\langle\boldsymbol{rr}'\rangle_i}a_{\boldsymbol{rr}'}=2\pi\bar{n}_i$$
$$=\begin{cases}\frac{10\pi}{9}, & i\in\text{ kagomé lattice},\\ 0, & i\in\text{ auxiliary sites},\end{cases} \quad (8)$$

where $\langle\boldsymbol{rr}'\rangle_i$ refers to the dual link around the original lattice site i. The $U(1)$ gauge flux distribution for the vortices is depicted in Fig. 2(a), and a gauge choice is given in Fig. 2(b). Moreover, the inhomogeneous spin exchange also transfers to the vortex hopping on the dual lattice such that one would crudely have $t'>t$ because $J'<J$ allows an easier vortex tunneling where t' is the vortex hopping on the bonds of the blue hexagon and t is the hopping on the remaining bonds, and this relation should extend to the ideal kagomé limit with $J'=0$. So far, the vortices are bosonic, and are strongly interacting. Due to the geometric frustration of the spin model, the vortices are at a finite density with a half filling per site.

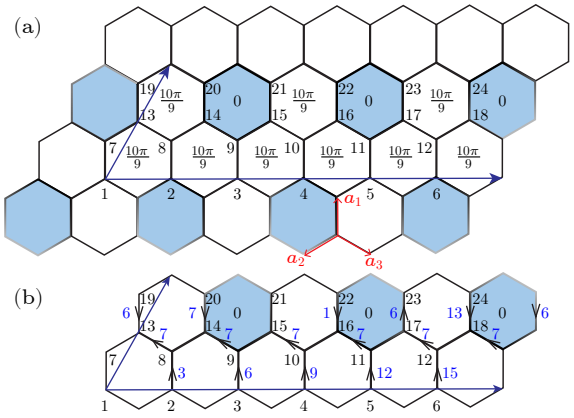

Fig. 2. (a) The $U(1)$ gauge flux distribution on the dual honeycomb lattice. The blue arrows define the enlarged unit cell once the gauge fixing procedure is implemented. The blank (blue) hexagon refers to $10\pi/9$ (0) background $U(1)$ gauge flux. The numbers 1, 2, ..., 24 are the sublattice indices after the gauge fixing. The nearest-neighbor vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ are set to unity. (b) The background $U(1)$ phase $\bar{a}_{ij}$ of each bond is given as $2\pi m_{ij}/18$, where m_{ij} is the blue integer on the bond ij in the plot, and the arrow is the direction of gauge link. The unmarked bonds have $m_{ij}=0$.

Fermionized Vortex Theory. The boson-vortex duality resolves the internal $U(1)$ gauge flux generation via the magnetization, but has not resolved the emergence of the fermions for the quantum oscillation. Here we aim at performing the fermionized vortex treatment based on the existing argument due to the strong vortex interaction. This procedure amounts to attaching 2π $U(1)$ gauge flux to each vortex via the Chern–Simons gauge field A with the condition

$$(\Delta\times A)_{\boldsymbol{r}}=2\pi N_{\boldsymbol{r}}. \quad (9)$$

With the attached flux, the vortex becomes fermionic. The resulting kinetic part of the fermionized vortices is then given as

$$H_{\text{ferm}}=-\sum_{\langle\boldsymbol{rr}'\rangle}t_{\boldsymbol{rr}'}d^\dagger_{\boldsymbol{r}'}d_{\boldsymbol{r}}e^{-i\bar{a}_{\boldsymbol{rr}'}-ia_{\boldsymbol{rr}'}-iA_{\boldsymbol{rr}'}}, \quad (10)$$

where the fermion operator $d^\dagger_{\boldsymbol{r}}$ $(d_{\boldsymbol{r}})$ creates (annihilates) a fermionized vortex at the site $\boldsymbol{r}$. The fermionized vortex is minimally coupled to the dual $U(1)$ gauge field via the

duality transformation and the Chern–Simons gauge field A via the flux attachment. The formulation is exact at this stage.

To analyze the interacting fermionized vortex theory, we consider a mean-field or saddle-point configuration for the $U(1)$ gauge fields and the Chern–Simons gauge fields. It is straightforward to incorporate the background $U(1)$ gauge flux via $\bar{a}_{\boldsymbol{rr}'}$ in Fig. 2(b). The attached 2π flux to each vortices is incorporated through a flux-smearing procedure.[10] One first smears the attached 2π gauge flux of the vortex on the dual lattice to the neighboring three hexagons. The total smeared flux from all six sites on each honeycomb turns out to be 0 (mod 2π). One can then set $A_{\boldsymbol{rr}'} \to 0$ in the flux-smeared mean-field treatment. The mean-field theory for the fermionized vortices is now written as

$$H_{\text{ferm,MF}} = -\sum_{\langle \boldsymbol{rr}' \rangle} t_{\boldsymbol{rr}'} d^{\dagger}_{\boldsymbol{r}'} d_{\boldsymbol{r}} e^{-i\bar{a}_{\boldsymbol{rr}'}}. \tag{11}$$

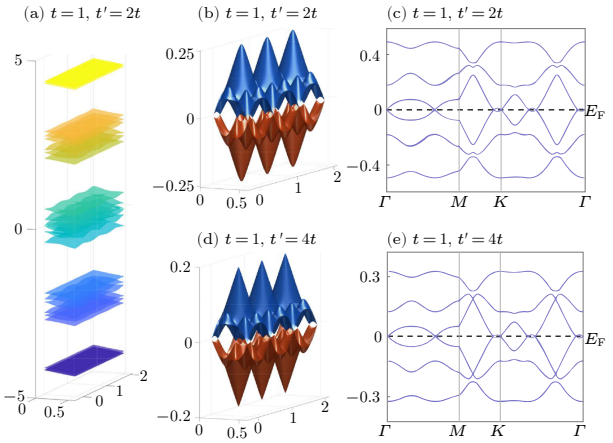

Fig. 3. The band structure of the fermionized vortices. (a) All the bands for the fermionized vortices where we choose with $t' = 2t$. [(b), (d)] The Dirac band touchings at the Fermi energy for $t' = 2t$ and $4t$. [(c), (e)] The bands near the Fermi energy for $t' = 2t$ and $4t$ along the Γ–M–K–Γ momentum line in the Brillouin zone.

With the enlarged magnetic unit cell due to the background gauge flux in Fig. 2, there exist 24 bands, $\omega_\mu(\boldsymbol{k})$ ($\mu = 1, 2, \ldots, 24$), for the dual vortices in the reduced Brillouin zone. At the half filling, the Fermi surface occurs at a few discrete Dirac points that are depicted in Figs. 3 and 4. The vortex bands near the Fermi level are further plotted along the Γ–M–K–Γ momentum lines in the reciprocal space in Fig. 3. Due to the deconfinement and the fractionalization, the single vortex excitation is not detectable in the spectroscopic measurements. Instead, the gapless Dirac vortex excitations give rise to a T^2 specific heat and contribute to the thermal conductivity κ_{xx}. Via the disorder scattering,[16] the Dirac vortices are expected to give a finite κ_{xx}/T at low temperatures.

Dynamic Spin Structure Factors. As mentioned above, the vortex band cannot be directly measured alone. In a spectroscopic measurement such as NMR and inelastic neutron scattering that will be discussed below, the vortices appear as the vortex–antivortex pair excitations in the spectrum. Due to the emergent gapless Dirac fermions near the Fermi level, it is illuminating for us to compute the the vortex–antivortex continuum as the physical observable first. Later on, we will connect with the spin correlation and the inelastic neutron scattering measurement.

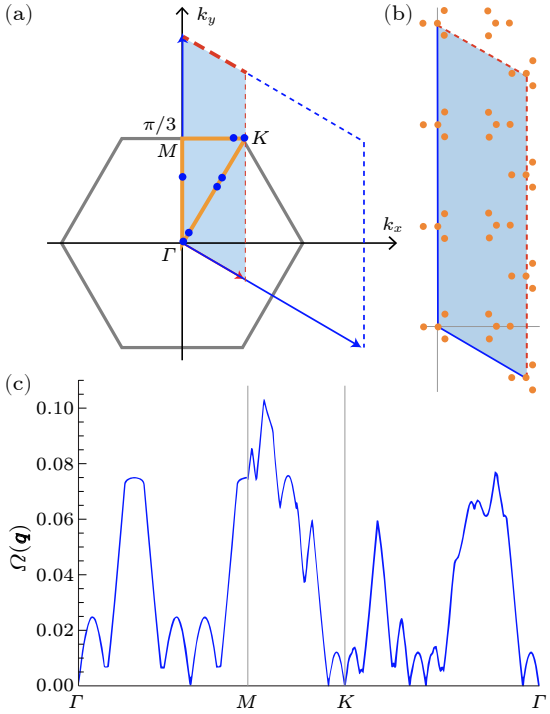

Fig. 4. (a) The Brillouin zone of the kagomé lattice and the reduced Brillouin zone (in blue) of the dual honeycomb lattice with the gauge choice in Fig. 2(b). (b) The enlarged version of the reduced Brillouin zone of (a). The oranges dots are the vortex Dirac points at the Fermi level. (c) The lower excitation edges of the vortex–antivortex continuum along Γ–M–K–Γ for $t' = 2t$, computed for 120×180 momentum points in the reduced Brillouin zone. The energy unit is t. The blue points along Γ–M–K–Γ in (a) are the gapless points in (c).

We first reveal the lower excitation edges $\Omega(\boldsymbol{q})$ of the vortex–antivortex continuum as

$$\Omega(\boldsymbol{q}) = \min_{\boldsymbol{k}}[\omega_\mu(\boldsymbol{k}+\boldsymbol{q}) - \omega_\nu(\boldsymbol{k})], \tag{12}$$

where $\omega_\mu(\boldsymbol{k})$ ($\omega_\nu(\boldsymbol{k})$) is the unfilled (filled) vortex dispersion. In Fig. 4, we plot the lower excitation edge of the vortex–antivortex continuum for $t' = 2t$. We have computed other parameters, and the structures are not quite different except the overall energy scales. The inter- and intra-Dirac-cone scattering events are most visible as the gapless excitations at various momentum points that are marked along the path Γ–M–K–Γ in Fig. 4(a). These gapless modes are important information for the experimental or numerical identification.

We return to the spin correlation functions that are directly measured in the inelastic neutron scattering experiments. The S^z operator not only relates to the internal gauge flux, but also has the contribution from the

vortex loop currents. The former indicates that the S^z–S^z correlation contains an important piece from the $U(1)$ gauge field correlation. Despite this useful relation, the gauge field ($\Delta \times a$) correlation suffers from the usual suppression of intensity at low energies.[11,17] As a comparison, for the scalar spin chirality description of the gauge flux in the slave-fermion construction, the S^z–S^z correlation cannot be directly related to the gauge field fluctuations. With the assistance from the Dzyaloshinskii–Moriya interaction, there can be a piece of gauge field fluctuation in the S^z–S^z correlation, but is further suppressed by $\sim \mathcal{O}(D_z/J)^2$, where D_z is the strength of the out-of-plane Dzyaloshinskii–Moriya interaction. The latter contribution from the vortex loop current to S^z indicates the presence of the vortex–antivortex continuum in the S^z–S^z correlation. This connection, together with the density of states for the vortex–antivortex pair, should in principle provide the information about the spectral intensity of the vortex continuum in the momentum space. Here, we examine the positions of the gapless excitations and other structures from the lower excitation edge of the S^z–S^z dynamic spin structure factor. This is likely to be the minimal spectral information that a scattering experiment can provide.

The S^+ operator changes the S^z quantum number and thus creates 2π gauge flux. Classically, with the introduced 2π background flux, each Dirac fermion has the quasi-localized zero-energy state near the introduced flux at the zero energy limit. A detailed symmetry analysis is required to establish the relation between S^+ and the monopole insertion operators in the low-energy field theory together with their momenta,[10] and this may be studied in the future.

Discussion. The Dirac spectrum of the fermionized vortices immediately leads to the prediction of specific heat $C_v \sim T^2$, and the thermal conductivity $\kappa \sim T$ at low temperatures.[18] As the variation of the internal $U(1)$ flux also generates the Berry curvature distribution of the relevant quasiparticles, quantum oscillation often implies the existence of thermal Hall effect, though the reverse is not true.[19] Here, due to the fractional flux (especially when the flux is varied), the vortex bands would generically have a finite Berry curvature distribution and give rise to the thermal Hall effect. However, this is not supposed to be a surprising nor unique effect. What can be a useful and important indication from the thermal Hall measurement is the emergent fermionic excitations. It was suggested that κ_{xy}/T could show universal behaviors (in the temperature dependence) at the finite temperatures that are related to the statistics of the emergent excitation.[20] The fermionized vortices are the energy carriers, and if they persist to finite temperatures, such a universal behavior could be useful. It may not confirm the fermionized vortices, but confirm the emergent fermionic statistics in the strongly interacting spin systems.

With the background $U(1)$ gauge flux, the magnetic unit cell is three times of the original unit cell, and the translation symmetry is realized projectively on the fermionized vortices for the flux-smeared mean-field state. The vortex–antivortex continuum would exhibit the translation symmetry fractionalization with the spectral periodicity enhancement.[21] Unfortunately, the crystal unit cell of the underlying material YCu_3-Br[1] already exhibits tripling compared to the kagomé lattice. Thus, this is probably not a useful signature for this system, but applies to other systems and/or the numerical calculation of the spin dynamics. Nevertheless, the rich structure of the spin excitation from the fermionized vortex theory is quite useful for the further study.

About the candidate material YCu_3-Br, the candidate zero-field spin liquid is expected to be quite different from the spin liquid proposal for the kagomé lattice Heisenberg model and the herbertsmithite.[3,4] The positions of scattering intensity, which are equivalent to the K points,[22] differ from the M points expected from Dirac spin liquid of Ref. [4], algebraic vortex liquid of Ref. [11], and the minima of gapped spin excitations in $\mathbb{Z}_2$ spin liquids.[23,24] This distinction might lie in the different structures and/or different spin models of the material. The puzzle is that the Heisenberg spin model in the field gives rises to 1/9 magnetization plateau,[6] but the Heisenberg spin model at zero field does not seem to give rise to the K point scattering intensity. While our proposal in this work does not strongly depend on the model, the actual spin model for YCu_3-Br needs a revisit.

In summary, the current work is inspired from the recent quantum oscillation result in the magnetized kagomé spin liquid and attempts to propose one interpretation from the fermionized dual vortex theory. In this interpretation, the magnetization serves as the emergent gauge flux, and the emergent fermions are the fermionized vortices. This interpretation provides a different induction mechanism of the internal $U(1)$ orbital flux through the external magnetic fields from the Motrunich's weak Mott insulating case. In the future, more theoretical analysis are needed for the low-energy effective theory of the $U(1)$ Dirac vortex liquid state, and the more quantitative connection to the experiments. The proposed state in this work merely takes one step along the line of dual vortex theory and is the starting point for further analysis and mutual feedback with experiments.

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