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Experimental Road to a Charming Family of Tetraquarks... and Beyond

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Discovery of the X(3872) meson in 2003 ignited intense interest in exotic (neither $q\bar{q}$ nor qqq) hadrons, but a $c\bar{c}$ interpretation of this state was difficult to exclude. An unequivocal exotic was discovered in the $Z_c(3900)^+$ meson—a *charged* charmonium-like state. A variety of models of exotic structure have been advanced but consensus is elusive. The grand lesson from heavy quarkonia was that heavy quarks bring clarity. Thus, the recently reported triplet of all-charm tetraquark candidates—$X(6600)$, $X(6900)$, and $X(7100)$—decaying to $J/\psi\,J/\psi$ is a great boon, promising important insights. We review some history of exotics, chronicle the road to prospective all-charm tetraquarks, discuss in some detail the divergent modeling of $J/\psi\,J/\psi$ structures, and offer some inferences about them. These states form a Regge trajectory and appear to be a family of radial excitations. A reported, but unexplained, threshold excess could hint at a fourth family member. We close with a brief look at a step beyond: all-bottom tetraquarks.

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1. Introduction. The quark model of hadrons celebrates its diamond jubilee in 2024. Way back in 1964 Gell-Mann[1] and Zweig[2] famously proposed that hadrons are composed of pairs, or triplets, of particles, whimsically named "quarks". Their idea triumphed in the early 1970s, and it stands at the irrevocable core of the Standard Model of particle physics. So it was with great excitement that the new millennium opened with the discovery of the "$X(3872)$" meson[3]—a charmonium-like particle that ushered in a new era of tetra- and penta-quark states.

In the ensuing years there was a veritable flood of exotic candidates,[4] but controversy persists over the nature of their internal structure, or even if some of them might instead be dynamical artifacts of production thresholds.[5–7] A significant handicap was that, as quark states, these candidates contained light (u, d, s) quarks, and these are notoriously challenging to model because their dynamics is in the non-perturbative regime of QCD. A boon to model builders would be exotic hadrons composed solely of heavy (c, b) quarks.[8]

This dream was realized in 2020 when the LHCb collaboration reported a peak around 6900 MeV in the $J/\psi\,J/\psi$ channel[9]—a prime candidate for an all-charm tetraquark. Confirmation came from the ATLAS[10] and CMS[11] collaborations. The bounty increased in 2022 when the latter also reported structures around 6600 and 7100 MeV.[12] A heavy triplet offers a qualitative and quantitative bonus over a lone state for model testing, and thus the $J/\psi\,J/\psi$ structures are a singular aid to theorists unraveling the nature of exotic hadrons.

This article will briefly review the winding road that led to the $X(3872)$, the experimental campaign towards all-heavy tetraquarks, and finally arriving at a family of all-charm tetraquark candidates. The current data on the $J/\psi\,J/\psi$ mass spectrum are reviewed. We bring coherence to the somewhat divergent results, and offer some implications, speculations, and perhaps even conclusions. We close with some future prospects.

1.1 Pealing Back the Layers of Matter. Since humans have had awareness, we have always asked ourselves where we come from, and how we are constructed. The Chinese saying "Tao begets One, One begets Two, Two begets Three, and Three begets all things" reflects humanity's deep curiosity about the fundamental components of our existence—and foreshadows today's vision. After a few millennia of such philosophical musings from the ancient Chinese, Greek, and Indian civilizations about the fundamental stuff of our material world, this spirit was finally transformed into a scientific theory of elementary and indestructible entities by John Dalton in 1803.[13] However, in the early decades of the 20th century it was learned that Dalton's structureless and indestructible atoms had structure (Rutherford in 1911[14]) and were being destructed (Cockcroft and Walton in 1932[15]). By mid-century doubts grew that the mounting collection of elementary particles might not be so elementary (e.g., Ref. [16] for early qualms).

A new and deeper level of structure was posited by Murray Gell-Mann[1] and George Zweig[2] in 1964: the quark model. Inspired by the 1956 model of Shoichi Sakata—where hadrons were composed of elementary p, n, and Λ^0[17]—Gell-Mann and Zweig instead postulated a triplet of quarks, u, d, and s, as constituents. Mesons are composed of a quark and antiquark ($q\bar{q}$), and baryons of three quarks (qqq). Quarks were odd beasts, they have

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fractional electrical charge, and it was perplexing how no one had stumbled upon fractionally charged matter. As a measure of the incredulity: Zweig was unable to publish his paper![18] It took almost a decade for physical quarks to become orthodoxy. The tide turned with deep-inelastic e–p scattering at SLAC in 1969, providing the first direct evidence of quark-like substructure of the proton.[19] The 1970s witnessed a prolific age of discovery: the J/ψ in 1974[20,21] and new world of charm hadron spectroscopy; and likewise for bottom hadrons starting in 1977;[22] particle jets from quarks;[23] formulation of the inter-quark force, quantum chromodynamics (QCD),[24] and its asymptotic freedom.[25,26] Later decades witnessed the ever narrowing of gaps, including tracking down top, the final quark, in 1995;[27,28] and the last missing ground-state meson, the B_c^+ in 1998.[29] At the close of the 20th century the quark model of hadrons was utterly triumphant.

Thus it was quite a surprise, only a few years into the 21st century, when the $X(3872)$ meson was discovered, sparking intense interest in "exotic" hadrons, i.e., states that are neither $q\bar{q}$ nor qqq. Despite the deep skepticism over the quark model prior to the 1970s, the history of exotics predates its acceptance. In 1949 Fermi and Yang considered $N\bar{N}$ bound states as a model of the pion.[16] In 1961 the deuteron was put in a dibaryon multiplet[30] using the $SU(3)$ symmetry,[31] and there was some evidence for a Λp-state.[32] Then, Gell-Mann and Zweig did note in their 1964 papers,[1,2] that their symmetry relations also permitted tetra- and penta-quark states.

With skepticism over the reality of quarks—versus mere mathematical constructs—it seems doubly ridiculous to entertain exotics. On the other hand, disbelief in quarks meant one was not chained to its restrictions. In the mid-1960s enhancements in KN scattering pointed to +1 strangeness baryons, implying $qqqq\bar{s}$.[33] Similarly, $K\bar{K}$ states were suggested to explain a low-mass enhancement in $\bar{p}p\to K\bar{K}\pi$.[34] And theoretically, duality arguments for baryon-antibaryon scattering via meson exchanges implied, in quark language, $qq\bar{q}\bar{q}$ systems.[35]

With the embrace of quarks, and QCD, in the early 1970s, the question of exotics shifted to a dynamical one: are any manifest in an observationally meaningful way? QCD opened the door to new types of exotics: "hybrids" with valence gluons, and "glueballs" without any quarks. Constructing a bag model, Jaffe and Johnson answered positively, but also argued some known 0^{++} mesons (f_0, a_0,...) were better viewed as $qq\bar{q}\bar{q}$ systems.[36–38] Later, a $K\bar{K}$ state was invoked to explain $\pi\pi\to f_0(980)\to K\bar{K}$ data,[39]... and so it went...

The prospects for exotics ebbed and flowed. In 1978 the Particle Data Group (PDG) introduced a dibaryon mini-review in its Particle Properties[40]—but dropped it in 1992.[41] Non-$q\bar{q}$ candidates fared better, with mini-reviews starting in 1988,[42] and continuing to this day. The century closed with some light exotic candidates, but crisp clear examples eluded searchers.[43] The basic difficulty is that light mesons rapidly broaden as excited, and potential exotics mix with their ordinary counterparts. Indeed, the PDG no longer separates light exotics but subsumes them under "Scalar mesons below 1 GeV". One could conclude that there very likely *were* light exotics, but the muddled soup of light mesons left one unsatisfied by the lack of a clear smoking gun. Experimentalists and theorists alike would continue to slog away trying to unravel light exotics.

Table 1. Summary of the principal players hunting for heavy exotic hadrons.

Accelerator	Type	$\sqrt{s}$(GeV)	Running years	Detectors	Location
Tevatron	$p\bar{p}$	1800(Run-I) 1960(Run-II)	1987–1996(Run-I) 2000–2011(Run-II)	CDF, DØ	Fermilab
KEKB	e^-e^+	10.58(8.0/3.5)	1998–2010	Belle	KEK
Super-KEKB	e^-e^+	10.58(7.0/4.0)	2018–?	Belle II	KEK
PEP-II	e^-e^+	10.58(9/3.1)	1999–2008	BABAR	SLAC
BEPCII	e^-e^+	1.84–4.95	2009–2023	BESIII	IHEP
LHC	pp	>7000(Run-I) 13000(Run-II) 13600(Run-III)	2009–2013(Run-I) 2015–2018(Run-II) 2022–?(Run-III)	CMS, ATLAS, ALICE, LHCb	CERN

As we continue on to the next phase of our exotic story, we summarize in Table 1 the principal experimental actors that we will encounter.

2. Discovery and Establishment of Heavy Exotic Hadrons. The new century began with a murky picture of potential light exotics lurking among the standard mesons, but then exotics were suddenly thrown in the limelight. At the October 2002 PANIC (Osaka) conference the LEPS collaboration reported a pentaquark: four quarks plus one antiquark. They saw a peak in $\Theta^+\to nK^+$ at 1540 ± 10 MeV (we adopt the convention, unless specified otherwise, that for experimental results a single uncertainty is statistical only, whereas dual uncertainties are statistical followed by systematic)[44]—being a baryon with +1 strangeness it was manifestly exotic. The DIANA collaboration rapidly followed with a $\Theta^0\to pK_S^0$ signal at 1539 ± 2 MeV.[45] There had been spurious pentaquark claims in the past, but these new results were followed by mounting confirmations in 2003 and beyond.[46]

Then on 10 August 2003 Belle caused a stir when they posted to the arXiv server their submission to the 21ST LEPTON AND PHOTON SYMPOSIUM reporting a peak in $J/\psi\pi^+\pi^-$ at $3871.8\pm0.7\pm0.4$ MeV.[47] This did not match expectations for standard charmonia—the world of exotics

had, maybe, graduated to heavy quarks.

One could cheer 2003 as shaping up to be an *annus mirabilis* for exotics. Unfortunately, as the dust settled, and despite *many* confirmations, the Θ pentaquarks evaporated.[46,48–50] What about Belle's peak?

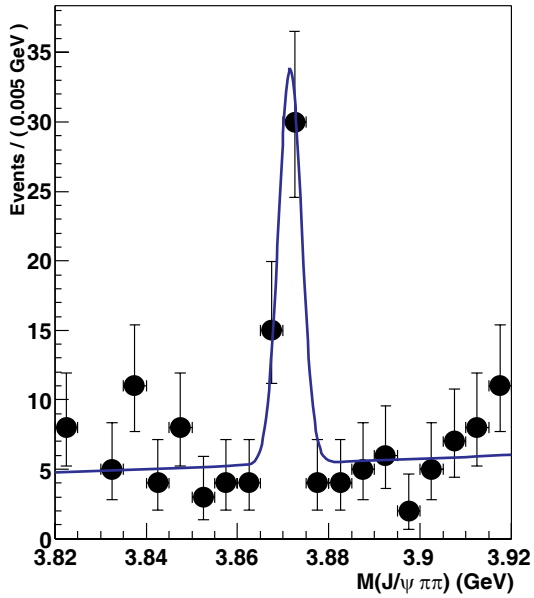

Fig. 1. Belle's $J/\psi\pi^+\pi^-$ mass distribution showing the $X(3872)$ peak.[3]

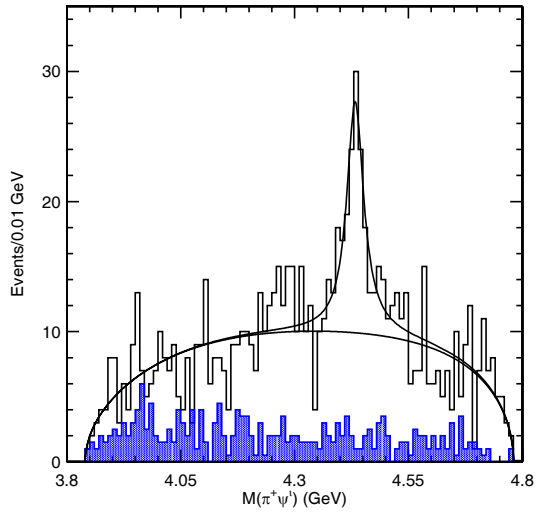

Fig. 2. The $Z_c(4430)^+$ in the $M[\psi(2S)\pi^+]$ distribution from Belle for events in the $M_{bc} - \Delta E$ signal region, where M_{bc} is the beam-constrained mass (defined as $M_{bc} = \sqrt{E^2_{\rm beam} - p^2_B}$, $E_{\rm beam}$ is the cms beam energy, and p_B is the vector sum of the cms momenta of the B decay products).[51]

2.1 Discovery of the $X(3872)$ Meson. In early 2003 a Belle analysis team was optimizing a selection of B decays to CP eigenstates, including $B \to \psi(2S)K$.[52] Fortuitously, the $J/\psi\pi^+\pi^-$ mass window was broad, and a small 3σ-ish bump around 3870 MeV was noticed. That summer a large dump of new data arrived, and the bump became an 8.6σ peak—the $X(3872)$ [Some argue a 1993 report by E705 (FNAL) of a $J/\psi\pi^+\pi^-$ peak at 3836 MeV may have been the $X(3872)$: the PDG separated the $X(3872)$ and $\psi(3836)$,[53] but later put both under the $X(3872)$.[54] E705 fired 300 GeV $\pi^\pm$'s into a ^{7}Li target, and reported 58 ± 21 candidates (local 2.8σ) at 3836 ± 13 MeV (width of 24 ± 5 MeV).[55] This seems unlikely to be the $X(3872)$. Aside from having the wrong mass—it is speculated that maybe the pion momentum scale was incorrect (but not noticed for the $\psi(2S)$)—their peak seems too large. The ratio of the raw $\psi(3836)$ yield to that of $\psi(2S)$ was 0.75 ± 0.34 ($\sqrt{s} = 24$ GeV), whereas the raw ratio from CDF's $X(3872)$ confirmation in $\bar{p}p$ ($\sqrt{s} = 2000$ GeV) was 0.100 ± 0.017.[56] (A ratio from LHCb for prompt production is $0.076 \pm 0.005 \pm 0.009$.[57]) Also, CDF's mass resolution was almost six times better than E705's, and with their resolution CDF would have had difficulty of seeing an $X(3872)$ in 2003. The evidence is against an $X(3872)$ interpretation, and with a 2.8σ local significance, there is little reason not to accept the $\psi(3836)$ as a spurious signal.]—just in time for August's Lepton-Photon symposium. Belle used 152M $B\bar{B}$ events and reported a peak of 34.4 ± 6.5 candidates in the $J/\psi\pi^+\pi^-$ channel, with a mass of $3871.8 \pm 0.7 \pm 0.4$ MeV and a width less than 3.5 MeV (Fig. 1).[47,58] However, this mass was about 60 MeV above potential model predictions for a $1D$ charmonium state. The striking "coincidence" that, within the uncertainties, the state was at the $D\bar{D}^{0*}$ threshold was highly suggestive that this might be a $D^0\bar{D}^{0*}$ "molecule".[59–62]

In contrast to the Θ's fate, CDF internally confirmed Belle's peak eight days after their arXiv posting. CDF already had a pre-selected and reconstructed Run II $J/\psi\pi^+\pi^-$ sample waiting on disk to continue an internal study of Run I data, where a weak signal around 3870 was seen in 1994. Using 220 pb^{-1} of $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV CDF found 730 ± 90 candidates (exceeding 10σ) and measured the mass to be $3871.3 \pm 0.7 \pm 0.4$ MeV. CDF presented its confirmation that September at the 2nd Workshop on Quarkonium,[63] and submitted a paper to PRL in December.[56] Confirmation papers from DØ[64] and BaBar[65] followed in May and June of 2004. All experiments were in good accord. This was an exciting discovery, but it was difficult to exclude a conventional explanation—maybe the charmonia mass predictions were off?

There was no clear consensus as to the nature of the $X(3872)$—evidence pointed in different directions:

- $\boldsymbol{m(D\bar{D}^{0*}) - m(3872)}$: It was striking that the $X(3872)$ mass was, within uncertainties, consistent with the $D\bar{D}^{0*}$ mass. Currently they agree to better than 200 keV (within 1σ),[66] pointing to a $D\bar{D}^{0*}$ molecule, or perhaps to an artifact of dynamic threshold effects.
- $\boldsymbol{m(\pi^+\pi^-)}$: Belle,[3] and later CDF,[67,68] found evidence that the dipion mass spectrum in $J/\psi\pi^+\pi^-$ comes (predominantly) from ρ mesons, whereas such isospin breaking is not natural for $c\bar{c}$ states.
- **Hadronic production:** The B-factories found the

$X(3872)$ in B decays, but hadron colliders may directly produce the particle. An immediate suspicion was that large non-B production at a hadron collider counted against an exotic interpretation.[69] In 2004 CDF found that its signal was dominantly directly produced: $16.1\pm4.9\pm2.0\%$ came from B's.[70] (Similarly, CMS found a fraction of $26.3\pm2.3\pm1.6\%$ in $\sqrt{s}=7\,\text{TeV}$ pp collisions.[71]) It was later quantified that CDF's direct $X(3872)$ production was orders of magnitude too large for a molecule.[72] But it has since been argued that the effects of $D^0\bar{D}^0$ rescattering of an S-wave threshold resonance were not accounted for, and this boosts the production by orders of magnitude, so it is claimed that a molecular interpretation can accommodate hadronic production data.[73,74]

There has been much progress since these early days. The $X(3872)$ is now seen in seven modes[66]—and some modes predicted by particular models have not been seen. It has also been observed in p-Pb[75] and Pb-Pb[76] collisions—providing a radically new arena to study its production. Over the years its J^{PC} has been whittled down to 1^{++}[77,78]—prompting its renaming to $\chi_{c1}(3872)$.[66]

LHCb found its natural width is extremely small: $0.96^{+0.19}_{-0.18}\pm0.21\,\text{MeV}$ with a Breit–Wigner lineshape,[79] or $0.22^{+0.07+0.11}_{-0.06-0.13}\,\text{MeV}$ with the Flatté formalism.[80] The lineshapes were not distinguishable by the data, but the analytic structure of the Flatté amplitude favored a quasi-bound $D\bar{D}^{0*}$, with a compact $c\bar{c}$ component $<33\%$.[79,80] This has been contested,[81,82] but the debate has moved away from either-or to what is the fraction of molecular vs compact.

2.2 A Theoretical Interlude. A fuller picture of the challenge in unravelling the nature of the $X(3872)$, and other heavy exotic candidates, may be appreciated by noting the profusion of exotic proposals. These fall into a few classes:

- Four-quark systems:
  - Meson-Meson molecules:[83,84] e.g. $X(3872)$ as loosely bound D–$\bar{D}^{0*}$ system;[59–62]
  - $c\bar{c}$ core + molecular component;[85]
  - Hadro-charmonium: compact $c\bar{c}$ embedded in an excited $q\bar{q}$ cloud;[86]
  - Tetraquarks: tightly bound $c\bar{c}q\bar{q}$ system, usually as a diquark pair cq-$\bar{c}\bar{q}$;[87–89]
- Hybrids: $c\bar{c}$ core + valance gluon;[36,90–94]
- Threshold effects: coupled-channel interactions;[5,95] threshold cusps;[6,96] triangle singularities;[6,97] and Pomeron exchange.[7]

The various four-quark options are, of course, idealizations of pure cases whereas there is really a dynamical continuum among them, and the wave function for an exotic is likely a mixture of components. A weakly bound molecular state, with some smallish compact $c\bar{c}$ component, seems to be a heavy favorite for the $X(3872)$.

Exotic speculations were triggered by the fact that the $X(3872)$ mass significantly conflicted with potential model expectations.[3] However, since those early days it has been better appreciated that these charmonium mass predictions can be substantially shifted by interaction with open flavor thresholds, and there have been continued efforts to interpret the $X(3872)$ as a conventional $c\bar{c}$ state.[98–100] Thus it has been difficult to rigorously exclude conventional interpretations.

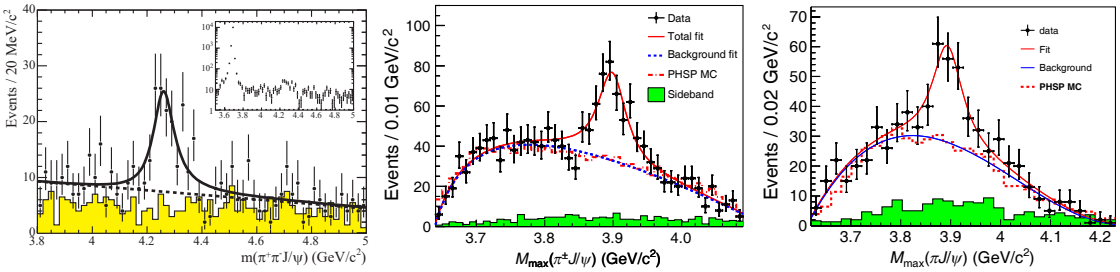

Fig. 3. The $J/\psi\pi^+\pi^-$ spectrum from BABAR showing the charmonium-like $Y(4260)$ (left).[101] The $J/\psi\pi^\pm$ spectrum from BESIII (middle),[102] and Belle (right),[103] showing the $Z_c^+(3900)$ peaks.

2.3 Establishment of Exotic Hadrons: Discovery of Charged Charmonium-Like States. Discovery of the $X(3872)$ meson opened the floodgates, and discovering exotic candidates seemed to become almost routine.[104] It was difficult to "prove" a state was exotic—unless the candidate's quantum numbers, such as its J^{PC}, are forbidden for $q\bar{q}$ or qqq systems. Charmonium-like states have one gross property prohibited in the quark model: $c\bar{c}$ states must be neutral. Nevertheless, charged charmonium-like states are not easy to come by.

An early search by BABAR in 2004 using 234 million $B\bar{B}$ decays came up empty handed.[105] In 2008 Belle laid claim to the first charged charmonium-like state in 657 million $B\bar{B}$ events with a narrow peak (6.5σ) in the $\psi(2S)\pi^+$ mass distribution from $\bar{B}^0\to\psi(2S)K^-\pi^+$ and $B^+\to\psi(2S)K^0_S\pi^+$ decays at $4433\pm4\pm2\,\text{MeV}$, with a width of $45^{+18+30}_{-13-13}\,\text{MeV}$, named $Z(4430)^+$ (Fig. 2).[51] However, BABAR investigated whether mass and angular reflections of the $K\pi^+$ system could describe the structure of the $\psi(2S)\pi^+$ mass distribution. While they could not exclude the possibility of a $Z(4430)^+$, they found no need for it.[106] Belle responded with a 2D Dalitz-plot analysis confirming their original mass but a much larger width: 4443^{+15+10}_{-12-13} and $107^{+86+74}_{-43-56}\,\text{MeV}$.[107] Then in 2013 Belle

performed a more sensitive full 4D-amplitude analysis on 2000 $B^0 \to \psi(2S)K^+\pi^-$ decays, which saw some further parameter shifts—$4485\pm22^{+28}_{-11}$ and 200^{+41+26}_{-46-35} MeV—and $J^P = 1^+$ was favored.[108] The next year LHCb confirmed Belle (also with 4D analysis, but with 25000 $\psi(2S)K^+\pi^-$ decays) with more precise values: $4475\pm7^{+15}_{-25}$ and $172 \pm 13^{+37}_{-34}$ MeV, and determined its true resonant character with an Argand plot.[109]

This fully settled the existence of the "4430" in $\psi(2S)\pi^+$, but before this state was resolved reports of a $Z_c(3930)^+$ state in $e^+e^- \to J/\psi\pi^+\pi^-$ appeared from both BESIII and Belle experiments in 2013. Years earlier BaBar saw a charmonium-like exotic candidate $Y(4260) \to J/\psi\pi^+\pi^-$ (Fig. 3, left) via initial state radiation (ISR), i.e. $e^+e^- \to \gamma_{\rm ISR} J/\psi\pi^+\pi^-$,[101] where its J^{PC} must be 1^{--}. Inspired by the suspicion of its exotic nature and J^{PC}, BESIII searched $e^+e^- \to J/\psi\pi^+\pi^-$ data at an energy of 4.26 GeV—i.e. at the Y(4260) mass. In 2013 they reported a $J/\psi\pi^+$ signal of 307 ± 48 candidates (8σ) for a peak with a mass and width of $3899.0 \pm 3.6 \pm 4.9$ and $46 \pm 10 \pm 20$ MeV (Fig. 3, center).[102] Similarly Belle found a signal of $159 \pm 47 \pm 7$ candidates ($> 5.2\sigma$) at a mass and width of $3894.5 \pm 6.6 \pm 4.5$ and $63 \pm 24 \pm 26$ MeV (Fig. 3, right).[103]

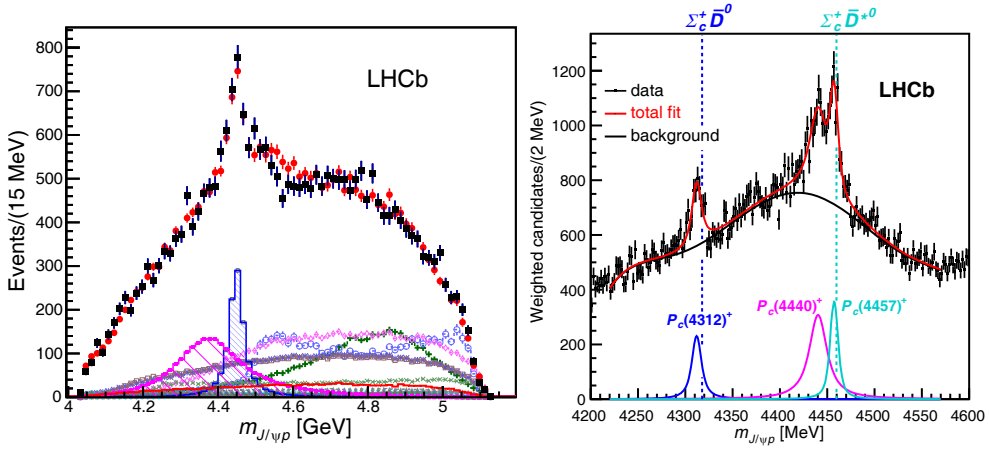

Fig. 4. Resonant structures in the $J/\psi p$ mass spectrum from LHCb. Two structures, named $P_c(4450)^+$ and $P_c(4380)^+$, were observed in a 2015 analysis with $3\,\text{fb}^{-1}$ (left).[110] A 2019 analysis with $6\,\text{fb}^{-1}$ (right) revealed that the $P_c(4450)^+$ was actually resolved into two overlapping narrow peaks, $P_c(4440)^+$ and $P_c(4457)^+$, and a new narrow state, $P_c(4312)^+$, was also seen in Ref. [111].

Now one finally had conclusive evidence for non-$q\bar{q}$ mesons, with many other charged charmonium-like states to follow: $Z_c(3900)^+$, $Z_c(4020)^+$, $Z_c(4050)^+ \ldots$; and even the bottomonium-like $Z_b(10610)^+$ and $Z_b(10650)^+$.[66]

2.4 Pentaquarks. Despite the Θ^+ fiasco, the potential for pentaquarks was a sound idea, especially in light of the discovery of the $X(3872)$ and its companions. In 2015, LHCb used $3\,\text{fb}^{-1}$ of $\Lambda_b^0 \to J/\psi K^- p$ decays, dominated by $\Lambda^* \to K^- p$ resonances, to search for $J/\psi p$ structures. Two peaks were identified at $4380 \pm 8 \pm 29$ and $4449.8 \pm 1.7 \pm 2.5$ MeV, with respective widths of $205 \pm 18 \pm 86$ and $39 \pm 5 \pm 19$ MeV (Fig. 4, left).[110] The significance of each of these P_c^+ peaks was more than 9σ.

With an increased signal efficiency for the Λ_b^0, LHCb obtained a sample of $\Lambda_b^0 \to J/\psi p K^-$ decays that was nine times larger than before. This larger sample revealed three hidden-charm pentaquark states: $P_c(4312)^+$, with a statistical significance of 7.3σ, and the previous $P_c(4450)^+$ was resolved into two narrow overlapping peaks, $P_c(4440)^+$ and $P_c(4457)^+$, with a significance of 5.4σ for the two-peak interpretation (Fig. 4, right).[111]

In 2021 LHCb reported the first evidence of a pentaquark with a strange quark, $P_{cs}(4459)^0$, in the $J/\psi\Lambda^0$ channel from an amplitude analysis of the $\Xi_b^- \to J/\psi\Lambda^0 K^-$ decay.[112] This structure had a mass of $4458.8 \pm 2.9^{+4.7}_{-1.1}$ MeV and a width of $17.3 \pm 6.5^{+8.0}_{-5.7}$ MeV. Then, in 2022, LHCb announced another strange pentaquark, named $P_{c\bar{c}s}(4338)^0$, in an amplitude analysis of the $B^- \to J/\psi\Lambda^0\bar{p}$ decay.[113] The mass and width of this candidate were measured to be $4338.2 \pm 0.7 \pm 0.4$ MeV and $7.0 \pm 1.2 \pm 1.3$ MeV, respectively.

However, LHCb could not confirm or refute the existence of the previously reported broad $P_c(4380)^+$ state. This was because the newly observed peaks were so narrow that broad structures, such as the $P_c(4380)^+$, were difficult to isolate in the new fit as they blended in with other contributions that vary more slowly across the $m(J/\psi p)$ range. The fits with and without the broad $P_c(4380)^+$ state both describe the data well, and so they could not discriminate between the two possibilities and the status of this broad structure remains open.

The states observed in the $J/\psi p$ system, namely $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$, have minimal quark content of $c\bar{c}uud$, consistent with hidden-charm pentaquarks. On the other hand, the $P_{cs}(4459)^0$ and $P_{c\bar{c}s}(4338)^0$ observed in the $J/\psi\Lambda$ have minimal quark content of $c\bar{c}uds$, indicating the presence of strangeness in this pentaquark. All candidates have, so far, only been reported by LHCb, and one looks forward to other experiments entering this arena.

Table 2. Summary of structures reported in the $J/\psi\phi$ mass spectrum. The significances are evaluated accounting for total (statistical-only) uncertainties.

Year experiment	Luminosity	Process and yield $B \to J/\psi\phi K$	Mass (MeV)	Width (MeV)	Signif.$[*\sigma]$
2009 CDF [114]	$2.7\,\text{fb}^{-1}$	58 ± 10	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$	3.8
2011 CDF [115,116]	$6.0\,\text{fb}^{-1}$	115 ± 12	$4143.4^{+2.9}_{-3.0} \pm 0.6$	$15.3^{+10.4}_{-6.1} \pm 2.5$	5.0
			$4274.4^{+8.4}_{-6.7} \pm 1.9$	$32.3^{+21.9}_{-15.3} \pm 7.6$	3.1
2012 LHCb [117]	$0.37\,\text{fb}^{-1}$	346 ± 20	4143.0 fixed	15.3 fixed	
2013 CMS [118]	$5.2\,\text{fb}^{-1}$	2480 ± 160	$4148.0 \pm 2.4 \pm 6.3$	$28^{+15}_{-11} \pm 19$	5.0
2013 DØ [119]	$10.4\,\text{fb}^{-1}$	215 ± 37	$4159.0 \pm 4.3 \pm 6.6$	$19.9 \pm 12.6^{+1.0}_{-8.0}$	3.1
2014 BaBar [120]	$422.5\,\text{fb}^{-1}$	189 ± 14	4143.4 fixed	15.3 fixed	1.6
2014 BESIII [121]		$e^+e^- \to \gamma\phi J/\psi$			
2015 DØ [122]	$10.4\,\text{fb}^{-1}$	$p\bar{p} \to J/\psi\phi + anything$	$4152.5 \pm 1.7^{+6.2}_{-5.4}$	$16.3 \pm 5.6 \pm 11.4$	4.7 (5.7)
2016 LHCb [123]	$3\,\text{fb}^{-1}$	4289 ± 151	$4146.5 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	8.4
			$4273.3 \pm 8.3^{+17.2}_{-3.6}$	$56 \pm 11^{+8}_{-11}$	6.0
			$4506 \pm 11^{+12}_{-15}$	$92 \pm^{+21}_{-20}$	6.1
			$4704 \pm 10^{+14}_{-2}$	$120 \pm 31^{+42}_{-33}$	5.6
2021 LHCb [124]	$9\,\text{fb}^{-1}$	24220 ± 170	$4118 \pm 11^{+19}_{-36}$	$162 \pm 21^{+24}_{-49}$	13 (16)
			$4146 \pm 18 \pm 33$	$135 \pm 28^{+59}_{-30}$	4.8 (8.7)
			$4294 \pm 4^{+3}_{-6}$	$53 \pm 5 \pm 5$	18 (18)
			$4474 \pm 3 \pm 3$	$77 \pm 6^{+10}_{-8}$	20 (20)
			$4626 \pm 16^{+18}_{-110}$	$174 \pm 27^{+134}_{-73}$	5.5 (5.7)
			$4684 \pm 7^{+13}_{-16}$	$126 \pm 15^{+36}_{-41}$	15 (15)
			$4694 \pm 4^{+16}_{-3}$	$87 \pm 8^{+16}_{-6}$	17 (18)

3. The Road to All-Heavy Exotic Mesons—An Intermediate Step. With unambiguous exotics in hand, and a flood of candidates, the theoretical challenge was how to understand their nature via their production, decay, and properties. Compounding the problem, some argued that (at least some of) the exotic candidates may be artifacts of dynamical effects of pair-production thresholds [5,7,125]—many exotic candidates are close to such thresholds. A basic difficulty in unraveling potential tetraquark structure is modeling systems containing light quarks. Just as the heavy charmonia spectrum brought clarity to the quark model of mesons, all-heavy tetraquarks hold similar promise.

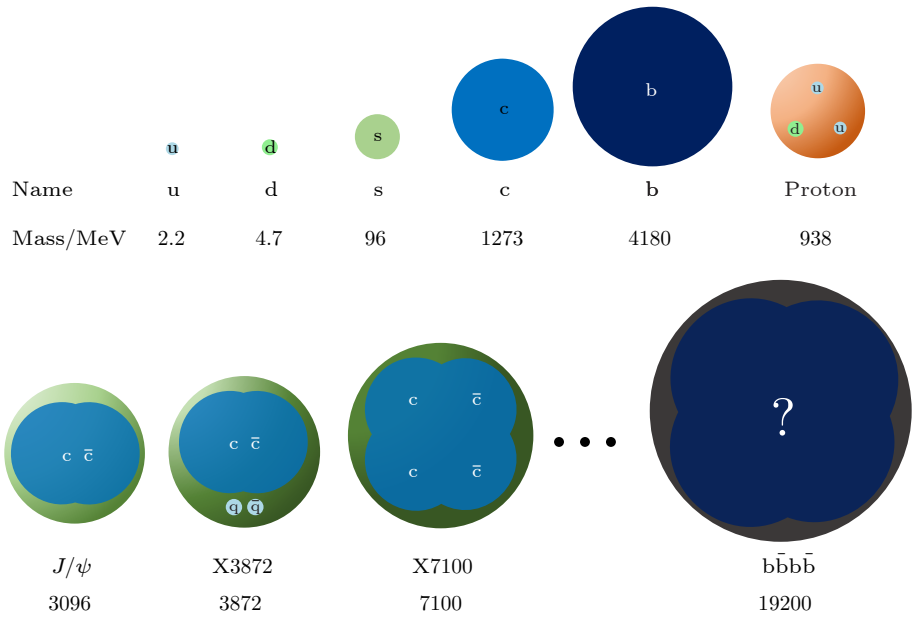

Fig. 5. Illustration of the scale of particle masses.

Figure 5 shows a schematic comparison of the masses of various quarks as sizes (except the top quark which is not spectroscopically relevant due to its rapid decay), the proton, $X(3872)$, $X(7100)$, and a hypothetical $4b$ state. Hadrons with light quarks are more complicated to model due to their highly relativistic dynamics and nonperturbative binding. This is trivially illustrated by the fact that a proton weighs 938 MeV, but its quarks collec-

tively weigh $\lesssim 10\,\text{MeV}$, i.e. the quarks inside only account for 10% of its mass. This is due to the strong quark binding and their relativistic velocities. In contrast, ≈70% of the $X(7100)$ mass is accounted for by the four quarks inside ($4 \times 1.28 = 5.1\,\text{GeV}$). An all-heavy quark structure is much simpler than a proton's.

However, hadrons with heavy quarks are more difficult to produce. The s quark is not heavy, but its mass ($\sim 100\,\text{MeV}$) is about 40 times that of the u and d quarks, and so is relatively heavy. An intermediate step is to study $c\bar{c}s\bar{s}$—a system with none of the lightest quarks—and hunt for the archetypical $J/\psi(c\bar{c})\phi(s\bar{s})$ channel. Establishing structure in $J/\psi\phi$ was somewhat convoluted—the inventory of candidates is tracked in Table 2.

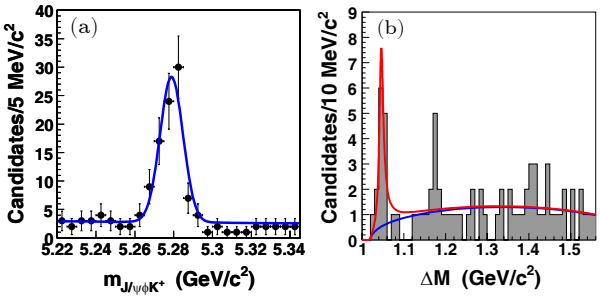

Fig. 6. The CDF reconstruction of $B^+ \to J/\psi\phi K^+$ ($\phi \to K^+K^-$), showing a signal of 75 ± 10 B's (left). The mass difference, ΔM, between $\mu^+\mu^-K^+K^-$ and $\mu^+\mu^-$, for events in the B^+ mass window (right). The peak (14 ± 5 candidates) at ΔM of $1046.3 \pm 2.9\,\text{MeV}$ translates into a mass of $4143.0 \pm 2.9 \pm 1.2\,\text{MeV}$.[114]

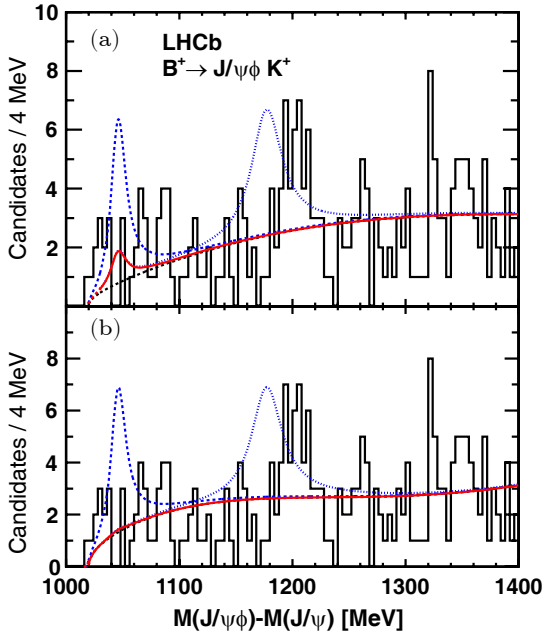

Fig. 7. The mass difference $M(J/\psi\phi) - M(J/\psi)$ in $B^+ \to J/\psi\phi K^+$ decays from LHCb. The (a) and (b) fits are different background functions.[117]

In 2009 CDF made the first advance with evidence of a $J/\psi\phi$ structure, labeled $Y(4140)$. A signal of 14 ± 5 candidates (3.8σ global signif.) was seen in $2.7\,\text{fb}^{-1}$ of $\bar{p}p$ data (Fig. 6), for a mass of $4143.0 \pm 2.9 \pm 1.2\,\text{MeV}$ and width of $11.7^{+8.3}_{-5.0} \pm 3.7\,\text{MeV}$.[114] The $Y(4140)$ is an analogue of the $X(3872)$ where a $q\bar{q}$ pair is replaced by the heavier $s\bar{s}$. Belle found no evidence for the $Y(4140)$ in $B^+ \to J/\psi\phi K^+$, but this did not contradict CDF.[126] Instead, they found evidence for $X(4350)$ in the $\gamma\gamma \to J/\psi\phi$ process.[127] In 2011, with $6\,\text{fb}^{-1}$, CDF confirmed the $Y(4140)$ with a significance of 5σ, and with parameters: $4143.4^{+2.9}_{-3.0} \pm 0.6$ and $15.3^{+10.4}_{-6.1} \pm 2.5\,\text{MeV}$.[115] A second structure, $Y(4274)$, was also identified with 22 ± 8 candidates (global significance $\approx 3.1\sigma$), with a mass and width of $4274.4^{+8.4}_{-6.7} \pm 1.9$ and $32.3^{+21.9}_{-15.3} \pm 7.6\,\text{MeV}$. However, in 2012, LHCb failed to confirm these states, their mass distributions are shown in Fig. 7.[117] These results diverged from CDF's results at the 2.4σ level, and publication of CDF's paper was delayed until 2017.[116] The situation at time was reviewed in Ref. [8].

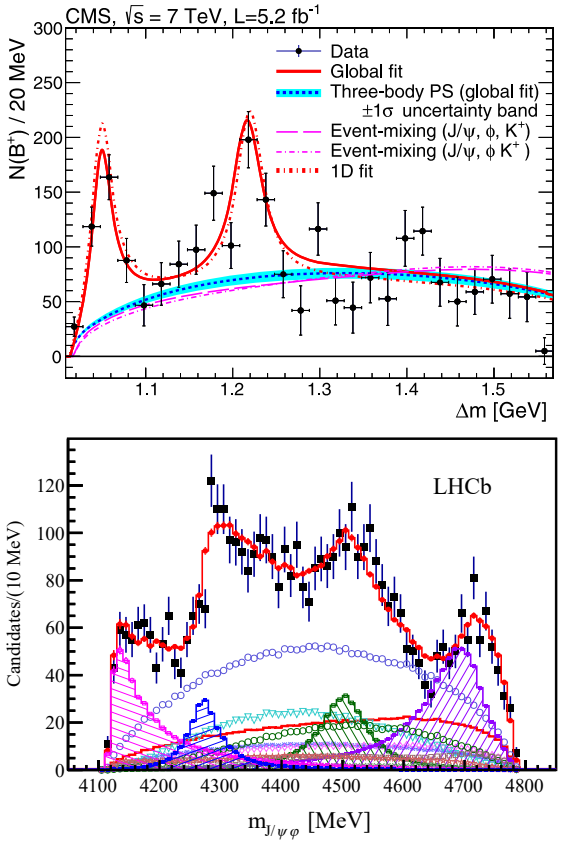

Fig. 8. The number of $J/\psi\phi$ candidates as a function of $\Delta m = m(\mu^+\mu^-K^+K^-) - m(\mu^+\mu^-)$ in CMS (top).[118] Distribution of $J/\psi\phi$ invariant mass for the $B^+ \to J/\psi\phi K^+$ data from LHCb (bottom).[123]

Resolution came in 2013, when CMS confirmed CDF's results using $5.2\,\text{fb}^{-1}$, and measured a mass and width: $4148.0 \pm 2.4 \pm 6.3$ and $28^{+15}_{-11} \pm 19\,\text{MeV}$ (Fig. 8, top).[118] Additionally, another structure was seen at a mass of $4313.8 \pm 5.3 \pm 7.3\,\text{MeV}$, but its statistical significance could not be reliably determined. In the same year, DØ offered further support for the $Y(4140)$ resonance (3.1σ) and indicated the presence of another one (47 ± 20 candidates) at a mass of $4328.5 \pm 12.0\,\text{MeV}$,[119] consistent

with a prior CMS result.[118] In contrast to these successes, BABAR searched for $J/\psi\phi$ in 2014 but found nothing significant.[120] BESIII and DØ also explored new processes for the $Y(4140)$. In 2014, BESIII searched $e^+e^- \to \gamma J/\psi\phi$ for the $Y(4140)$ at different center-of-mass energies, but no significant signal was observed.[121] In 2015, DØ searched for the inclusive production of the $Y(4140)$ in hadronic collisions and found strong evidence for its direct production.[122]

In 2016, LHCb performed an amplitude analysis of $B^+ \to J/\psi\phi K^+$, and established the $Y(4140)$ and $Y(4274)$.[123] However, the width of $Y(4140)$ was found to be substantially larger than before. The high $J/\psi\phi$ mass region was investigated for the first time with good sensitivity, revealing two significant 0^{++} resonances: $X(4500)$ and $X(4700)$; and renamed all $J/\psi\phi$ states as X states. In 2021, LHCb improved their amplitude analysis using a signal six times larger than before, providing further insights into the $Y(4140)$ and other structures.[124] The study reaffirmed the previous $X \to J/\psi\phi$ states with higher significance and confirmed quantum number assignments. An additional 1^+ state, $X(4685)$, was identified with a relatively narrow width around 125 MeV and high significance. Additionally, a novel state $X(4630)$ was unveiled with a significance of 5.5σ. This state favors a spin-1 assignment over 2 at 3σ, rejecting other J^P at 5σ. LHCb's pioneering study also made the first observation of states decaying to $J/\psi K^+$—charged states with the new quark content $c\bar{c}u\bar{s}$. We now have a large bounty of charmonium-like states with strange quarks for model builders to study.

4. Structures in the Di-Charmonium Channel. The dream of all-heavy tetraquarks may finally have turned into reality in 2020 with LHCb's observation of $X(6900) \to J/\psi\, J/\psi$ decays.[9–11] It may be surprising that consideration of such a rare beast as an all-charm tetraquark dates all the way back to Yoichi Iwasahi in 1975,[87] just months after the J/ψ discovery! This was founded on a misguided interpretation that the excited ψ states might be $c\bar{c}q\bar{q}$ tetraquarks, which logically also lead to predicting an all-charm tetraquark around 6.2 GeV. Iwasahi seemed (briefly) vindicated by interpreting the E288 (FNAL) experiment's newly discovered "Upsilon" (as known as "Oops-Leon") particle around 6 GeV[128] as a four-charm state.[129] Alas, this "Upsilon" fizzled.[130] Another early stab at $4c$ states was by Kuang-Ta Chao who, in 1981, speculated that apparent structure in the elevated cross sections seen in the e^+e^- continuum [the $R(e^+e^- \to \text{hadrons})$ ratio] over the 6–7 GeV range[131] might be due to P-wave (cc)-$(\bar{c}\bar{c})$ states.[132]

The $J/\psi\, J/\psi$ channel is a prime discovery route to four-charm states. Exploration of the di-J/ψ channel was pioneered by the NA3 experiment at CERN in 1982,[133,134] where they reported 13 $J/\psi\, J/\psi$ events (Fig. 9). A new era opened with the advent of high luminosity hadron colliders where the relentless growth of $J/\psi\, J/\psi$ samples—116 pairs for LHCb in 2012,[135] 1000 for CMS in 2014[136] and LHCb in 2017[137]—enabled studies of $J/\psi\, J/\psi$ production. CMS and LHCb had also shown crude $J/\psi\, J/\psi$ mass distributions, but the samples were still small and the binning of the spectra was too large (> 1 GeV) to be spectroscopically interesting.

Finally, in 2020, with thirty times the data, LHCb found the $X(6900)$ structure[9]—providing the first candidate for an all-heavy exotic. It was confirmed by ATLAS[10] and CMS.[11] Despite this consensus, the di-J/ψ spectrum presents a more interesting and complex situation: with two additional structures claimed by CMS, $X(6600)$ and $X(7100)$,[11] and an unexplained threshold feature. This spectrum promises more lessons.

4.1 The $J/\psi\, J/\psi$ Mass Spectrum. The $J/\psi\, J/\psi$ samples used were: 140 fb^{-1} (2015–2018) at $\sqrt{s} = 13$ TeV for ATLAS, 135 fb^{-1} (2016–2018) at $\sqrt{s} = 13$ TeV for CMS, and 9 fb^{-1} (2011–2018) at $\sqrt{s} = 7$, 8, and 13 TeV for LHCb. The integrated luminosity for LHCb is smaller than the others but it still has similar spectroscopic reach as the other two because LHCb is designed for small-angle forward physics where p_T cuts are much lower, and the production cross sections are larger. Figure 10 shows the $J/\psi\, J/\psi$ spectra from the three experiments. The same broad features are seen—with structure, of *some* sort, immediately apparent. The data rapidly rise at threshold, forming a broad "hump" peaking around 6500 MeV, then dropping into a pronounced "dip" around 6750 MeV before rising to a relatively narrow peak around 6900 MeV, and then tapering off at high masses. There is also a small sign of a possible structure around 7200 MeV in the CMS and LHCb spectra.

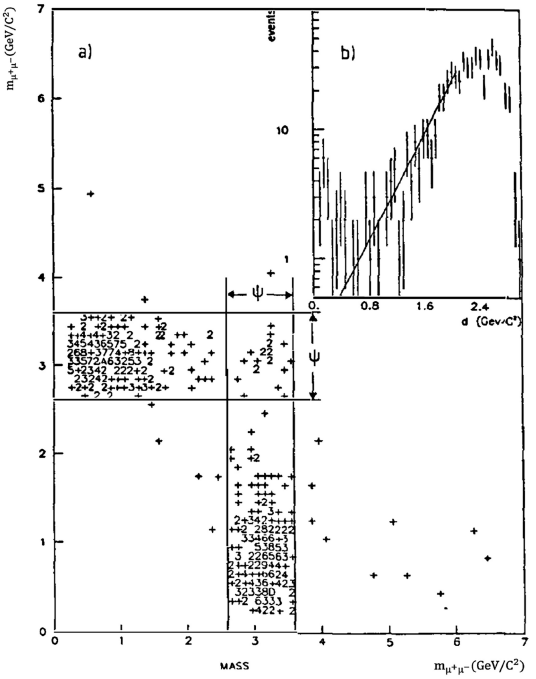

Fig. 9. Distribution of the two dimuon mass combinations in $\mu^+\mu^-\mu^+\mu^-$ events from NA3 in 1985.[134]

The main background processes are (non-resonant) single-parton scattering (NRSPS), and double-parton scattering (DPS).[138–140] The NRSPS process is the conventional single parton-parton interaction in a single proton-

proton collision. At the very high LHC luminosities, the normally rare process of two parton interactions in a single proton-proton collision can become a significant part of triggers, like $J/\psi\,J/\psi$ production. Within the fit ranges, DPS dominates at high $J/\psi\,J/\psi$ masses, and NRSPS at low masses. There is also a small "combinatoric" background where at least one J/ψ was "fake". These backgrounds are common to all three experiments, but are modeled in somewhat different ways based on a combination of Monte Carlo and data.

Constructing fit models is usually straightforward and uncontroversial. This was not the case for the $J/\psi\,J/\psi$ spectra, as each experiment offered two different principal models, resulting in three major types of incongruent fit models, and thereby offering a variety of different potential masses for roughly the same features. We use LHCb's fitting models as the starting point for our discourse, as they pioneered modeling the $J/\psi\,J/\psi$ spectrum.

4.1.1 Simple Multi-Resonance Fits. The Breit–Wigner (BW) lineshape was the function of choice to model the data—for physical resonances and as an *ad hoc* shape for features of indeterminant origin. It is clear (Fig. 10) that component(s) are needed to account for the broad "hump" around 6500 MeV, and for the 6900 MeV peak. LHCb excluded the null hypothesis by 6.0σ. They tried to describe each of these structures with a single BW (along with the backgrounds), but found a fit χ^2 probability of 1.2%.[141] The fit poorly described the threshold turn-on and high mass tail of the broad hump. LHCb also described the hump with two BW's: this three resonance fit, Model I (Fig. 11, right; Table 3),[9] was one of their two principal models. The relatively narrow peak was fit with a mass and width of $6905 \pm 11 \pm 7$ and $80 \pm 19 \pm 33$ MeV, from 252 ± 63 candidates. The significance of this signal exceeds 5σ. The fitted values of the two lower BW's were not reported, but their masses are around 6250 and 6500 MeV by visual inspection. The threshold turn-on is described well, but the "dip" around 6750 MeV was not. The fit probability was only 4.6%. LHCb also performed a supplementary four-BW fit with a BW for a small possible excess of events around 7200 MeV, but no parameters were reported as its significance was too low.[141]

ATLAS also performed a three-BW fit, but they did not publish a plot or list resonance parameters.[10] However they stated this model was excluded at a confidence level exceeding 95%. Given the similarity of spectra seen in Fig. 10, we presume the fit's low probability is mainly due to the 6750 MeV dip.

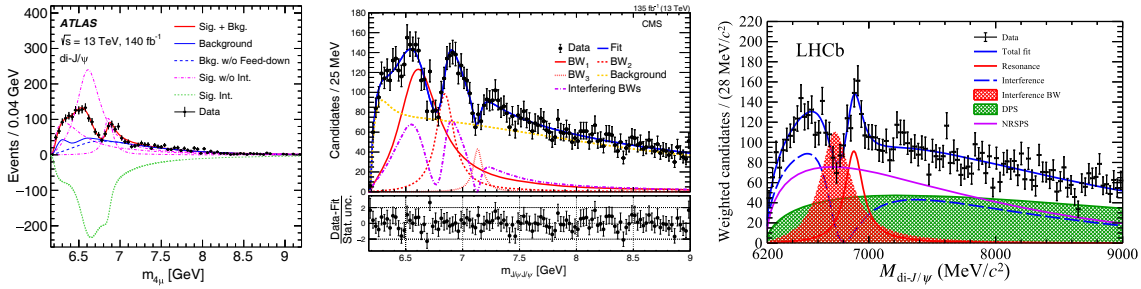

Fig. 10. The $J/\psi\,J/\psi$ mass spectra from ATLAS with Model A fit (left),[10] CMS interference fit (center),[11] and LHCb's model Ⅱ fit (right).[9] These are the experiments' interference fits, which their data prefer.

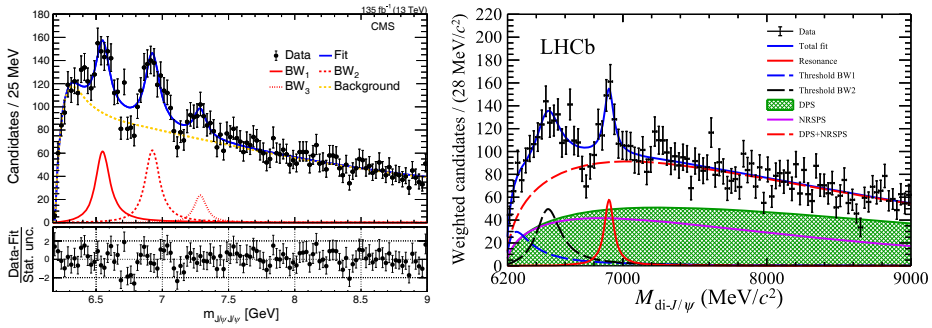

Fig. 11. The CMS[11] (left) and LHCb[9] (right) $J/\psi\,J/\psi$ spectra fit without interference (i.e., their respective "no-interf." model and Model Ⅱ).

CMS reported a four-BW fit as one of their main results: two BW's described the threshold region, and one each for the peaks around 6900 and 7200 MeV—CMS included a BW for any structure with a local significance above 3σ. This fit is listed in Table 3 as "No-Interf" (Fig. 11, left). The $X(6900)$ signal had 492^{+78}_{-73} candidates, and parameters consistent with LHCb's. The CMS threshold turns on very fast, and evidently forms a brief shoulder before rising to a peaking structure around 6600 MeV, presenting a striking pattern of a total of three peaks, which CMS called $X(6600)$, $X(6900)$, and $X(7100)$. LHCb's spectrum (Fig. 11, right) does not appear inconsistent with this, but the effect is washed-out by their slower turn-on making their spectrum more amenable to alternate interpretations. CMS's fit also had a dip discrepancy around 6750 MeV like LHCb, but there is another smaller dip discrepancy around 7150 MeV. The parameters for the threshold BW were not reported because, as discussed

later, CMS included it in the background. The fit χ^2 only for masses below 7.8 GeV yields a 9% probability. This fit cannot be dismissed statistically, but the 6750 MeV dip is the principal discrepancy.

In summary, using multi-BW models all experiments found similar fit behavior. Notably the description of the threshold hump seems to require two BW's, and the dip around 6750 MeV was poorly described across experiments. The depth of the difficulty in describing the dip may be appreciated by noting that the dip in the data descends to about the same level as the fitted background (Fig. 11): i.e. there is little room for additive contributions from neighboring BW structures. One potential way to ameliorate this problem is to use multiple narrow BW's to model individual peaking structures, so that the BW's could fall more steeply into the dip. CMS alluded to this possibility,[11] but since no such results were reported, we pursue it no further.

4.1.2 Interference with Background Fits. To produce a dip in the fit to the level of the background, LHCb invoked quantum mechanical interference. A minimal way to employ interference is to have a *single* BW interfering with the NRSPS background. It is unusual, but possible, for some NRSPS processes to have the same quantum numbers as, and maintain coherence with, a resonant structure. LHCb performed such a fit, but it only had a χ^2 probability of 2.8%.[141] Destructive interference successfully generated a dip around 6750 MeV, but the threshold hump and the large and narrow 6900 MeV peak were poorly described. More components are necessary to model their data.

Table 3. The principal results of fitting the $J/\psi\,J/\psi$ mass spectra by ATLAS, CMS, and LHCb. The experiments offer disparate models, and the parameters in one model cannot necessarily be compared to those in another. To organize the results we take a phenomenological approach and categorize reported structures which are conflicted according to the range of their mass as "Threshold" or "Intermediate" BW's, along with the unambiguous "$X(6900)$" and "$X(7100)$". Masses and widths are in MeV.

		ATLAS[10]		CMS[11]		LHCb[9]	
		Model A	Model B	No-Interf.	Interf.	Model I	Model Ⅱ
Thresh.	m	$6410 \pm 80^{+80}_{-30}$	–	Unreported	Unreported	Unreported	–
	Γ	$590 \pm 350^{+120}_{-200}$	–	Unreported	Unreported	Unreported	–
Interm.	m	$6630 \pm 50^{+80}_{-10}$	$6650 \pm 20^{+30}_{-20}$	$6652 \pm 10 \pm 12$	6638^{+43+16}_{-38-31}	Unreported	6741 ± 6[141]
	Γ	$350 \pm 110^{+110}_{-40}$	$440 \pm 50^{+60}_{-50}$	$124^{+32}_{-26} \pm 33$	$449^{+230+110}_{-200-240}$	Unreported	288 ± 16[141]
$X(6900)$	m	$6860 \pm 30^{+10}_{-20}$	$6910 \pm 10 \pm 10$	$6927 \pm 9 \pm 4$	6847^{+44+48}_{-28-20}	$6905 \pm 11 \pm 7$	$6886 \pm 11 \pm 11$
	Γ	$110 \pm 50^{+20}_{-10}$	$150 \pm 30 \pm 10$	$122^{+24}_{-21} \pm 18$	191^{+66+25}_{-49-17}	$80 \pm 19 \pm 33$	$168 \pm 33 \pm 69$
$X(7100)$	m	–	–	$7287^{+20}_{-18} \pm 5$	7134^{+48+41}_{-25-15}	–	–
	Γ	–	–	$95^{+59}_{-40} \pm 19$	97^{+40+29}_{-29-26}	–	–

LHCb's Model Ⅱ had BW's for the 6900 MeV peak and an auxiliary BW which interfered with the NRSPS. The results are shown in Fig. 10 (right), and in Table 3.[9] The auxiliary BW fits to a mass of 6741 ± 6 MeV—the location of the dip. The interference is maximal at the BW mass, and it contributes to the threshold hump by constructive interference below the peak. This gives a good description of the dip, with a fit probability of 15.5%. The $X(6900)$ does not participate in the interference, but since the "background" underneath it shifts, the resonance mass is lowered to $6886 \pm 11 \pm 11$ MeV, and the yield is increased to 784 ± 148 candidates.

ATLAS replicated LHCb's Model Ⅱ as their Model B.[10] The results are given in Table 3. The fit describes their data well, but the auxiliary BW parameters are not very consistent with LHCb's: the ATLAS mass is almost 100 MeV lower, and the width is ∼50% larger. Tension among $X(6900)$ parameters is more modest.

CMS did not adopt Model Ⅱ, but for comparisons they did fit their data with it.[11,12,142] The Model Ⅱ fit described CMS's 6750 MeV dip, and resulted in the masses and widths: 6736 ± 38 and 439 ± 6, and 6918 ± 10 and 187 ± 40 MeV. CMS's auxiliary mass matches LHCb's Model Ⅱ, but CMS's width was much broader (but it agrees with ATLAS's Model B). CMS's $X(6900)$ mass is higher than LHCb's, but agrees with ATLAS's Model B. This situation appears to arise because the threshold turn-on of CMS, and to a lesser extant ATLAS, is faster than for LHCb (compare in Fig. 10), and without a threshold BW the CMS fit must pull up the NRSPS to describe the turn-on (which is still poorly described for CMS up to 6.5 GeV: see pulls plot in Fig. 2 (right) of Ref. [12]); but then to compensate for the large NRSPS the auxiliary BW must be increased and broadened [e.g. compare the backgrounds in Fig. 2 (right) of Ref. [12] to Fig. 10 (left)]. That BW parameters are sensitive to how detectors sculpt the data indicates that the model is probably not a good physical representation.

4.1.3 Interfering Resonance Fits. Interference is able to describe the 6750 MeV dip, but interference with NRSPS is not the only option. Arguably it is more natural to arrange interference between the well-defined quantum states of resonances.

ATLAS's Model A had interference among three resonances, two for the threshold hump and one for the $X(6900)$. The results are shown in Fig. 10 (left), and listed in Table 3 (each peak $> 5\sigma$). The data is described well, and the $X(6900)$ parameters are consistent with LHCb's Model Ⅱ (NRSPS interference), and CMS (below).

CMS also adopted a multi-resonance interference fit that mirrors the structure of their "no-interf." model: a non-interfering "background" threshold BW plus three interfering BW's. The fit result is shown in Fig. 10 (center), and Table 3 under "Interf". CMS advanced three physi-

cal structures, $X(6600)$, $X(6900)$, and $X(7100)$, with local significances of 7.9σ, 9.8σ, and 4.7σ respectively. The $X(6900)$ parameters are consistent with ATLAS's Model A and LHCb's Model Ⅱ, but the latter has some tension with the CMS width. The 6750 MeV dip is well described, but the fit also favors an interference dip around 7100 MeV.

4.2 The $J/\psi\psi(2S)$ Spectrum. It is natural to suspect that sufficiently massive $J/\psi\,J/\psi$ structures will also appear in the $J/\psi\psi(2S)$ channel. Estimates of the $X(6900)$ $\to J/\psi\psi(2S)$ branching fraction relative to $J/\psi\,J/\psi$ vary from ∼10–110%.[143,144]

ATLAS searched the $\mu^+\mu^-\mu^+\mu^-$ channel and reported a substantial threshold excess, but again offered two alternate fit models.[10] In Model α a threshold BW is fixed to the $X(6900)$ parameters from their $J/\psi\,J/\psi$ data (Model A), which is added to the background (plus the tails of the two sub-threshold BW's fixed to Model A parameters), and finally, and additional BW is included to describe a possible peak around 7200 MeV. The result is shown in Fig. 12 (left). The collective significance of the excess was 4.7σ; and for the second peak only, the local significance was 3.0σ. That peak has a mass and width of $7220\pm30\pm^{+10}_{-30}$ and $90\pm60^{+60}_{-30}$ MeV. Without the 7200 MeV peak, Model β was a single BW, shown in Fig. 12 (right), with a significance of 4.3σ. The mass and width were $6960\pm50\pm30$ and $510\pm170^{+110}_{-100}$ MeV—the width is much larger than in $J/\psi\,J/\psi$.

This result is substantial evidence for a $J/\psi\psi(2S)$ excess, but ATLAS did not prefer a particular fit, and out of caution declined to attribute the excess to any particular origin.

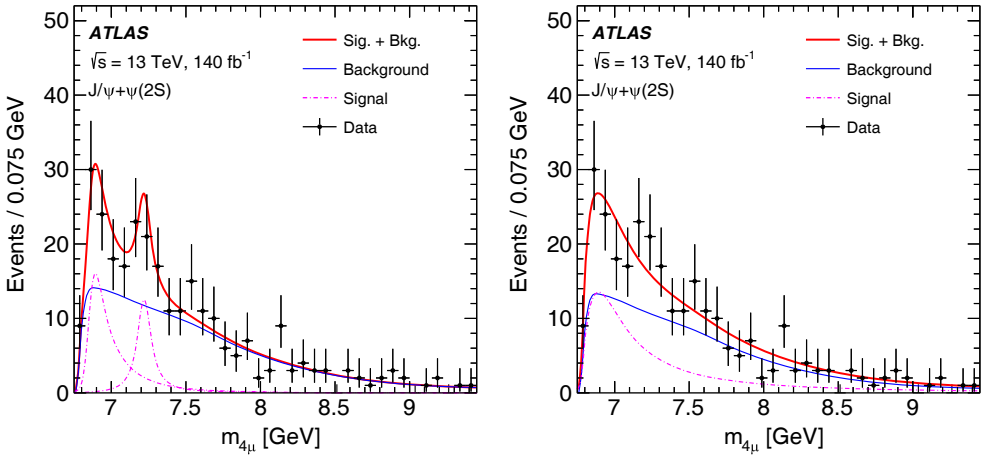

Fig. 12. The $J/\psi\psi(2S)$ mass spectra from ATLAS with Model α (left) and Model β (right) fits.[10] In addition to background, Model α consists of two signal BW's, one fixed to the parameters of the $X(6900)$ from their $J/\psi\,J/\psi$ data and the second floats freely; Model β has a single floating BW.

5. Di-Charmonia: Inferences, Speculations, and Explanations? The $X(6900)$ structure is firmly established with ATLAS, CMS, and LHCb all in unison, although parameters depend upon the fit model chosen. Likewise, the experiments concur on the gross features of the remainder of the mass spectrum, including a dip around 6750 MeV, which favors interference—but different modeling choices resulted in a somewhat confusing collection of potential structures. Up to four BW's are needed to describe the $J/\psi\,J/\psi$ spectrum. The experimental picture may seem muddled, especially for theoretical model builders.

There is large body of theoretical literature on the $J/\psi\,J/\psi$ structures,[145] but it is not our aim to review this work. Rather we look at some basic features of the data, and some simple and gross physics considerations.

5.1 Some Inferences. To bring some coherence of the diversity of $J/\psi\,J/\psi$ modeling, we assess each of the four BW's:

Threshold BW: All experiments see evidence that the threshold requires some component to account for the rapid threshold turn-on. All experiments were reluctant to assign any particular origin to it: tetraquark, feeddown, rescattering of dicharmonia production, ... A BW seems to provide a viable *ad hoc* description. CMS chose to explicitly relegate it to the background. What this structure is stands as an enigma.

***X*(6600)**: Given its position and broadness there are nagging concerns about contributions from potential feeddown, or that it could be composed of multiple BW's.

***X*(6900)**: All experiments observe this structure, but with somewhat varying masses. Its existence is not questioned but its nature is. Whether tetraquark or dynamical effect, the $X(6900)$ is accepted as a physics signal.

***X*(7100)**: CMS found a significant (4.7σ) peak around 7200 MeV in the $J/\psi\,J/\psi$ channel, and hints were seen by ATLAS and LHCb. CMS data favor $X(7100)$ interference, but the hints from ATLAS and LHCb are too weak to discern such an effect. We provisionally adopt the $X(7100)$ as a physical structure.

In summary, we adopt the $X(6600)$, $X(6900)$, and $X(7100)$ structures as physical, and remain agnostic about the threshold BW. We also accept that these three structures likely interfere with each other.

5.2 Some Speculations. We will consider some of the

broad empirical features of the data and what general lessons they might suggest. We will on occasion draw on the diquark model of tetraquarks as a backdrop for these reflections.

One immediate conclusion: interference among the states, if confirmed, implies they all have the same J^{PC}.

Next we consider mass differences: they are relatively large (>250 MeV), larger than is generally characteristic of spin or orbital mass splittings. As a reference, we compare CMS's X mass splittings to that of the Υ family in Fig. 13. We plot $\Delta m_i(\Upsilon) = m[\Upsilon(\{i+1\}S)] - m[\Upsilon(iS)]$,[66] with similar values for $\Delta m_i(X)$ superimposed. The indices of the three X's are shifted to $i = 3$, 4, and 5 so that the Δm_i values of similar magnitude for the Υ's and X's overlap. The X mass splittings are roughly similar to those for the $2S$–$4S$ Υ's.

A popular approach to modeling heavy quarkonia is to use some refinement of the Cornell Potential,[146,147] which in its original, and simplest, form is the sum of a Coulomb-like attraction ($1/r$) for one-gluon exchange and a long-range linear (r) confining force. The two components have the same magnitude around 0.3 fm.[148] The Upsilon sizes for the $1S$, $2S$, and $3S$ have been estimated to be 0.24, 0.51, and 0.73 fm.[149] Thus the large Υ mass splitting for $1S$–$2S$ is because the $1S$ state is deeper in the $1/r$ part of the potential, whereas the higher radial excitations are increasingly dominated by the linear potential. The X splittings, by analogy with the Upsilons, suggest that the triplets are in, or near, the linear regime of the potential.

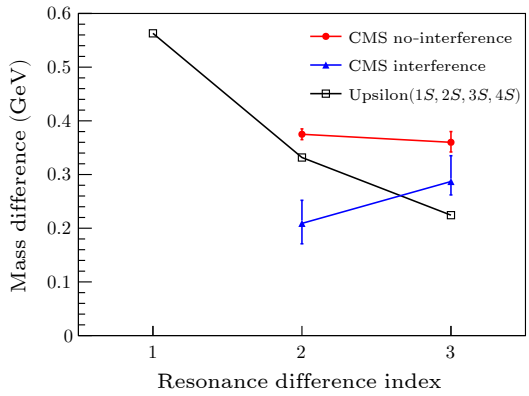

Fig. 13. The mass differences between neighboring states as a function of state index. The black line is $\Delta m_i(\Upsilon) = m[\Upsilon(\{i+1\}S)] - m[\Upsilon(iS)]$, where i is their radial index. Similarly, the blue (red) line is $\Delta m_i(X)$ for CMS interference (no-interf.) fits, where the $X(6600)$, $X(6900)$, and $X(7100)$ are assigned indices $i = 2$, 3, and 4. The plotted uncertainties for $\Delta m_i(X)$ are only statistical, and do not include the $X(6900)$ contribution in order to avoid a correlation in uncertainties between the points.

Cornell-type potentials have also been used to model tetraquarks.[150,151] In some models the diquark structure is exploited to convert the four-body problem into three two-body problems. Calculations find that the c-c separation in a diquark is around $\sim$0.4–0.6 fm.[144,150–152] [Calculations seem to display a curious feature that ground-state diquark separations in the tetraquark, $\sim$0.2–0.4 fm,[144,145] are *smaller* than the c-c separation within the diquark. This suggests that the approximations that use diquarks to convert the four-body problem into two-body problems are based on an invalid assumption,[150] but four-body calculations seem to arrive at similar spacings.[144] But this may suggest that approximating the pair of diquarks as a two-body problem may place them deeper, i.e. more tightly bound, in the Coulombic part of the potential, resulting in lower tetraquark masses.] If correct, this prevents the diquark pair from inhabiting much of the Coulombic potential, and would be aligned with the X states displaying splittings *on par* with the higher Υ states. These crude considerations suggest the X triplet to be a family of radial excitations (see Ref. [153] for not-crude considerations); and from a diquark perspective, that the diquarks have a significant separation and reside in, or near, the linear confining part of the potential.

Going a step further, we look at the absolute masses. A Regge trajectory of the X's as m^2 versus a (presumed) radial index (n_r)[154] is plotted in Fig. 14 for both CMS's no-interference and interference fits. The triplet masses in both models are well-described by a linear relationship. This has several implications: it supports CMS's identification of these structures as physical states; the triplet does indeed look like a family; and it implies any contaminating effects such as feeddown, or multiple resonances, have not greatly distorted the masses. This is especially striking for the no-interference values where uncertainties are much smaller.

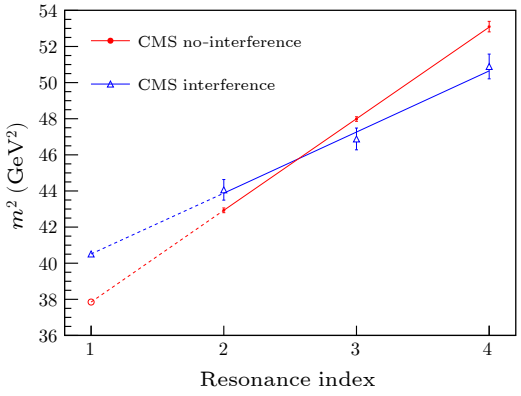

Fig. 14. A Regge-like plot of the square of the CMS $X(6600)$, $X(6900)$, and $X(7100)$ masses as a function of their index $i = 2$, 3, and 4. The line is a χ^2 fit of the data. The dashed line with open points is an extrapolation for a hypothetical neighboring structure. The uncertainties are only statistical.

The Regge relationship $n_r = \beta m^2 + \beta_0$ was used to fit the known charmonium states, and then to extrapolate them to Regge parameters for tetracharm trajectories.[154] We compare them to our fits of CMS data in Table 4.

The slope from CMS's no-interference fit is compatible with the projections, the interference fit disagrees by almost 3σ (but recall the CMS uncertainties here are only statistical). The mass predictions in Ref. [154] for the two heaviest states of the 0^{++} (1^+ diquark) trajectory are 6883 ± 27 and 7154 ± 22 MeV, which agree well with CMS's

interference model, but the intermediate state prediction of 6555^{+36}_{-37} MeV is about 100 MeV below the interference value—but agrees with the no-interference fit. CMS included the effect of feeddown in the systematic uncertainties of $X(6600)$, but its central value can be biased by this. A somewhat lower $X(6600)$ mass would actually improve the linear fit for the interference trajectory (Fig. 14). For the no-interference fit the $X(6600)$ mass has the same problem, but also the $X(7100)$ mass is 130 MeV higher than the projection. This Regge comparison is not conclusive, but both fits seem well-aligned with a tetraquark interpretation of the triplet.

Table 4. Comparison between theoretical Regge parameters for tetracharm trajectories [154] and CMS data fit results.

State	Diquark	β (GeV^{-2})	β_0	CMS model	β (GeV^{-2})	β_0
0^{++}	1^+	0.206 ± 0.013	-7.74 ± 0.56	No Interf.	0.197 ± 0.005	-7.46 ± 0.24
0^{++}	0^+	0.212 ± 0.013	-7.76 ± 0.57	Interf.	0.289 ± 0.025	-11.70 ± 1.23
2^{++}	1^+	0.211 ± 0.013	-8.00 ± 0.58			

Another interesting feature of the triplet is their natural widths. The widths from the interference fit significantly shrink as one goes to higher states: $\Gamma_{6900}/\Gamma_{6600} = 0.43^{+0.26}_{-0.22}$ and $\Gamma_{7100}/\Gamma_{6900} = 0.51^{+0.27}_{-0.20}$, which are compatible with a common value. The diquark model offers a simple intuitive possibility: at higher excitations the diquark separation increases, reducing the overlap of diquark wave functions, and thus suppressing decay rates. S-wave states still have significant overlap at the origin, so the large decrease could point to a P-wave triplet. In dramatic contrast, the widths from CMS's non-interference fit are consistent with being equal.

The Regge interpretation of the $X(6900)$ as $n_r = 3$ leaves the $n_r = 1$ state missing. We can extrapolate our fit to find this mass—irrespective of the particular interpretation of the triplet. Our Regge fit projects a structure at 6359 ± 58 (6152 ± 20) MeV for the interference (no-interference) fit. The no-interference mass is below threshold, but perhaps not implausibly so. ATLAS reported a mass for their threshold-BW in Model A as $6410 \pm 80^{+80}_{-30}$ MeV, consistent with our interference projection. Other experiments can make a similar report from their existing fits. However, it is a delicate matter to determine precise resonance parameters with confidence because of the (unknown) feedown contamination and potential complications from being near threshold. This is an encouraging first glance at the threshold structure, and perhaps a way to help unravel its mystery.

One may also look in the other direction based on a Regge projection: where might a higher structure lie? The next structure, if there is one, would be around 7598 ± 57 $(7951 \pm 28$ MeV) for interference (non-interference) models. These masses are well above the $\Xi_{cc}^+\Xi_{cc}^-$ and $\Xi_{cc}^{++}\Xi_{cc}^{--}$ thresholds (~ 7240 MeV [66]), possibly leaving the $X(7100)$ as the last member of the $J/\psi\,J/\psi$ family. This is a prime issue for future LHC studies.

5.2.1 Explanations? The $J/\psi\,J/\psi$ mass spectrum observed at the LHC has proven very fruitful. The discovery of the $X(6900)$ marks a major milestone in hadronic spectroscopy, and the addition of the $X(6600)$ and $X(7100)$ structures introduces critical information to understand the $J/\psi\,J/\psi$ spectrum. While the remainder of the spectrum has engendered some puzzlement, CMS's picture of a triplet of states finds additional support in their orderly pattern of masses, which are in fair agreement with a theoretical projections for the Regge trajectories of $c\bar{c}c\bar{c}$ systems. [154] The Regge character of CMS's triplet lends support for the three structures having the same J^{PC}, as implied by the interference model. The success of the Regge trajectory model also implies that the mysterious threshold BW makes a plausible candidate for a first radial state.

This tidy tetraquark picture weights against dynamical explanations. But, it is hard to dismiss the dynamical approach given a degree of success in their modeling of the $J/\psi\,J/\psi$ spectrum, [7,125,155] and the "coincidence" of the proximity of exotic structures to pair-production thresholds, e.g., $X(6600)$ around $\eta_c\chi_{c1}$, h_cJ/ψ, $\eta_c\psi(2S)$, ...; $X(6900)$ around $J/\psi\psi(2S)$, $J/\psi\psi(3686)$, $J/\psi\psi(3770)$,...; $X(7100)$ around $\chi_{c2}\chi_{c2}$, $\chi_{c1}\eta_c(2S)$, $\Xi_{cc}\bar{\Xi}_{cc}$,.... Still, it seems odd that purely dynamical threshold structures would follow a Regge trajectory.

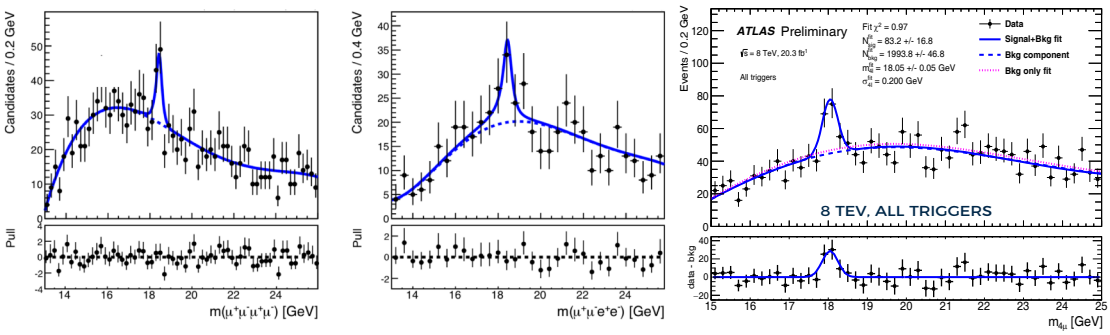

Fig. 15. The Υl^+l^- spectrum for $\mu^+\mu^-\mu^+\mu^-$ (left) and $\mu^+\mu^-e^+e^-$ (center) channels using CMS 7 and 8 TeV CMS data. [156,157] The ATLAS $\mu^+\mu^-\mu^+\mu^-$ distribution for 20 fb^{-1} of 8 TeV data (right). [158–160]

6. Searches in the Di-Upsilon Spectra. The $J/\psi\,J/\psi$ channel opened the world of all-heavy exotics, but there is much more in this world, in particular $c\bar{c}b\bar{b}$ and $b\bar{b}b\bar{b}$, and non-symmetric combinations like $c\bar{b}b\bar{b}$. These are obvious targets for the ever increasing data sets expected from the LHC. But, in fact, searches for the ultimate all-heavy exotic, $b\bar{b}b\bar{b}$, have already been formally or informally reported.

The first observation of Υ-pair production was reported by CMS with $21\,\text{fb}^{-1}$ of 8 TeV data in 2017.[161] Only a year later, a conference report of a Ph.D. thesis described structure in the Υl^+l^- channel using the $\mu^+\mu^-\mu^+\mu^-$ ($\sim 25\,\text{fb}^{-1}$) and $\mu^+\mu^-e^+e^-$ ($\sim 20\,\text{fb}^{-1}$) modes with CMS data at 7 and 8 TeV.[156,157,162] A preliminary signal was seen in each channel as shown in Fig. 15 (left and center). With a 4μ signal of 44 ± 13 candidates (3.9σ local significance) the mass was $18.4 \pm 0.1 \pm 0.2$ GeV. In the $2e2\mu$ mode, 35 ± 13 candidates (3.2σ) measured a mass of $18.5 \pm 0.2 \pm 0.2$ GeV. The two modes agree, and when combined yield a mass of $18.4 \pm 0.1 \pm 0.2$ GeV, for a local significance of 4.9σ, and globally 3.6σ.

These are fairly significant results, but they were challenged by another Υl^+l^- search by LHCb with $6\,\text{fb}^{-1}$ of data at 7, 8, and 13 TeV in 2018, where nothing was seen in a 17.5–20.0 GeV window.[163] In due course CMS published a result in this channel with $36\,\text{fb}^{-1}$ of 13 TeV data from 2016—and now also found no signal.[164] This was not much more data than CMS analyzed before, but it was at almost twice the energy.

Energy is not a very plausible excuse for a signal to vanish, (While an energy difference is not a very likely explanation, higher energies come with larger cross sections, typically increased instantaneous luminosities, and higher pileup background, which usually forces triggers and selections to be tightened. It is conceivable that an Υl^+l^- signal was lost through the increased contamination of pileup events and the tightened trigger selection. This might be unravelled if the signal resurfaces, and experimenters wished to do some reconstructive archaeology.) so the tale becomes even more bizarre with a 2023 conference report by ATLAS where they seemed to confirm CMS's 18 GeV peak—just before unconfirmed it![158–160] With $20\,\text{fb}^{-1}$ of 8 TeV data ATLAS reported an 18.05 ± 0.05 GeV peak in the 4μ channel with a 5.4σ local significance (Fig. 15, right). But in 13 TeV data from 2015–17 ($52\,\text{fb}^{-1}$) the "signal" was an insignificant 1.9σ with the mass was fixed to 18.05 GeV; and then with data from 2018 ($59\,\text{fb}^{-1}$, with a different trigger) they were unable to confirm any structure: there was no upward "fluctuation" seen at all! Weird... but an object lesson on the insidious nature of vanishing signals.

The LHC was not alone. A surprising entry in the all-bottom tetraquark sweepstakes comes from heavy-ion collisions. In 2019 a preprint by the A$_N$DY collaboration at RHIC claimed a peak with a dijet signature.[165] Jet mass reconstructions have been the domain of very heavy particles, like W, Z, and top—and not for hadron spectroscopy. A$_N$DY looked at dijets triggered in Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV recorded in 2012 (The data was collected for a calorimeter test, and as a result the luminosity was not measured.) and found a peak in the forward dijet mass of $18.12 \pm 0.15 \pm 0.6$ GeV, with significance over 9σ. The peak is several GeV wide, and thus presumably would be a collective effect of multiple (many?) hypothetical tetraquarks. Unfortunately this paper remains unpublished, and there seems to be no further reports.

Finally, it should also be noted that some theoretical expectations, such as lattice QCD,[166] predict that there are no all-b tetraquark states below the $\eta_b\eta_b$ threshold, i.e. 18.76 GeV. In any case, the LHC is churning out more data, and a compelling demonstration of all-bottom structures might not be too far off?

7. Summary and Conclusions. The quark model has become an irrevocable, but not immutable, core of the Standard Model of particle physics. Originally perceived as a crazy and bizarre idea in 1964, quarks became real in the 1970s, and the model was followed by a relentless march of successes. At the pinnacle of its success at the turn of the century the model was confronted by a new twist: particles not conforming to the conventional quark configurations—exotic hadrons. This was not entirely a surprise, as some investigators had been toiling at the periphery chasing nature's occasional teasers of exotics, but also stumbling over a number of false alarms. The discovery of the $X(3872)$ meson in 2003 ignited an explosion of exotic hadrons. The oddities included pentaquarks and charged charmonium-like states (Z^+'s). Despite this bounty of states, there continues much debate as to what is their exact natures: molecular, diquark, hybrid, or dynamical threshold effects.

The old lesson of simplicity with heaviness learned from the ψ and Υ families pointed down the road to seek systems composed entirely of heavy quarks. It took six years to find the first $s\bar{s}q\bar{q}$ candidate in $J/\psi\phi$, but not without several years of controversy. Then, finally in 2020, LHCb found the $X(6900)$ structure in $J/\psi\,J/\psi$—the ideal candidate for an all-charm tetraquark. The $J/\psi\,J/\psi$ spectrum was more complex than this single structure, and CMS reported observing the $X(6600)$ structure in excess of 5σ, opening the door to study of the *spectrum* of tetracharm states. A bonus was their $X(7100)$, which had a substantial significance even if below the 5σ benchmark. It is by virtue of having a triplet that all of our speculations and explanatory musings, right or wrong, become possible.

But having reached this pot of gold, we are confronted with the question: exactly what is their nature?

We reviewed the experimental results from ATLAS, CMS, and LHCb on the $J/\psi\,J/\psi$ spectrum, and sifted through the various fit models to winnow all the hypothetical Breit–Wigner "resonances" down to a basic set of physical candidates. The data paints a picture of at least three structures, $X(6600)$, $X(6900)$, and $X(7100)$, and possibly (probably?) a fourth at threshold. The triplet states appear to mutually interfere—thereby implying the same

J^{PC}'s. The mass splittings are fairly large (>250 MeV), suggesting a family of radial excitations. This picture is endorsed by the X masses conforming to a Regge trajectory of radial excitations. The pattern of significantly shrinking natural widths in the triplet prompts a simple appeal to a diquark model.

This picture is by no means conclusive. While resorting to interference has been universal and is strongly motivated, the data is not statistically conclusive and the possibilities of non-interference modelling has not been fully exhausted (e.g. multi-resonance peaks). It is noteworthy that the no-interference fit of CMS data also conforms to a Regge trajectory—but this too implies common quantum numbers. Issues of interference, and the possibility of further states, will be addressed with the ongoing collection of larger data samples at the LHC, as well as new types of measurements will become possible, such as production cross sections, new modes, branching fractions, and spin-parity determinations. More data also means all-heavy tetraquarks containing bottom quarks, with signatures like $J/\psi\Upsilon$, $\Upsilon\Upsilon$, $\Upsilon\Upsilon^*$ (Υ^* is off-shell Υ decaying to dileptons), may also be in reach soon. One can also expect searches in new directions and modes, such as Belle's look for structure in $\eta_c J/\psi$ directly from e^+e^- annihilation.[167]

The $X(3872)$ sparked a revolution in spectroscopy which as has been echoing to this day. The coming years promise exciting new chapters in the physics of exotic hadrons, and the exploration of QCD in this new domain.

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