Dark Contributions to $h \to \mu^+\mu^-$ in the Presence of a μ-Flavored Vector-Like Lepton

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A simple extension of the standard model (SM) with a μ-flavored vector-like lepton (VLL) doublet and a real singlet scalar can have an interesting implication to the $h \to \mu^+\mu^-$ decay while offering the simplest possible explanation for the dark matter (DM) phenomenology. Assuming the real singlet scalar to be a viable DM candidate, it has been shown that the muon Yukawa coupling can have a negative contribution at the one-loop order if the 2$^{\rm nd}$ generation SM leptons are allowed to couple with the VLL doublet. The stringent direct detection bounds corresponding to a real singlet scalar DM can easily be relaxed if the SM quark sector was augmented with a dimension-6 operator at some new physics (NP) scale $\Lambda_{\rm NP}$. Thus, this model presents a significant phenomenological study where the muon Yukawa coupling can be corrected within a real singlet scalar DM framework. The considered parameter space can be tested/constrained through the high luminosity run of the LHC (HL-LHC) and future direct detection experiments.

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The observation of a standard model (SM)-like Higgs boson ($\sim 125\,\text{GeV}$) at the LHC[1,2] and the subsequent measurements of its couplings to the other SM particles have played a crucial role in understanding the possibilities beyond the SM. The couplings of the SM Higgs to the 3$^{\rm rd}$ generation fermions and the vector bosons have already been probed within 10%–20% of their SM predictions.[3] A recent CMS search has recorded the first direct evidence of the SM Higgs decaying to a pair of muons,[4] and thus constraining the relative coupling strength of the 2$^{\rm nd}$ generation leptons to the Higgs. However, the measurements indicate that still sufficient room is available there to accommodate new physics (NP) contributions which can connect other beyond standard model (BSM) phenomena to the muon Yukawa coupling, i.e., dark matter (DM) phenomenology being a notable possibility.

Significant cosmological and astrophysical evidences[5–7] exist there, which have already confirmed the presence of the DM through its gravitational interactions. Assuming the DM to be a weakly interacting massive particle (WIMP)[8] a number of BSM formulations have been proposed, where the SM Higgs acts as a portal particle between the dark sector and the standard model, i.e., extending the SM with a real singlet scalar (protected through some discrete symmetry)[9] of mass$\sim \mathcal{O}(10^2)\,\text{GeV}$ being the simplest theoretical construction of this kind which can produce the correct relic abundance. Although the H-portal model with a real singlet scalar DM has already been ruled out for $M_{\rm DM} \in [100, 1000]\,\text{GeV}$ by the current direct detection (DD) bounds,[10–12] a recent study[13] has shown that it can be revived within an effective field theory (EFT) framework if one extends the SM quark sector with a particular dimension-6 effective operator corresponding to an NP scale $\Lambda_{\rm NP}$. In contrast to the earlier studies,[14–17] the present study explicitly uses this idea to describe a DD-allowed real singlet scalar DM model which, in the presence of a dark μ-flavored vector-like lepton (VLL), can result in a remarkable negative contribution to the muon Yukawa coupling at one-loop order. VLLs are phenomenologically well motivated[18–22] for describing an anomaly free UV complete theory,[23] whereas the flavor-specific choice helps to explain the non-observation of the lepton flavor violating processes, e.g., $h \to \ell_i\ell_j$, $\ell_j \to \ell_i\gamma$, and $\ell_j \to 3\ell_i$. The presented results can be tested at the HL-LHC provided that the weak interaction of the DM has been verified through the future direct search experiments.

In this Letter, firstly the mathematical formulation is set up to describe the proposed model. Secondly, the collider bounds are enlisted on various components of the parameter space. Thirdly, the DM phenomenology with a real singlet scalar WIMP is discussed. Fourthly, the correction to $h\bar{\mu}\mu$ coupling is studied with a parameter space constrained through the DM observables. Finally, the results are summarized.

The BSM Formulation. The proposed model considers a simple extension of the SM containing a vector-like lepton doublet f having $+1$ unit of muonic lepton number and a real singlet scalar S. Both f and S are odd under a discrete $\mathbb{Z}_2$ symmetry which does not introduce any non-trivial transformation for the SM fields. Moreover, the model framework follows a lepton number conserving structure. Table 1 enlists the complete particle content of this model along with their gauge quantum numbers.

In addition to the BSM particles mentioned above, let us assume an effective dimension-6 operator $\Delta\mathcal{L}^{Q}_{\rm Eff}$ arising at an NP scale $\Lambda_{\rm NP}$, which can alter the linear alignment of mass and Yukawa coupling in the SM quark sector. Thus, the complete Lagrangian for this model reads

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm SM} + \Delta\mathcal{L}^{Q}_{\rm Eff} + \bar{f}(i\not{\mathcal{D}} - m_f)f + \frac{1}{2}(\partial^\mu S)(\partial_\mu S) \\ - \mathcal{V}(H,S) - [\xi_\ell\,\bar{\ell}_{\rm L} S f + \text{h.c.}],\end{aligned} \tag{1}$$

where $\mathcal{D}_\nu \equiv \partial_\nu + ig_1\frac{Y}{2}B_\nu + ig_2\frac{\sigma^i}{2}W^i_\nu$ denotes the $SU(2)_{\rm L}\times$

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$U(1)_Y$ covariant derivative, g_1 and g_2 being the $U(1)_Y$ and $SU(2)_{\rm L}$ couplings, respectively; m_f corresponds to the physical mass of the BSM lepton f. The scalar potential in Eq. (1) can be cast as

$$\mathcal{V}(H,S) = \frac{1}{2}\mu_{\rm S}^2 S^2 + \lambda_{\rm HS}(H^\dagger H)S^2, \tag{2}$$

where $\lambda_{\rm HS}$ is a dimensionless coupling. Note that, there can be S^4 term as well but here it has been dropped from the phenomenological point of view. Further, the proposed dimension-6 operator can be defined as

$$\Delta\mathcal{L}^Q_{\rm Eff} = \frac{H^\dagger H}{\Lambda^2_{\rm NP}}\left[\mathcal{Y}^u_{\rm H}\bar{Q}_{\rm L}\tilde{H}U_{\rm R} + \mathcal{Y}^d_{\rm H}\bar{Q}_{\rm L}HD_{\rm R}\right], \tag{3}$$

where $\mathcal{Y}^{u(d)}_{\rm H}$ are the Wilson coefficients associated with the up (down)-type quarks, respectively. $\tilde{H} = i\tau_2 H^*$ is the conjugate Higgs field. Note that the energy scale $\Lambda_{\rm NP}$ should be sufficiently higher than $\mathcal{O}(m_f)$ for representing an effective theory within this proposed BSM formulation. The phenomenological impact of this effective operator will be discussed in the following. After electroweak symmetry breaking (EWSB) only the SM Higgs $H = \frac{1}{\sqrt{2}}\binom{0}{h+v}$ develops a VEV, $v = 246\,\text{GeV}$. Thus, the physical mass term for S can be defined as

$$M_{\rm S} = \sqrt{\mu_{\rm S}^2 + \lambda_{\rm HS}v^2}. \tag{4}$$

Within this minimally extended framework $\xi_\ell\,\bar{\ell}_{\rm L}Sf$ [last term of Eq. (1)] arises as a new Yukawa contribution which connects the SM leptons with the BSM sector. However, since S does not acquire any VEV, the mass matrix in the $(\ell\ f)^{\rm T}$ basis remains diagonal. Further, the lepton number conservation demands $\xi_e, \xi_\tau = 0, f$ being a μ-flavored vector-like lepton. Thus, in the presence of a term like $\xi_\mu\,\bar{\mu}Sf^-$, one can have an additional one-loop contribution to the $h\bar{\mu}\mu$ coupling. Moreover, with the assumption that $M_{\rm S} < m_f$ over the entire parameter space, the proposed $\mathbb{Z}_2$ symmetry protects S as the lightest kinematically stable odd particle and thus makes it a viable candidate for being Dark Matter. In the following, it will be shown that the DM phenomenology can play a significant role in modifying the effective $h\bar{\mu}\mu$ coupling.

Table 1. Particles and their transformations under $\mathcal{G}_{\rm SM} = SU(3)_C \times SU(2)_{\rm L} \times U(1)_Y$ and $\mathbb{Z}_2$.

Fields	Generation & spin	$\mathcal{G}_{\rm SM}$	$\mathbb{Z}_2$
$\ell_{\rm L} = (\nu_\ell\ \ell)^{\rm T}_{\rm L}$	(3, 1/2)	(**1**, **2**, −1/2)	+1
$\ell_{\rm R} = (e_{\rm R}, \mu_{\rm R}, \tau_{\rm R})$	(3, 1/2)	(**1**, **1**, −1)	+1
$Q_{\rm L} = (u_{\rm L}\ d_{\rm L})^{\rm T}$	(3, 1/2)	(**3**, **2**, 1/6)	+1
$U_{\rm R} = (u_{\rm R}, c_{\rm R}, t_{\rm R})$	(3, 1/2)	(**3**, **1**, 2/3)	+1
$D_{\rm R} = (d_{\rm R}, s_{\rm R}, b_{\rm R})$	(3, 1/2)	(**3**, **1**, −1/3)	+1
$H = \left(H^+\ H^0\right)^{\rm T}$	(1, 0)	(**1**, **2**, 1/2)	+1
$f = (f^0\ f^-)^{\rm T}$	(1, 1/2)	(**1**, **2**, −1/2)	−1
S	(1, 0)	(**1**, **1**, 0)	−1

Experimental Bounds. A recent CMS analysis performed using $p-p$ collision data at $\sqrt{s} = 13\,\text{TeV}$ for an integrated luminosity (IL) of $137\,\text{fb}^{-1}$ provides with the first direct evidence of the SM Higgs decaying to a pair of muons. The signal strength relative to the SM prediction has been recorded as [4]

$$\kappa_\mu = 1.19^{+0.40}_{-0.39}\ (\text{stat})\ ^{+0.15}_{-0.14}\ (\text{syst}). \tag{5}$$

However, the HL-LHC project with an IL of $3000\,\text{fb}^{-1}$, which aims to collect the collision data at $\sqrt{s} = 14\,\text{TeV}$, will narrow down the uncertainty in κ_μ to 3–4% with the 5σ level of significance. [24]

For vector-like lepton doublets coupling to the 3rd generation SM leptons, a dedicated CMS search using an IL of $77.4\,\text{fb}^{-1}$ at $\sqrt{s} = 13\,\text{TeV}$ has excluded the mass regime 120–790 GeV. [25] However, this limit is not directly applicable to a μ-flavored VLL as has been considered here. Another recent CMS based analysis (for $77.4\,\text{fb}^{-1}$ IL at $\sqrt{s} = 13\,\text{TeV}$) excludes VLL doublets up to 800 GeV with a major focus on the 4ℓ final states. [26] Thus, in this study, $m_f \geq 1000\,\text{GeV}$ will be considered in general to evade the collider constraints.

DM Phenomenology. As proposed above, the real singlet scalar S being the lightest $\mathbb{Z}_2$-odd particle can be a suitable candidate for explaining the DM phenomenology.

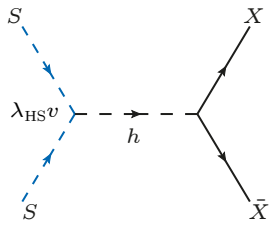

Fig. 1. Annihilation of S (DM) to the SM particles (X) through the Higgs boson.

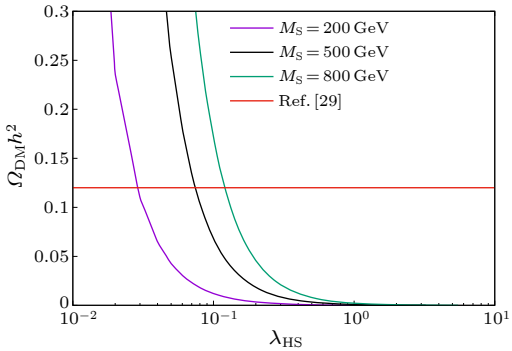

Fig. 2. Variation of the relic density as a function of $\lambda_{\rm HS}$ for $M_{\rm S} = 200\,\text{GeV}$ (purple), 500 GeV (black) and 800 GeV (green).

The Boltzmann equation corresponding to S is given by

$$\frac{dn}{dt} + 3\mathcal{H}n = -\langle\sigma_{\mathcal{A}}v_r\rangle(n^2 - n_{\rm eq}^2), \tag{6}$$

where $\mathcal{H}$ denotes the Hubble parameter, $\langle\sigma_{\mathcal{A}}v_r\rangle$ is the thermal averaged annihilation cross section times the relative velocity of the WIMP annihilating to the SM sector; n and $n_{\rm eq}$ signify the number density of S and its equilibrium value, respectively. Figure 1 depicts the annihilation process of $SS \leftrightarrow X\bar{X}$, where X is a generic notation for the SM fields. For numerically analyzing the DM observables, micrOMEGAs [27] has been used. Figure 2 shows the variation of relic abundance of S as a function of the coupling $\lambda_{\rm HS}$ for three different $M_{\rm S}$ values: 200 GeV (purple line), 500 GeV (black line), and 800 GeV (green line).

Figure 2 reveals that the correct relic abundance, i.e., $\Omega_{\rm DM}h^2 = 0.1198 \pm 0.0012$ (red line) [28,29] can be obtained

at

(i) $\lambda_{\rm HS} = 0.03$ for $M_{\rm S} = 200\,{\rm GeV}$,

(ii) $\lambda_{\rm HS} = 0.075$ for $M_{\rm S} = 500\,{\rm GeV}$,

(iii) $\lambda_{\rm HS} = 0.12$ for $M_{\rm S} = 800\,{\rm GeV}$.

Noticeably, the phenomenological possibility of a real singlet scalar to be a viable DM candidate has already been ruled out for $M_{\rm DM} \sim \mathcal{O}(10^2)\,{\rm GeV}$ through the direct detection searches. However, this stringent constraint can easily be evaded in the presence of the dimension-6 operator described by Eq. (3). It can be readily shown that if the SM is augmented with $\Delta\mathcal{L}^{Q}_{\rm Eff}$, the physical SM quark masses and Yukawa couplings can be recast as

$$\begin{aligned} m_q &= \frac{v}{\sqrt{2}}\Big(Y_q - \frac{\beta}{2}\mathcal{Y}^q_{\rm H}\Big), \\ y_q &= \frac{1}{\sqrt{2}}\Big(Y_q - \frac{3\beta}{2}\mathcal{Y}^q_{\rm H}\Big) = \frac{m_q}{v} - \frac{\beta}{\sqrt{2}}\mathcal{Y}^q_{\rm H}, \end{aligned} \tag{7}$$

where $\beta = (v/\Lambda_{\rm NP})^2$. For simplicity, a flavor-diagonal structure for the NP Yukawa couplings has been assumed, i.e., $\mathcal{Y}^q_{\rm H} = \mathbb{I}_{3\times3}$. Thus, Eq. (7) enhances the degree of freedom to alter the quark Yukawa couplings without deviating the quark masses from their observed physical values. Thus, following Ref. [13] if one assumes non-SM-like negative quark Yukawa couplings for the 2$^{\rm nd}$ generation quarks, the null results in the direct search experiments can easily be explained irrespective of the value of $\lambda_{\rm HS}$. Therefore, assuming $y_{\rm c} = -1.7\,y^{\rm SM}_{\rm c}$ and $y_{\rm s} = -0.7\,y^{\rm SM}_{\rm s}$ for charm and strange quark Yukawa couplings respectively, the variation of the spin-independent (SI) direct detection cross section of S as a function of $M_{\rm S}$ has been shown in Fig. 3, with the purple, black, and red lines standing for $\lambda_{\rm HS} = 0.03$, 0.075, and 0.12, respectively.

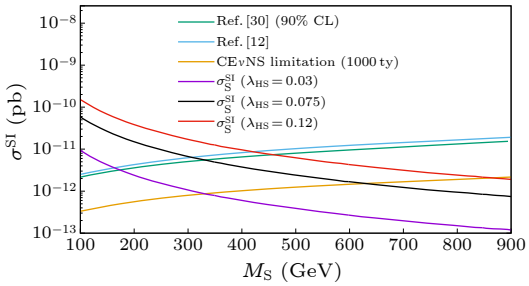

Fig. 3. Variation of $\sigma^{\rm SI}_{\rm S}$ as a function of $M_{\rm S}$ for $\lambda_{\rm HS} = 0.03$ (purple), 0.075 (black), and 0.12 (red).

The sky blue and green lines signify the direct detection bound from the LZ collaboration [12] and the projected limit from the XENONnT experiment, [30] respectively. The yellow line corresponds to the neutrino floor. Thus, for the considered data set, i.e., ($M_{\rm S} = 200\,{\rm GeV}$, $\lambda_{\rm HS} = 0.03$), ($M_{\rm S} = 500\,{\rm GeV}$, $\lambda_{\rm HS} = 0.075$) and ($M_{\rm S} = 800\,{\rm GeV}$, $\lambda_{\rm HS} = 0.12$), $\sigma^{\rm SI}_{\rm S}$ is well above the neutrino floor, making it a testable theory for the future direct detection searches.

New Contribution to $H \to \mu^+\mu^-$. As already stated, within this proposed framework one can have a new contribution to the muon Yukawa coupling due to the $\xi_\mu\,\bar{\mu}Sf^-$ term at one-loop level. Note that the same NP coupling $\lambda_{\rm HS}$, which plays a crucial role in the thermal production of the DM S, can also regulate the one-loop correction to muon Yukawa coupling through Fig. 4.

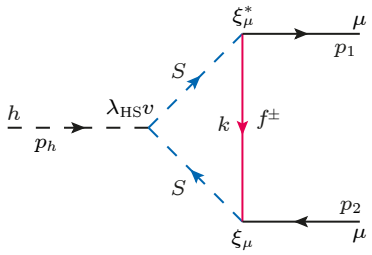

Fig. 4. One-loop BSM contribution to the muon Yukawa coupling. All the arrows represent momentum direction.

The coupling correction corresponding to Fig. 4 can be parametrized as,

$$\begin{aligned} \Delta y_\mu = i\int \frac{d^4k}{(2\pi)^4}\Bigg[&\xi_\mu P_{\rm R}\Big(\frac{\not{k}+m_f}{k^2-m_f^2}\Big) \\ &\cdot \xi^*_\mu P_{\rm L}\frac{1}{(k+p_1)^2 - M_{\rm S}^2}\,\lambda_{\rm HS}v \\ &\cdot \frac{1}{(k+p_2)^2 - M_{\rm S}^2}\Bigg], \end{aligned} \tag{8}$$

where $P_{\rm L,R}$ are the left and right chiral projection operators, respectively. Feynman parametrization followed by the integration over loop momentum reduces the above expression to

$$\Delta y_\mu = -\frac{|\xi_\mu|^2\lambda_{\rm HS}v}{16\pi^2}\Big(\frac{m_\mu}{M_{\rm S}^2}\Big)\int_0^1 dx\int_0^{1-x} dy\;\frac{(1-x)}{\mathcal{M}(x,y)}, \tag{9}$$

where x, y are the Feynman parameters, m_μ is the mass of muon, and $\mathcal{M}(x,y) = xr_f + (1-x) - y(1-x-y)r_h$ with $r_f = (m_f/M_{\rm S})^2$ and $r_h = (M_h/M_{\rm S})^2$, M_h being the mass of the SM Higgs. Here, an on-shell $h \to \mu^+\mu^-$ decay has been considered, such that $p_h^2 \approx -2p_1\cdot p_2 = M_h^2$, $(m_\mu/M_{\rm S})^2 \to 0$ being a valid approximation. Further, Eq. (9) can be recast in terms of Passarino–Veltman (PV) integrals as

$$\Delta y_\mu = -\frac{|\xi_\mu|^2\lambda_{\rm HS}v\,m_\mu}{16\pi^2}\sum_{i=1}^{2} C_i(0, M_h^2, 0, m_f^2, M_{\rm S}^2, M_{\rm S}^2), \tag{10}$$

where $C_{1,2}$ are the standard 3-point PV integrals. [31] Thus, the muon Yukawa coupling modifier can be defined as

$$\kappa_\mu = \Big(1 + \frac{\Delta y_\mu}{y^{\rm SM}_\mu}\Big), \tag{11}$$

where $y^{\rm SM}_\mu = m_\mu/v$. Figure 5(a) shows the variation of κ_μ as a function of ξ_μ for $M_{\rm S} = 500\,{\rm GeV}$ and $\lambda_{\rm HS} = 0.075$.

As indicated by the plotted variation, the proposed framework generates a negative contribution to the muon Yukawa coupling, reducing its effective value from the SM prediction. However, the variation is well within the experimental bounds [see Eq. (5)]. With increasing m_f, the deviation from $y^{\rm SM}_\mu$ decreases, as can be seen from Fig. 5(a). The ξ_μ has been varied randomly within the perturbative bounds. However, in practice, the condition of perturbative unitarity restricts $\xi_\mu \sim \mathcal{O}(1)$. [32] Figure 5(b) depicts a similar variation for ($M_{\rm S} = 200\,{\rm GeV}$, $\lambda_{\rm HS} = 0.03$) [purple line], ($M_{\rm S} = 500\,{\rm GeV}$, $\lambda_{\rm HS} = 0.075$) [red line], and ($M_{\rm S} = 800\,{\rm GeV}$, $\lambda_{\rm HS} = 0.12$) [green line] with $m_f = 1\,{\rm TeV}$. The apparently non-trivial variation between the purple

and the red line is a consequence of the fact that $|\Delta y_\mu|$ increases in proportion to $\lambda_{\rm HS}$ but at the same time decreases as $1/M_{\rm S}^2$.

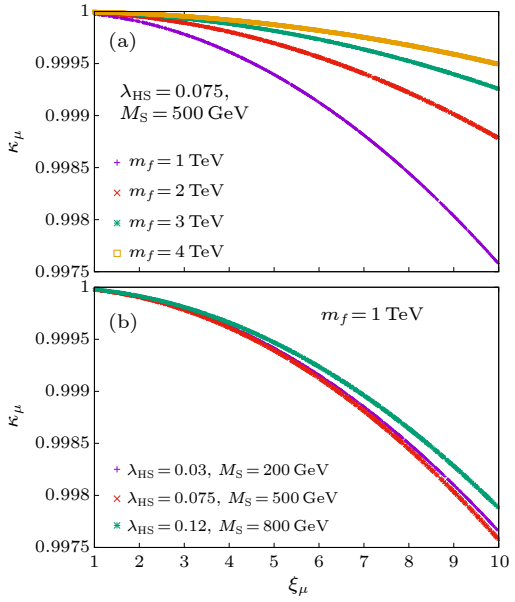

Fig. 5. Variation of κ_μ as a function of ξ_μ for (a) $m_f = 1$ TeV (purple), 2 TeV (red), 3 TeV (green), and 4 TeV (yellow) with $M_{\rm S} = 500$ GeV and $\lambda_{\rm HS} = 0.075$, and (b) $M_{\rm S} = 200$ GeV, $\lambda_{\rm HS} = 0.03$ (purple), $M_{\rm S} = 500$ GeV, $\lambda_{\rm HS} = 0.075$ (red), and $M_{\rm S} = 800$ GeV, $\lambda_{\rm HS} = 0.12$ (green) with $m_f = 1$ TeV.

Note that, within this model the same Yukawa term $\xi_\mu \bar{\mu} S f^-$ can also introduce a BSM contribution to $(g-2)_\mu$ at the one-loop order. However, the contribution in the chosen parameter space is not sufficient to explain the observed discrepancy.[33,34] For example, with $M_{\rm S} = 500$ GeV, $m_f = 1$ TeV and $\xi_\mu = 2$, one obtains $\Delta a_\mu \simeq 3.184 \times 10^{-11}$. Further, due to lepton number conservation the non-observation of the charged lepton flavor violating processes does not add any extra constraint on the parameter space.

In summary, the SM has been extended with a real singlet scalar S and a μ-flavored vector-like lepton doublet f. Both of these fields being odd under an imposed $\mathbb{Z}_2$ symmetry, S can be a potential DM candidate with the condition $M_{\rm S} < m_f$, valid over the entire parameter space. Further, the presence of an effective dimension-6 operator has been assumed corresponding to an NP scale $\Lambda_{\rm NP}$, which only augments the SM quark sector and breaks the linear alignment between the quark mass and Yukawa coupling after EWSB. It has been numerically shown that the proposed framework not only is able to satisfy the observed relic abundance but also can explain the null results in the direct detection searches. Moreover, the same fields can lead to a negative contribution to the muon Yukawa coupling at one-loop order. Though the shift is beyond the reach of present experiments but can definitely be tested at the HL-LHC as it will reduce the uncertainty in κ_μ to 3–4% with a discovery-level of significance (5σ). Any such observed negative shift in y_μ, thus, can be considered as an indirect evidence of a dark sector coupling with the 2nd generation leptons, where a real singlet scalar can possibly explain the DM phenomenology. Therefore, to completely test this model, the $h \to \mu^+\mu^-$ search sensitivity at the HL-LHC must be supported by a positive signal from the future direct detection experiments.

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