$Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ in a Multiquark Color Flux-Tube Model

Yi-Heng Wang(王意恒), Jia Wei(韦佳), Chun-Sheng An(安春生)*, and Cheng-Rong Deng(邓成荣)*

School of Physical Science and Technology, Southwest University, Chongqing 400715, China

(Received 14 November 2022; accepted manuscript online 18 January 2023)

We systematically investigate the mass spectrum, spatial configuration and magnetic moment of the ground and p-wave states $[cu][\bar{c}\bar{s}]$ with various color-spin configurations in a multiquark color flux-tube model. Numerical results indicate that the state $Z_{cs}(4000)^+$ can be described as the compact state $[cu][\bar{c}\bar{s}]$ with 1^3S_1. Its main color-spin configuration is $[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$ and its magnetic moment is $0.73\mu_N$. The state $Z_{cs}(4220)^+$ can be depicted as the compact state $[cu][\bar{c}\bar{s}]$ with 1^1P_1 (or 1^3P_1). Its main color-spin configuration is $[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$ (or $[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$) and its magnetic moment is $0.12\mu_N$ (or $0.64\mu_N$). The physical state should be the mixture of these two different color-spin configurations and deserves further investigation. In addition, we also predict the properties of the states $[cu][\bar{c}\bar{s}]$ with other quantum numbers in the model.

DOI: 10.1088/0256-307X/40/2/021201

In 2020, the BESIII collaboration studied the processes of $e^+e^- \to K^+D_s^-D^{*0}$ and $K^+D_s^{*-}D^0$ reactions and observed an enhancement near the thresholds of $D_s^-D^{*0}$ and $D_s^{*-}D^0$,[1] called $Z_{cs}(3985)^-$. It is the first candidate for the charged hidden charm tetraquark state with strangeness and its mass and width in MeV are

$$Z_{cs}(3985)^-:\ M = 3982.5 \pm 2.1^{+1.8}_{-2.6},\quad \Gamma = 12.8^{+5.3}_{-4.4} \pm 3.0.$$

It was suggested as a spin parity of 1^+. However, other spin-parity assignments cannot be excluded. In 2021, the LHCb collaboration reported two exotic structures $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ in the $J/\psi K^+$ invariant mass spectrum of the $B^+ \to J/\psi\phi K$ decay.[2] The mass and width of the state $Z_{cs}(4000)^+$ were measured to be

$$Z_{cs}(4000)^+:\ M = 4003 \pm 6^{+4}_{-14},\quad \Gamma = 131 \pm 15 \pm 26.$$

Its spin parity was determined to be $J^P = 1^+$ with a high significance. The states $Z_{cs}(3985)^-$ and $Z_{cs}(4000)^+$ have comparable mass while their widths differ by an order of magnitude. The mass and width of the state $Z_{cs}(4220)^+$ in MeV are

$$Z_{cs}(4220)^+:\ M = 4216 \pm 24^{+43}_{-30},\quad \Gamma = 233 \pm 52^{+97}_{-73}.$$

Its spin parity may be either 1^+ or 1^-. The smallest quark content is $c\bar{c}s\bar{u}$ for the state $Z_{cs}(3985)^-$ and is $c\bar{c}u\bar{s}$ for the states $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$.

The investigation on their structure and property could help us to improve the understanding of the strong interactions and its underlying theory, quantum chromodynamics (QCD). Prior to the BESIII and LHCb experiments, the states had been explored with the hadronic molecular picture,[3] the compact tetraquark picture,[4,5] the hadro-quarkonium picture,[6] and the initial-single-chiral-particle-emission mechanism.[7] After the experiments, the states have been further researched within various theoretical frameworks.[8–16] Especially, whether the states $Z_{cs}(3895)^-$ and $Z_{cs}(4000)^+$ are two different states or not and how to understand their inner structures have attracted wide attention from the theoretical side.[17–20] Their most popular interpretation is describing them as two different pictures: compact tetraquark states with different J^P (Refs. [8,11]) or molecular states DD_s^* and D^*D_s.[16,18] More comprehensive descriptions on the states can be found in the latest review of the new hadron states.[21]

In this work, we prepare to perform a systematic investigation on the mass spectrum, spatial configuration and magnetic moment of the ground and p-wave states $[cu][\bar{c}\bar{s}]$ within the framework of the multiquark color flux-tube model (MCFTM). We anticipate to broaden the insight of the property and structure of the states $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ from the perspective of the MCFTM. We also hope that this work can improve the understanding of the mechanism of the low-energy strong interaction.

In this Letter, first we present the descriptions of the multiquark color flux-tube model. Then, we provide the trial wave functions of the state $[cu][\bar{c}\bar{s}]$. The numerical results and discussions are given. Finally, a brief summary is presented.

Multiquark Color Flux-Tube Model. The MCFTM has been developed on the basis of the color flux-tube picture in the lattice QCD[22] and the chiral quark model.[23] The model Hamiltonian includes one-gluon-exchange (OGE), one-boson-exchange (OBE), σ-meson exchange, and a multibody confinement potential depending on the color flux-tube structure. The complete Hamiltonian for mesons and the tetraquark states reads

$$\begin{aligned} H_n &= \sum_{i=1}^{n}\left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i}\right) - T_c + \sum_{i>j}^{n} V_{ij} + V^{\rm CON}(n), \\ V_{ij} &= V_{ij}^{\rm OGE} + V_{ij}^{\rm OBE} + V_{ij}^{\sigma}. \end{aligned} \tag{1}$$

In the state $[cu][\bar{c}\bar{s}]$, the codes of c, u, $\bar{c}$ and $\bar{s}$ are assumed to be 1, 2, 3, and 4, respectively; $\boldsymbol{p}_i$ and m_i are the momentum and mass of the i-th quark (antiquark),

*Corresponding authors. Email: ancs@swu.edu.cn; crdeng@swu.edu.cn
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respectively; T_c is the center-of-mass kinetic energy of the states and should be deducted. From the non-relativistic reduction of the OGE diagram in QCD for the point-like quarks, one gets

$$V_{ij}^{\rm OGE} = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c\Big(\frac{1}{r_{ij}} - \frac{2\pi\delta(\boldsymbol{r}_{ij})\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j}\Big), \tag{2}$$

where $\boldsymbol{\lambda}_i$ and $\boldsymbol{\sigma}_i$ are the color $SU(3)$ Gell-Mann matrices and the Pauli matrices, respectively. The Dirac $\delta(\boldsymbol{r}_{ij})$ function should be regularized in the form [23]

$$\delta(\boldsymbol{r}_{ij}) \to \frac{1}{4\pi r_{ij} r_0^2(\mu_{ij})} e^{-\frac{r_{ij}}{r_0(\mu_{ij})}}, \tag{3}$$

where $r_0(\mu_{ij}) = r_0/\mu_{ij}$, r_0 is an adjustable model parameter, and μ_{ij} is the reduced mass of two interacting particles i and j. The quark-gluon coupling constant α_s adopts an effective scale-dependent form

$$\alpha_s(\mu_{ij}^2) = \frac{\alpha_0}{\ln(\mu_{ij}^2/\Lambda_0^2)}, \tag{4}$$

where Λ_0 and α_0 are adjustable model parameters.

The origin of the constituent quark mass can be traced back to the spontaneous breaking of chiral symmetry. Chiral symmetry breaking suggests dividing quarks into two different sectors: light quarks (u, d, and s) where the chiral symmetry is spontaneously broken and heavy quarks (c and b) where the symmetry is explicitly broken. The OBE interactions only occur in the light quark sector. The central parts of the interactions originating from chiral symmetry breaking can be resumed as follows: [23]

$$\begin{aligned}
V_{ij}^{\rm OBE} =& V_{ij}^{\pi}\sum_{k=1}^{3}\boldsymbol{F}_i^k\boldsymbol{F}_j^k + V_{ij}^{K}\sum_{k=4}^{7}\boldsymbol{F}_i^k\boldsymbol{F}_j^k \\
&+ V_{ij}^{\eta}(\boldsymbol{F}_i^8\boldsymbol{F}_j^8\cos\theta_P - \sin\theta_P), \\
V_{ij}^{\chi} =& \frac{g_{\rm ch}^2}{4\pi}\frac{m_\chi^3}{12m_i m_j}\frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j \\
&\times\Big[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3}Y(\Lambda_\chi r_{ij})\Big], \quad \chi = \pi,\ K,\ \eta, \\
V_{ij}^{\sigma} =& -\frac{g_{\rm ch}^2}{4\pi}\frac{\Lambda_\sigma^2 m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2}\Big[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\Big].
\end{aligned} \tag{5}$$

The mass parameters m_χ take their experimental values, while the cutoff parameters Λ_χ and the mixing angles θ_P take the values from Ref. [23]. The mass parameter m_σ can be determined with the PCAC relation $m_\sigma^2 \approx m_\pi^2 + 4m_{u,d}^2$. [24] The chiral coupling constant $g_{\rm ch}$ can be obtained from the πNN coupling constant by

$$\frac{g_{\rm ch}^2}{4\pi} = \Big(\frac{3}{5}\Big)^2\frac{g_{\pi NN}^2}{4\pi}\frac{m_{u,d}^2}{m_N^2}. \tag{6}$$

Finally, any model imitating QCD should incorporate the nonperturbative color confinement effect. For an ordinary meson, the quark and anti-quark are connected by a three-dimensional color flux tube. Its confinement potential can be written as

$$V^{\rm CON}(2) = kr^2, \tag{7}$$

where r is the separation of the quark and anti-quark, and k is the stiffness of a three-dimensional color flux tube.

According to the double Y-shaped color flux-tube structure of the tetraquark state $[c_1u_2][\bar{c}_3\bar{s}_4]$, the four-body quadratic confinement potential instead of the linear one used in the lattice QCD can be written as

$$\begin{aligned}
V^{\rm CON}(4) =& k[(\boldsymbol{r}_1 - \boldsymbol{y}_{12})^2 + (\boldsymbol{r}_2 - \boldsymbol{y}_{12})^2 + (\boldsymbol{r}_3 - \boldsymbol{y}_{34})^2 \\
&+ (\boldsymbol{r}_4 - \boldsymbol{y}_{34})^2 + \kappa_d(\boldsymbol{y}_{12} - \boldsymbol{y}_{34})^2],
\end{aligned} \tag{8}$$

where $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, $\boldsymbol{r}_3$, and $\boldsymbol{r}_4$ are the particle's positions; $\kappa_d k$ is the stiffness of a d-dimensional color flux tube. [25] Two Y-shaped junctions $\boldsymbol{y}_{12}$ and $\boldsymbol{y}_{34}$ are variational parameters, which can be determined by taking the minimum of the confinement potential $V^{\rm CON}(4)$. Taking the values of $\boldsymbol{y}_{12}$ and $\boldsymbol{y}_{34}$ into the $V^{\rm CON}(4)$, one can express the minimum as

$$V^{\rm CON}(4) = k(\boldsymbol{R}_1^2 + \boldsymbol{R}_2^2 + \frac{\kappa_d}{1+\kappa_d}\boldsymbol{R}_3^2), \tag{9}$$

where $\boldsymbol{R}_i$ are a set of canonical coordinates and have the following forms:

$$\begin{aligned}
\boldsymbol{R}_1 &= \frac{1}{\sqrt{2}}(\boldsymbol{r}_1 - \boldsymbol{r}_2), \quad \boldsymbol{R}_2 = \frac{1}{\sqrt{2}}(\boldsymbol{r}_3 - \boldsymbol{r}_4), \\
\boldsymbol{R}_3 &= \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2 - \boldsymbol{r}_3 - \boldsymbol{r}_4), \\
\boldsymbol{R}_4 &= \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \boldsymbol{r}_3 + \boldsymbol{r}_4).
\end{aligned} \tag{10}$$

Compared to the chiral quark model, the MCFTM merely modifies the sum of two-body confinement potential to a multi-body quadratic one. Relative to the lattice QCD, we replace the linear potential with the quadratic one. For the ground and low excited states, their sizes are generally less than or around 1 fm, in which the difference between the quadratic potential and the linear one is not obvious. The difference can be further diluted by the adjustable stiffness of color flux tube. Moreover, the quadratic confinement potential can greatly simplify the numerical calculation in the dynamical investigation on the multiquark states.

Wave Functions. Within the framework of the diquark–antiquark configuration, the trial wave function of the state $[cu][\bar{c}\bar{s}]$ with $I(J^P)$ can be constructed as a sum of the following direct products of color φ_c, isospin φ_i, spin φ_s, and spatial ϕ terms:

$$\begin{aligned}
\boldsymbol{\Phi}_{IJ}^{[cu][\bar{c}\bar{s}]} =& \sum_\alpha c_\alpha\Big\{\Big[[\phi_{l_a m_a}(\boldsymbol{r}_a)\varphi_{s_a}]_{J_a}[\phi_{l_b m_b}(\boldsymbol{r}_b)\varphi_{s_b}]_{J_b}\Big]_{J_{ab}} \\
&\times\phi_{LM}(\boldsymbol{\rho})\Big\}_{JM_J}^{[cu][\bar{c}\bar{s}]}[\varphi_{i_a}\varphi_{i_b}]_{IM_I}^{[cu][\bar{c}\bar{s}]}[\varphi_{c_a}\varphi_{c_b}]_{\mathbf{1}_c}^{[cu][\bar{c}\bar{s}]}.
\end{aligned} \tag{11}$$

The subscripts a and b stand for the diquark $[cu]$ and antidiquark $[\bar{c}\bar{s}]$, respectively. The square brackets imply all possible Clebsch–Gordan couplings. The summation index α represents all of possible channels and the coefficient c_α is determined by the model dynamics.

The color representations of the diquark $[cu]$ can be antisymmetric $\bar{\mathbf{3}}_c$ and symmetric $\mathbf{6}_c$. Those of the antidiquark $[\bar{c}\bar{s}]$ can be antisymmetric $\mathbf{3}_c$ and symmetric $\bar{\mathbf{6}}_c$. The color configuration of the state $[cu][\bar{c}\bar{s}]$ can be written as

$$\begin{aligned}
(\bar{\mathbf{3}}_c \oplus \mathbf{6}_c)\otimes(\mathbf{3}_c \oplus \bar{\mathbf{6}}_c) =& \underbrace{(\bar{\mathbf{3}}_c \otimes \mathbf{3}_c)}_{\mathbf{1}_c\oplus\mathbf{8}_c} \oplus \underbrace{(\bar{\mathbf{3}}_c \otimes \bar{\mathbf{6}}_c)}_{\mathbf{8}_c\oplus\overline{\mathbf{10}}_c} \\
&\oplus \underbrace{(\mathbf{6}_c \otimes \mathbf{3}_c)}_{\mathbf{8}_c\oplus\mathbf{10}_c} \oplus \underbrace{(\mathbf{6}_c \otimes \bar{\mathbf{6}}_c)}_{\mathbf{1}_c\oplus\mathbf{8}_c\oplus\mathbf{27}_c}.
\end{aligned} \tag{12}$$

According to the overall colorless requirement, only two coupling modes, $[[cu]_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]_{\mathbf{3}_c}]_{\mathbf{1}_c}$ and $[[cu]_{\mathbf{6}_c}[\bar{c}\bar{s}]_{\bar{\mathbf{6}}_c}]_{\mathbf{1}_c}$, are permitted. In general, the state should be the mixture of two modes. Both the diquark $[cu]$ and antidiquark $[\bar{c}\bar{s}]$ can be in the spin singlet or triplet. The state $[cu][\bar{c}\bar{s}]$ with the total spin S can be denoted as $[[cu]^{s_b}[\bar{c}\bar{s}]^{s_b}]^S$, where $S = s_a \oplus s_b$. In general, the diquark and antidiquark are a spatially extended compound with various color-flavor-spin-space configurations.[26] The Pauli principle does not act on the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$ because of no identical particles. Therefore, various color-spin configurations of the state $[cu][c\bar{s}]$ can be expressed as

$$
\begin{aligned}
S=0:&\begin{cases}\left[[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{q}]^1_{\mathbf{3}_c}\right]^0_{\mathbf{1}_c}, \left[[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}\right]^0_{\mathbf{1}_c},\\ \left[[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}\right]^0_{\mathbf{1}_c}, \left[[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}\right]^0_{\mathbf{1}_c},\end{cases}\\
S=1:&\begin{cases}\left[[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}\right]^1_{\mathbf{1}_c}, \left[[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}\right]^1_{\mathbf{1}_c}, \left[[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}\right]^1_{\mathbf{1}_c},\\ \left[[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}\right]^1_{\mathbf{1}_c}, \left[[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}\right]^1_{\mathbf{1}_c}, \left[[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}\right]^1_{\mathbf{1}_c},\end{cases}\\
S=2:&\quad \left[[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}\right]^2_{\mathbf{1}_c}, \left[[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}\right]^2_{\mathbf{1}_c}.
\end{aligned}
$$

In the spatial parts, we assume that both the diquark $[cu]$ and antidiquark $[\bar{c}\bar{s}]$ are in the ground states, i.e., $l_a = l_b = 0$, and the angular excitation only occurs between them, denoted as L. Therefore, the p-parity of the state $[cu][\bar{c}\bar{s}]$ can be determined by $(-1)^L$. In order to obtain reliable numerical results, the precision numerical method is indispensable. The Gaussian expansion method (GEM),[27] which has been proven to be rather powerful to solve the few-body problem, is therefore used in the present work. According to the GEM, the relative motion wave function can be written as

$$\phi^G_{lm}(\boldsymbol{x}) = \sum_{n=1}^{n_{\max}} c_n N_{nl} x^l e^{-\nu_n x^2} Y_{lm}(\hat{\boldsymbol{x}}), \tag{13}$$

where $\boldsymbol{x}$ denotes the Jacobian coordinates $\boldsymbol{r}_a$, $\boldsymbol{r}_b$, and $\boldsymbol{\rho}$,

$$
\begin{aligned}
&\boldsymbol{r}_a = \boldsymbol{r}_1 - \boldsymbol{r}_2, \quad \boldsymbol{r}_b = \boldsymbol{r}_3 - \boldsymbol{r}_4,\\
&\boldsymbol{\rho} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2}{m_1+m_2} - \frac{m_3\boldsymbol{r}_3 + m_4\boldsymbol{r}_4}{m_3+m_4}.
\end{aligned} \tag{14}
$$

The Gaussian sizes ν_n are taken as geometric progression

$$\nu_n = \frac{1}{r_n^2},\ r_n = r_1 d^{n-1},\ d = \left(\frac{r_{\max}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{15}$$

More details about the GEM can be found in Ref. [27]. In the present work, we can obtain the convergent results of the tetraquark state $[cu][\bar{c}\bar{s}]$ by taking $n_{\max} = 7$, $r_1 = 0.1\,\text{fm}$, and $r_{\max} = 2.0\,\text{fm}$.

Numerical Results and Discussions. Here we focus on the $[cu][\bar{c}\bar{s}]$ spectrum and candidate of $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$. Using the GEM, we can obtain the adjustable model parameters by solving the two-body Schrödinger equation to fit the ground state meson spectrum in the MCFTM. The values of the parameters and the ground state meson spectrum are presented in Tables 1 and 2, respectively.

Table 1. Adjustable model parameters: quark mass and Λ_0 in units of MeV, k in units of MeV·fm^{-2}, r_0 in units of MeV·fm, and dimensionless α_0.

Parameter	$m_{u,d}$	m_s	m_c	k	α_0	Λ_0	r_0
Value	280	459	1564	316	2.68	41.78	51.21

Table 2. Ground state meson spectrum in the MCFTM, in units of MeV.

State	π	ρ	ω	K	K^*	ϕ	$D^\pm$
MCFTM	145	802	731	486	932	1046	1859
PDG	139	775	783	496	896	1020	1869
State	D^*	$D_s^\pm$	D_s^*	η_c	J/Ψ		
MCFTM	1985	1984	2106	2997	3095		
PDG	2007	1968	2112	2980	3097		

In the following, we concentrate on the natures of the ground and p-wave states $[cu][\bar{c}\bar{s}]$ with various color-spin combinations in the MCFTM. With the well-define trial wave functions, we can obtain the masses of the states by solving the four-body Schrödinger equation, as listed in Table 3. In order to realize the model dynamic effects, we calculate the contributions coming from each part in the model Hamiltonian using the corresponding eigenvectors, which are also listed in Table 3. The contributions originating from the π-and K-meson exchange interaction are universally equal to zero and therefore omitted. In addition, we also calculate the probability of each color-spin configuration $[cu]^{s_a}_{\mathbf{c}_a}[\bar{c}\bar{s}]^{s_b}_{\mathbf{c}_b}$ in the coupled channels, the average distances and magnetic moments.

It can be found from Table 3 that the confinement potential $\langle V^{\rm CON}\rangle$ and meson-exchange interactions $\langle V^\eta\rangle + \langle V^\sigma\rangle$ do not change obviously in each color-spin configuration. The differences among different color-spin configurations mainly come from the kinetic energy $\langle E_k\rangle$ and the OGE $\langle V^{\rm C}\rangle + \langle V^{\rm CM}\rangle$. The stronger the OGE, the greater the kinetic energy, and the lower the mass of the state $[cu][\bar{c}\bar{s}]$. The differences are about in the range of several tens to one hundred of MeV through the competition between the kinetic energy and the OGE. After considering the coupling of all color-spin configurations, the lowest configuration is further decreased with several tens of MeV mainly because of the OGE. Therefore, the C.C. mass is much lower ($\sim$100–200 MeV) than those in the specific color-spin configuration. C.C. represents the coupling of all possible configurations.

Table 3. Mass of the state $[cu][\bar{c}\bar{s}]$ in units of MeV, the ratio of each color-spin configuration $[cu]^{s_a}_{\mathbf{c}_a}[\bar{c}\bar{s}]^{s_b}_{\mathbf{c}_c}$, the average distance in units of fm, and the magnetic moment in units of μ_N. $J = L \oplus S$ and $S = s_a \oplus_b$, L and $\langle \boldsymbol{r}^2_{cu,\bar{c}\bar{s}}\rangle^{\frac{1}{2}}$ are the angular momentum and average distance between the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$, respectively; μ_s, μ_l, and μ stand for the spin magnetic moment, orbit magnetic moment, and total magnetic moment, respectively; $\langle E_k\rangle$, $\langle V^{\rm CON}\rangle$, $\langle V^{\rm CM}\rangle$, $\langle V^{\rm C}\rangle$, $\langle V^{\eta}\rangle$, and $\langle V^{\sigma}\rangle$ strand for the average value of the kinetic energy, confinement potential, color-magnetic interaction, Coulomb interaction, η- and σ-meson exchange interactions, respectively.

$n^{2S+1}L_J$	J^P	$[cu]^{s_a}_{\mathbf{c}_a}[\bar{c}\bar{s}]^{s_b}_{\mathbf{c}_b}$	Mass	Ratio	$\langle E_k\rangle$	$\langle V^{\rm CON}\rangle$	$\langle V^{\rm CM}\rangle$	$\langle V^{\rm C}\rangle$	$\langle V^{\eta}\rangle$	$\langle V^{\sigma}\rangle$	$\langle \boldsymbol{r}^2_{\bar{c}\bar{s}}\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{r}^2_{cu}\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{r}^2_{c\bar{c}}\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{r}^2_{u\bar{s}}\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{r}^2_{u\bar{c}}\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{r}^2_{c\bar{s}}\rangle^{\frac{1}{2}}$	$\langle \boldsymbol{r}^2_{cu,\bar{c}\bar{s}}\rangle^{\frac{1}{2}}$	μ_s	μ_l	μ
		$[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$	4051	27%	653	250	−55	−649	0	−16	0.74	0.83	0.62	1.08	0.93	0.83	0.59	0.00	0.00	0.00
		$[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}$	4128	6%	587	285	14	−611	0	−14	0.84	0.93	0.55	1.13	0.95	0.83	0.50	0.00	0.00	0.00
1^1S_0	0^+	$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4086	10%	610	262	−13	−623	−2	−15	0.77	0.86	0.61	1.11	0.94	0.84	0.57	0.00	0.00	0.00
		$[cs]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4023	57%	721	238	−100	−682	−3	−18	0.77	0.85	0.50	1.04	0.87	0.76	0.45	0.00	0.00	0.00
		C.C.	3945		825	213	−184	−752	−4	−20	0.72	0.80	0.48	0.98	0.82	0.72	0.43	0.00	0.00	0.00
		$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$	4086	15%	613	264	−17	−626	0	−15	0.75	0.88	0.63	1.11	0.96	0.84	0.59	1.32	0.00	1.32
		$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}$	4117	12%	599	279	3	−617	0	−15	0.84	0.91	0.55	1.12	0.94	0.83	0.49	1.32	0.00	1.32
		$[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4080	16%	616	262	−23	−627	0	−15	0.79	0.83	0.63	1.10	0.94	0.86	0.59	0.14	0.00	0.14
1^3S_1	1^+	$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4100	11%	593	269	1	−614	−1	−15	0.78	0.87	0.62	1.12	0.96	0.85	0.58	0.73	0.00	0.73
		$[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4120	12%	596	280	6	−615	0	−15	0.82	0.92	0.55	1.12	0.95	0.82	0.49	0.14	0.00	0.14
		$[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4068	34%	657	257	−47	−649	−1	−16	0.80	0.88	0.52	1.08	0.90	0.79	0.47	0.73	0.00	0.73
		C.C.	4006		713	236	−95	−693	−2	−18	0.76	0.83	0.52	1.04	0.87	0.77	0.48	0.66	0.00	0.66
		$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4128	90%	563	282	26	−597	1	−14	0.79	0.89	0.65	1.15	0.98	0.87	0.61	1.46	0.00	1.46
1^5S_2	2^+	$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4144	10%	569	291	28	−598	1	−14	0.84	0.93	0.56	1.15	0.96	0.84	0.51	1.46	0.00	1.46
		C.C.	4124		567	284	30	−611	1	−14	0.81	0.90	0.63	1.15	0.98	0.86	0.58	1.46	0.00	1.46
		$[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$	4227	90%	670	316	−50	−566	0	−11	0.77	0.86	0.85	1.25	1.11	1.02	0.82	0.00	0.13	0.13
		$[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}$	4369	< 1%	590	365	11	−453	0	−10	0.90	0.99	0.77	1.31	1.13	1.02	0.72	0.00	0.11	0.11
1^1P_1	1^-	$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4269	< 1%	621	333	−2	−538	−1	−11	0.81	0.90	0.84	1.27	1.13	1.03	0.81	0.00	0.12	0.12
		$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4311	9%	650	333	−49	−477	−2	−12	0.86	0.94	0.73	1.25	1.07	0.97	0.69	0.00	0.11	0.11
		C.C.	4216		690	308	−71	−567	−1	−12	0.78	0.86	0.82	1.23	1.10	1.00	0.79	0.00	0.12	0.12
		$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$	4258	22%	635	330	−16	−548	0	−11	0.77	0.90	0.85	1.27	1.14	1.03	0.82	1.32	0.11	1.43
		$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}$	4360	2%	597	359	2	−455	0	−11	0.90	0.97	0.76	1.30	1.11	1.01	0.72	1.32	0.10	1.42
		$[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4253	60%	638	328	−21	−549	0	−11	0.81	0.86	0.85	1.27	1.12	1.04	0.82	0.14	0.12	0.26
$1^3P_{0,1,2}$	$0^-,\ 1^-,\ 2^-$	$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4277	11%	612	337	5	−535	−1	−11	0.81	0.90	0.85	1.28	1.14	1.04	0.82	0.73	0.11	0.84
		$[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4363	1%	595	361	5	−454	0	−10	0.89	0.99	0.77	1.30	1.12	1.01	0.72	0.14	0.10	0.24
		$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4333	4%	625	344	−25	−466	−1	−11	0.88	0.96	0.75	1.27	1.09	0.99	0.71	0.73	0.10	0.83
		C.C.	4242		658	319	−41	−549	−1	−11	0.80	0.87	0.83	1.25	1.10	1.02	0.80	0.52	0.12	0.64
		$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	4291	98%	597	346	19	−528	1	−10	0.82	0.91	0.87	1.30	1.15	1.06	0.84	1.46	0.12	1.58
$1^5P_{1,2,3}$	$1^-,\ 2^-,\ 3^-$	$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	4373	2%	583	366	15	−447	1	−10	0.90	0.98	0.78	1.31	1.13	1.02	0.73	1.46	0.12	1.58
		C.C.	4290		598	345	18	−529	1	−10	0.82	0.90	0.87	1.30	1.15	1.05	0.83	1.46	0.12	1.58

Table 4. Color matrix elements, $\hat{\boldsymbol{O}}^c_{ij} = \boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j$.

Color	$\langle \hat{\boldsymbol{O}}^c_{12}\rangle$	$\langle \hat{\boldsymbol{O}}^c_{34}\rangle$	$\langle \hat{\boldsymbol{O}}^c_{13}\rangle$	$\langle \hat{\boldsymbol{O}}^c_{24}\rangle$	$\langle \hat{\boldsymbol{O}}^c_{14}\rangle$	$\langle \hat{\boldsymbol{O}}^c_{23}\rangle$
$\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$	$-\frac{8}{3}$	$-\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$
$\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$

Table 5. Color-spin matrix elements in the state $[cu][\bar{c}\bar{s}]$ with 1^1S_0, $\hat{\boldsymbol{O}}^{cs}_{ij} = \boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j \boldsymbol{\sigma}^c_i \cdot \boldsymbol{\sigma}^c_j$.

Color-spin	$\langle \hat{\boldsymbol{O}}^{cs}_{12}\rangle$	$\langle \hat{\boldsymbol{O}}^{cs}_{34}\rangle$	$\langle \hat{\boldsymbol{O}}^{cs}_{13}\rangle$	$\langle \hat{\boldsymbol{O}}^{cs}_{24}\rangle$	$\langle \hat{\boldsymbol{O}}^{cs}_{14}\rangle$	$\langle \hat{\boldsymbol{O}}^{cs}_{23}\rangle$
$[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$	8	8	0	0	0	0
$[cu]^0_{\mathbf{6}_c}[\bar{c}\bar{s}]^0_{\bar{\mathbf{6}}_c}$	−4	−4	0	0	0	0
$[cu]^1_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$	$-\frac{8}{3}$	$-\frac{8}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	$\frac{8}{3}$	$\frac{8}{3}$
$[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{20}{3}$	$\frac{20}{3}$	$\frac{20}{3}$	$\frac{20}{3}$

In Table 3, the $\langle \boldsymbol{r}^2_{cu}\rangle^{\frac{1}{2}}$ and $\langle \boldsymbol{r}^2_{\bar{c}\bar{s}}\rangle^{\frac{1}{2}}$ represent the size of the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$, respectively. The $\langle \boldsymbol{r}^2_{cu,\bar{c}\bar{s}}\rangle^{\frac{1}{2}}$ stands for the distance between the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$. Other distances between the quark (c or u) and the antiquark ($\bar{c}$ or $\bar{s}$) are also presented. All of the distances are less than or around 1 fm so that the states $[cu][\bar{c}\bar{s}]$ should be a compact spatial configuration in the MCFTM. Such compact spatial configuration mainly comes from the dynamics of the systems: the color flux tubes shrink the distance between any two connected particles to a distance as short as possible to decrease the confinement potential energy, while the kinetic motion expands the distance between any two quarks to a distance as long as possible to reduce the kinetic energy: the compact spatial configuration meets these requirements. The four-body confinement potential based on the color flux tubes plays a crucial role in the formation of the compact spatial configuration.

It can be found from Table 3 that the state $[cu][\bar{c}\bar{s}]$ with 1^1S_0 has four color-spin configurations. Although the di-

quark $[cu]^1_{\bar{\mathbf{6}}_c}$ and the antidiquark $[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$ are not a "good" configuration,[28] the color-spin configuration $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$ is dominant, reaching 57%, which is determined by the strong Coulomb interaction $\langle V^{\rm C}\rangle$ and color-magnetic interaction $\langle V^{\rm CM}\rangle$ because they can provide strong attraction. The interactions between the $[cu]^1_{\bar{\mathbf{6}}_c}$ and the $[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$, especially the Coulomb interaction, are stronger than those of other color-spin configurations, which can be understood by means of the color matrix elements in Table 4 and color-spin matrix elements in Table 5. The stronger the interactions, the smaller the average distance between the $[cu]^1_{\bar{\mathbf{6}}_c}$ and the $[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$, see $\langle \boldsymbol{r}^2_{cu,\bar{c}\bar{s}}\rangle^{\frac{1}{2}}$. The configuration $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$ has the lowest mass of 4023 MeV. After considering the coupling of the four configurations, the mass can be further decreased to 3945 MeV.

The state $[cu][\bar{c}\bar{s}]$ with 1^3S_1 has six different color-spin configurations. Similar to the state $[cu][\bar{c}\bar{s}]$ with 1^1S_0, the dominant color-spin configuration is still the $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$, reaching 34%, and has a mass of 4068 MeV in the MCFTM. The coupled mass of the six configurations is 4006 MeV, which is in good agreement with the data of the state $Z_{cs}(4000)^+$ reported by the LHCb collaboration. Therefore, the state $Z_{cs}(4000)^+$ can be described as the state $[cu][\bar{c}\bar{s}]$ with 1^3S_1 in the MCFTM. Its main component is the color-spin configuration $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$. On the whole, the color configuration $\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$ prevails over $\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$ in the state $[cu][\bar{c}\bar{s}]$ with 1^3S_1.

The color-spin configurations of the p-wave states $[cu][\bar{c}\bar{s}]$ are exactly same as the ground states if they have the same spin structures. In strong contrast to the ground state $[cu][\bar{c}\bar{s}]$ with 1^1S_0, the strong Coulomb interaction and color-magnetic interaction rapidly decrease, especially the Coulomb interaction, in the state $[cu][\bar{c}\bar{s}]$ with 1^1P_1 because the orbit excitation occurs between the $[cu]$ and $[\bar{c}\bar{s}]$, see Table 3. Relative to the $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$, the strong Coulomb interaction and color-magnetic interaction in the $[cu]^0_{\mathbf{3}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$ decreases slowly. Therefore, the mass of the $[cu]^0_{\mathbf{3}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$ is much lower than that of the $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$. After the coupling of four color-spin configurations, the state $[cu][\bar{c}\bar{s}]$ with 1^1P_1 has a mass of 4216 MeV and its main color-spin configuration is $[cu]^0_{\mathbf{3}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$ in the MCFTM. The mass is highly consistent with that of the state $Z_{cs}(4220)^+$ so that the state $[cu][\bar{c}\bar{s}]$ with 1^1P_1 could be the candidate of the state $Z_{cs}(4220)^+$.

In the p-wave states with $1^3P_{0,1,2}$, the spin-orbit interaction is not taken into account because it is just several MeV and does not change the qualitative conclusions.[29] Similar to the state with 1^1P_1, the main color-spin configuration is $[cu]^1_{\mathbf{3}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$ in the states. The coupled mass is about 4242 MeV, which is also consistent with the value of the state $Z_{cs}(4220)^+$ within the range of errors. Alternatively, the state $Z_{cs}(4220)^+$ can be described as the state $[cu][\bar{c}\bar{s}]$ with 1^3P_1 in the MCFTM if its J^P is identified as 1^-. If so, the physical state should be the mixture of the 1^1P_1 and 1^3P_1 states. In the model study, the mixing can be induced by the spin-orbit interactions proportional to $\hat{S}_-$ in the Hamiltonian,[23,30] which is left for the further study in the near future.

Both the ground and p-wave states $[cu][\bar{c}\bar{s}]$ with $S=2$ are also predicted in the MCFTM. In the ground state with 2^+, the color-magnetic interactions in the $[cu]^1_{\mathbf{3}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$ and $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$ are repulsive and their difference is very small, see Table 3. Similarly, the differences of their Coulomb interactions are also very small. The kinetic and confinement potential in the $[cu]^1_{\mathbf{3}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$ are slightly lower than those of the $[cu]^1_{\bar{\mathbf{6}}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$. Therefore, their main component is the color-spin configuration $[cu]^1_{\mathbf{3}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$. In the p-wave states with $1^5P_{1,2,3}$, their main color-spin configuration is also $[cu]^1_{\mathbf{3}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$. The coupled masses of the ground and p-wave states $[cu][\bar{c}\bar{s}]$ with $S=2$ are about 4124 MeV and 4290 MeV, respectively, which are far away from the states $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$.

The magnetic moments encode some useful information about the distributions of the charge and magnetization inside the hadrons, which can help us to understand their geometric configurations. The magnetic moment of a compound system is the sum of that of its constituents including spin and orbital contributions:[31,32]

$$\boldsymbol{\mu}=\sum_i\boldsymbol{\mu}_i=\sum_i(g_i\boldsymbol{s}_i+\boldsymbol{l}_i)\mu_i; \tag{16}$$

g_i and μ_i are, respectively, the Lande factor and the magneton of the i-th quark or antiquark,

$$\mu_i=\frac{e_i}{2m_i}, \tag{17}$$

where e_i and m_i are the charge and effective mass of the i-th quark or antiquark, respectively.

For the simplicity of calculating orbital magnetic moment, we take the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$ as the compound bosons so that the state $[cu][\bar{c}\bar{s}]$ can be simplified to a two-body system.[31,32] The effective masses of the diquark and the antidiquark are assumed to be approximately equal to the sum of the quark static masses, namely $M_{[cu]}=m_c+m_u$ and $M_{[\bar{c}\bar{s}]}=m_c+m_s$. The charges of the diquark $[cu]$ and antidiquark $[\bar{c}\bar{s}]$ are $+\frac{4e}{3}$ and $-\frac{e}{3}$, respectively, with e being the charge unit. Adopting the same procedure with that of Refs. [31,32], we can express the total orbital magnetic moment of the state $[cu][\bar{c}\bar{s}]$ as

$$\mu_l\boldsymbol{l}=\mu_{[cu]}\boldsymbol{l}_{[cu]}+\mu_{[\bar{c}\bar{s}]}\boldsymbol{l}_{[\bar{c}\bar{s}]}, \tag{18}$$

where $\boldsymbol{l}$ represents the relative angular excitation between the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$; $\boldsymbol{l}_{[cu]}$ and $\boldsymbol{l}_{[\bar{c}\bar{s}]}$ are the effective orbit excitation of the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$, respectively,

$$\boldsymbol{l}_{[cu]}=\frac{M_{[c\bar{c}\bar{s}]}}{M_{[cu]}+M_{[\bar{c}\bar{s}]}}\boldsymbol{l},\quad \boldsymbol{l}_{[\bar{c}\bar{s}]}=\frac{M_{[cu]}}{M_{[cu]}+M_{[\bar{c}\bar{s}]}}\boldsymbol{l}, \tag{19}$$

$\mu_{[cu]}$ and $\mu_{[\bar{c}\bar{s}]}$ are the magneton of the diquark $[cu]$ and the antidiquark $[\bar{c}\bar{s}]$, respectively,

$$\mu_{[cu]}=\frac{2e}{3M_{[cu]}},\quad \mu_{[\bar{c}\bar{s}]}=-\frac{e}{6M_{[\bar{c}\bar{s}]}}. \tag{20}$$

Finally, we can calculate the magnetic moments of the states $[cu][\bar{c}\bar{s}]$ with various color-spin configurations using the eigen wave function,

$$\mu=\langle\Phi_{IJ}^{[cu][\bar{c}\bar{s}]}|\boldsymbol{\mu}|\Phi_{IJ}^{[cu][\bar{c}\bar{s}]}\rangle, \tag{21}$$

their numerical results are presented in Table 3.

One can see from Table 3 that the magnetic moment of the ground state $[cu][\bar{c}\bar{s}]$ with 1^+, the candidate of the state $Z_{cs}(4000)^+$, is about $0.73\mu_N$ in the MCFTM. Within

the framework of the light-cone QCD sum rules, the magnetic of moment of the state $Z_{cs}(4000)^+$ is $0.73^{+0.28}_{-0.26}\mu_N$,[33] which is highly consistent with our result. The magnetic moments of the states $[cu][\bar{c}\bar{s}]$ with 1^1P_1 and 1^3P_1, the candidate of the state $Z_{cs}(4220)^+$ in the MCFTM, are $0.12\mu_N$ and $0.64\mu_N$, respectively. The latter is close to the result, $0.77^{+0.27}_{-0.25}\mu_N$, of the state $Z_{cs}(4220)^+$ in Ref. [33]. The experimental measurements of the magnetic moments of the states $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ are expected, which is helpful to understand the substructure of the states by comparing with the theoretical results.

In summary, we have systematically studied the mass spectrum, spatial configuration and magnetic moment of the ground and p-wave states $[cu][\bar{c}\bar{s}]$ with various color-spin configurations in the multiquark color flux-tube model. The state $Z_{cs}(4000)^+$ can be described as the compact state $[cu][\bar{c}\bar{s}]$ with 1^3S_1 in the model. Its main color-spin configuration is $[cu]^1_{\mathbf{6}_c}[\bar{c}\bar{s}]^1_{\bar{\mathbf{6}}_c}$ and its magnetic moment is $0.73\mu_N$. The state $Z_{cs}(4220)^+$ can be depicted as the compact state $[cu][\bar{c}\bar{s}]$ with 1^1P_1 (or 1^3P_1). Its main configuration is $[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^0_{\mathbf{3}_c}$ (or $[cu]^0_{\bar{\mathbf{3}}_c}[\bar{c}\bar{s}]^1_{\mathbf{3}_c}$) and its magnetic moment is $0.12\mu_N$ (or $0.64\mu_N$). The physical state should be the mixture of these two different color-spin configurations and deserves further investigation. Hopefully, the systematical investigation will be helpful for the understanding of the properties of the exotic states $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$.

Note that our model conclusion just serves as one of possible theoretical suggestions. Molecular state description of the states $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ is also possible. The states should be the linear combinations of the compact tetraquark states and loose molecular states and it is beneficial to understand the difference between the states $Z_{cs}(4000)^+$ and $Z_{cs}(3895)^-$.

Acknowledgments. This work was partly supported by the Chongqing Natural Science Foundation (Grant No. cstc2021jcyj-msxmX0078), and the Fundamental Research Funds for the Central Universities (Grant No. SWU118111).

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