Global Positioning Scheme via Quantum Teleportation

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Quantum teleportation scheme is undoubtedly an inspiring theoretical discovery as an amazing application of quantum physics, which was experimentally realized several years later. For the purpose of quantum communication via this scheme, an entangled ancillary pair shared by Alice and Bob is the essential ingredient, and a quantum memory in Bob's system is necessary for him to keep the quantum state until the classical message from Alice arrives. Yet, the quantum memory remains a challenge in both technology and rationale. Here we show that quantum teleportation provides fresh perspectives in terms of an alternative scheme for global positioning system. Referring to fixed locations of Bob and Charlie, Alice can determine her relative position by comparing quantum states before and after teleporting around via Bob and Charlie successively. This may open up a new scene in the stage of the application of quantum physics without quantum memories.

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As is well known, the conventional global positioning system (GPS) relies on the measurement of the distance between the user and reference satellites. This scheme requires ultra-high accuracy of the timer in the system and a sufficient number of satellites in outer space. Is there an alternative to the current GPS scheme? It will be interesting to explore along a new route for the purpose of positioning on a sphere without timer. In this Letter, we show that quantum teleportation could be a promising candidate.

Along with the development of quantum technologies,[1,2] quantum communication[3–5] is at the frontier of scientific research at present. The pivotal task in quantum communication is the transmission of quantum signals over long distances.[6–10] In this context, quantum teleportation[11–14] serves as a primitive operation scheme where the Bell states are regarded as a resource that relies on the generation of nearly perfect entangled states between distant locations. Signal amplification provides an effective solution for long-distance classical communications, but it fails in the quantum case. In quantum communication, signals are transmitted over long distances in a way that fulfills the principles of quantum mechanics. The simple amplification strategy is hindered by the so-called quantum non-cloning theorem[15] due to the linearity of quantum mechanics. To avoid the problem of exponentially decayed fidelity caused by photon loss and detector noise,[12] the crucial concept of a quantum repeater[16–22] was introduced to transmit quantum information over long distances. The basic idea of the quantum repeater is to partition the transmission channel into several segments of which the length scale is comparable to the channel attenuation length. This procedure is called entanglement swapping and it allows the creation of entanglement over distances at which direct transmission is infeasible. After establishing long-distance entanglement, one can teleport quantum states between two remote places. Since such a quantum teleportation scheme[11] requires a classical message to be sent from the sender to the receiver, the receiver needs to store the message for a sufficiently long time, which means that an effective quantum memory[23–25] is necessary for handling the quantum-based information from the remote sender.

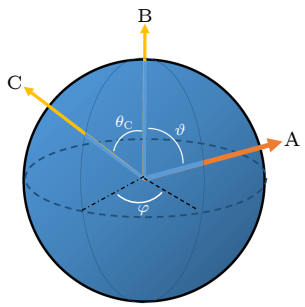

Fig. 1. Location parameters on a sphere exhibiting their relative latitudes and longitudes. Here A, B, and C denote Alice, Bob, and Charlie, respectively.

In our proposed global positioning scheme, we consider that Alice, Bob, and Charlie stay at different locations on a spherical surface (see Fig. 1). In terms of their local coordinate frames, the Bell bases are given by

$$|\psi_L^{\mp}\rangle_{\ell,\ell+1} = \frac{1}{\sqrt{2}}\big(|\uparrow_L\rangle_\ell|\downarrow_L\rangle_{\ell+1} \mp |\downarrow_L\rangle_\ell|\uparrow_L\rangle_{\ell+1}\big), \quad (1)$$

$$|\phi_L^{\mp}\rangle_{\ell,\ell+1} = \frac{1}{\sqrt{2}}\big(|\uparrow_L\rangle_\ell|\uparrow_L\rangle_{\ell+1} \mp |\downarrow_L\rangle_\ell|\downarrow_L\rangle_{\ell+1}\big), \quad (2)$$

with $L = \mathrm{A}$, B, and C representing Alice, Bob, and Charlie, respectively, and ℓ labeling the particles. Suppose that Alice and Bob share an Einstein–Podolsky–Rosen (EPR)

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pair, the Bell state $|\psi_{\rm A}^-\rangle_{12}$ of particles 1 and 2, Bob and Charlie share a Bell state $|\psi_{\rm B}^-\rangle_{34}$ of particles 3 and 4, and Charlie and Alice share a Bell state $|\psi_{\rm C}^-\rangle_{56}$ of particles 5 and 6 (see Fig. 2). These states are applied to faithfully transmit quantum states via the quantum teleportation scheme.[11] Their explicit expressions are given as follows:

$$|\psi_{\rm A}^-\rangle_{12} = \frac{1}{\sqrt{2}}\big(|\uparrow_{\rm A}\rangle_1|\downarrow_{\rm A}\rangle_2 - |\downarrow_{\rm A}\rangle_1|\uparrow_{\rm A}\rangle_2\big), \tag{3a}$$

$$|\psi_{\rm B}^-\rangle_{34} = \frac{1}{\sqrt{2}}\big(|\uparrow_{\rm B}\rangle_3|\downarrow_{\rm B}\rangle_4 - |\downarrow_{\rm B}\rangle_3|\uparrow_{\rm B}\rangle_4\big), \tag{3b}$$

$$|\psi_{\rm C}^-\rangle_{56} = \frac{1}{\sqrt{2}}\big(|\uparrow_{\rm C}\rangle_5|\downarrow_{\rm C}\rangle_6 - |\downarrow_{\rm C}\rangle_5|\uparrow_{\rm C}\rangle_6\big). \tag{3c}$$

Here, spin-up and spin-down are defined in different local frames; therefore, the subscripts A, B, and C are introduced to specify Alice, Bob, and Charlie's frames, respectively.

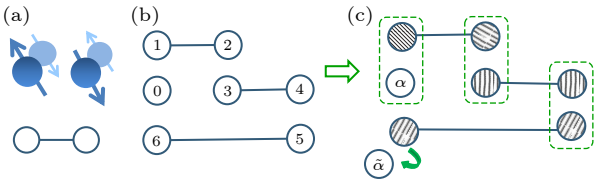

Fig. 2. Diagrammatic illustration of the teleportation scheme of seven particles. (a) Entangled state is diagrammatically notated by circles connected by a line. (b) Alice's Schrödinger's cat state $|\alpha\rangle$ and three EPR pairs shared by Alice, Bob, and Charlie are realized by particles labeled by 0, 1, 2,..., and 6. (c) Bell-state measurements indicated by dashed-line boxes cause wave-package collapse leading to the output state $|\tilde{\alpha}\rangle_6$ that Alice obtains. Here the Bell bases expressed in different frames are specified by different pencil-stripe orientations.

Firstly, Alice initially prepares a Schrödinger cat state

$$|\alpha\rangle_0 = \frac{1}{2}(|\uparrow_{\rm A}\rangle_0 + |\downarrow_{\rm A}\rangle_0),$$

in her lab with a particle named particle-0 for convenience. Then Alice takes a measurement with Bell bases of particle-0 and particle-1 with respect to the direct product state $|\alpha\rangle_0|\psi_{\rm A}^-\rangle_{12}$. This is illustrated in Fig. 2 by the dashed-line frame. If a measurement outcome corresponding to state $|\psi_{\rm A}^-\rangle_{01}$ is obtained, she takes a note A1, otherwise, if $|\psi_{\rm A}^+\rangle_{01}$, $|\phi_{\rm A}^-\rangle_{01}$ or $|\phi_{\rm A}^+\rangle_{01}$ is obtained, she takes the note A2, A3, or A4, correspondingly. Similarly, Bob and Charlie play the role of quantum repeaters to make measurements by Bell bases of particles 2 and 3, and of particles 4 and 5, respectively [see Figs. 2(b) and 2(c)]. Bob takes notes B1, B2, B3, or B4, while Charlie takes notes C1, C2, C3, or C4 according to their concrete measurement outcomes in terms of Bell bases. In this way, after a cyclic quantum teleportation (i.e., Alice → Bob → Charlie → Alice) of the initial state $|\alpha\rangle_0$, Alice will obtain a final state $|\tilde{\alpha}\rangle_6$. It will be very interesting for Alice to compare $|\alpha\rangle$ and $|\tilde{\alpha}\rangle$. In flat space, such a comparison seems meaningless; however, it will become geometrically meaningful if they are on a sphere.

As shown in Fig. 1, we choose Bob's location as the "north pole" of the sphere (i.e., $\theta_{\rm B} = \phi_{\rm B} = 0$). His local z-axis is along the normal vector of the surface, which is geometrically opposite to the direction of local gravity physically. As second reference site, Charlie's location is assumed at $\theta_{\rm C}$ and $\phi_{\rm C}$. For calculation simplicity, we adopt vanishing $\phi_{\rm C} = 0$ in counting the azimuthal angle. In Fig. 1, Alice is assumed to not know her precise position relative to Bob and Charlie at beginning. Using the quantum teleportation scheme, she is able to determine where she is. Let $\theta_{\rm A}$ and $\phi_{\rm A}$ (hereafter, we omit the subscript and simply use ϑ and φ without ambiguity) denote the polar and azimuthal angles that specify Alice's location on the sphere. In other words, Alice, Bob, and Charlie are positioned at various locations on a sphere, whose coordinates are given by $\boldsymbol{r}_{\rm A}$, $\boldsymbol{r}_{\rm B}$, and $\boldsymbol{r}_{\rm C}$, respectively, i.e., $\boldsymbol{r}_{\rm A} = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$, $\boldsymbol{r}_{\rm B} = (0, 0, 1)$, and $\boldsymbol{r}_{\rm C} = (\sin\theta_{\rm C}, 0, \cos\theta_{\rm C})$. In calculations, we use $|\uparrow_{\rm B}\rangle = |\uparrow\rangle$ and $|\downarrow_{\rm B}\rangle = |\downarrow\rangle$ together with Alice-site eigenkets:

$$|\uparrow_{\rm A}\rangle = \begin{pmatrix} \cos\frac{\vartheta}{2}\, e^{-i\varphi/2} \\ \sin\frac{\vartheta}{2}\, e^{i\varphi/2} \end{pmatrix}, \tag{4}$$

$$|\downarrow_{\rm A}\rangle = \begin{pmatrix} \sin\frac{\vartheta}{2}\, e^{-i\varphi/2} \\ -\cos\frac{\vartheta}{2}\, e^{i\varphi/2} \end{pmatrix}, \tag{5}$$

as well as the Charlie-site eigenkets:

$$|\uparrow_{\rm C}\rangle = \begin{pmatrix} \cos\frac{\theta_{\rm C}}{2} \\ \sin\frac{\theta_{\rm C}}{2} \end{pmatrix}, \tag{6}$$

$$|\downarrow_{\rm C}\rangle = \begin{pmatrix} \sin\frac{\theta_{\rm C}}{2} \\ -\cos\frac{\theta_{\rm C}}{2} \end{pmatrix}. \tag{7}$$

There are four Bell-state measurement outcomes for each of Alice, Bob, and Charlie. They are denoted, respectively, by Ai, Bi, and Ci for $i = 1, 2, 3, 4$ (denoted by A_i, B_i, and C_i in the following Tables for clarity in expression). These measurements imply successive projections by

$$\Pi_{\rm A} = \sum_i |\Phi_{\rm A}^i\rangle_{01}\langle\Phi_{\rm A}^i|_{01},$$

$$\Pi_{\rm B} = \sum_i |\Phi_{\rm B}^i\rangle_{23}\langle\Phi_{\rm B}^i|_{23},$$

$$\Pi_{\rm C} = \sum_i |\Phi_{\rm C}^i\rangle_{45}\langle\Phi_{\rm C}^i|_{45},$$

with respect to the tensor-product state $|\alpha\rangle_0|\psi_{\rm A}^-\rangle_{12}|\psi_{\rm B}^-\rangle_{34}$ $|\psi_{\rm C}^-\rangle_{56}$. Here the four Bell states in Eqs. (1) and (2) are renamed as $|\Phi_L^1\rangle_{\ell,\ell+1} = |\psi_L^-\rangle_{\ell,\ell+1}$, $|\Phi_L^2\rangle_{\ell,\ell+1} = |\psi_L^+\rangle_{\ell,\ell+1}$, $|\Phi_L^3\rangle_{\ell,\ell+1} = |\phi_L^-\rangle_{\ell,\ell+1}$, and $|\Phi_L^4\rangle_{\ell,\ell+1} = |\phi_L^+\rangle_{\ell,\ell+1}$ with $\ell = 0, 2, 4$ and $L = $ A, B, C for convenience in notations. There is a total of 64 cases occurring for them. We calculate all 64 quantum teleportation situations from Alice to Bob, then to Charlie, and finally back to Alice. We find that the results can be classified into three groups: there are 32 situations (see Table 1) that imply Alice's angle parameters together with Charlie's; there are 24 situations that do not imply Charlie's angle parameter $\theta_{\rm C}$ (see Table 2) and there are eight situations where no angle parameters appear (see Table 3). Furthermore, taking nonzero $\phi_{\rm C}$ we recalculate the eight situations listed in Table 3 and find that only four of them remain uninformative. Hence we relist them in Table 4 and Table 5, respectively.

Table 1. Thirty-two informative cases with $\theta_{\rm C}$.

Inner product $8\langle\alpha\|\widetilde{\alpha}\rangle$	Outcomes of Bell-state measurement			
$\sin\vartheta\sin\theta_{\rm C}\cos\varphi+\cos\vartheta\cos\theta_{\rm C}+i\sin\theta_{\rm C}\sin\varphi$	$A_4B_4C_3$	$A_2B_4C_3$		
$-\sin\vartheta\sin\theta_{\rm C}\cos\varphi-\cos\vartheta\cos\theta_{\rm C}-i\sin\theta_{\rm C}\sin\varphi$	$A_4B_1C_2$	$A_2B_1C_2$		
$-\sin\theta_{\rm C}\cos\varphi-i\sin\vartheta\sin\theta_{\rm C}\sin\varphi$	$A_4B_3C_3$	$A_2B_3C_3$		
$\sin\theta_{\rm C}\cos\varphi+i\sin\vartheta\sin\theta_{\rm C}\sin\varphi$	$A_4B_2C_2$	$A_2B_2C_2$		
$-\sin\vartheta\cos\theta_{\rm C}+\cos\vartheta\sin\theta_{\rm C}\cos\varphi$	$A_3B_4C_3$	$A_1B_1C_2$		
$\sin\vartheta\cos\theta_{\rm C}-\cos\vartheta\sin\theta_{\rm C}\cos\varphi$	$A_3B_1C_2$	$A_1B_4C_3$		
$-\sin\vartheta\sin\theta_{\rm C}-\cos\vartheta\cos\theta_{\rm C}\cos\varphi$	$A_3B_4C_2$	$A_3B_1C_3$		
$\sin\vartheta\sin\theta_{\rm C}+\cos\vartheta\cos\theta_{\rm C}\cos\varphi$	$A_1B_4C_2$	$A_1B_1C_3$		
$-\cos\theta_{\rm C}-i\cos\vartheta\sin\theta_{\rm C}\sin\varphi$	$A_3B_3C_3$	$A_1B_2C_2$		
$\cos\theta_{\rm C}+i\cos\vartheta\sin\theta_{\rm C}\sin\varphi$	$A_3B_2C_2$	$A_1B_3C_3$		
$-\sin\theta_{\rm C}+i\cos\vartheta\cos\theta_{\rm C}\sin\varphi$	$A_3B_3C_2$	$A_3B_2C_3$		
$\sin\theta_{\rm C}-i\cos\vartheta\cos\theta_{\rm C}\sin\varphi$	$A_1B_3C_2$	$A_1B_2C_3$		
$-\sin\vartheta\cos\theta_{\rm C}\cos\varphi+\cos\vartheta\sin\theta_{\rm C}-i\cos\theta_{\rm C}\sin\varphi$	$A_4B_4C_2$	$A_2B_4C_2$	$A_4B_1C_3$	$A_2B_1C_3$
$\cos\theta_{\rm C}\cos\varphi+i\sin\vartheta\cos\theta_{\rm C}\sin\varphi$	$A_4B_3C_2$	$A_2B_3C_2$	$A_4B_2C_3$	$A_2B_2C_3$

Table 2. Twenty-four informative cases without $\theta_{\rm C}$.

Inner product $8\langle\alpha\|\widetilde{\alpha}\rangle$	Outcomes of Bell-state measurement			
$-\cos\varphi-i\sin\vartheta\sin\varphi$	$A_4B_4C_1$	$A_2B_4C_1$		
$\cos\varphi+i\sin\vartheta\sin\varphi$	$A_4B_1C_4$	$A_2B_1C_4$		
$\sin\vartheta\cos\varphi+i\sin\varphi$	$A_4B_3C_1$	$A_2B_3C_1$		
$-\sin\vartheta\cos\varphi-i\sin\varphi$	$A_4B_2C_4$	$A_2B_2C_4$		
$-i\cos\vartheta\sin\varphi$	$A_3B_4C_1$	$A_1B_1C_4$		
$i\cos\vartheta\sin\varphi$	$A_3B_1C_4$	$A_1B_4C_1$		
$\cos\vartheta\cos\varphi$	$A_3B_3C_1$	$A_1B_2C_4$		
$-\cos\vartheta\cos\varphi$	$A_3B_2C_4$	$A_1B_3C_1$		
$-\sin\vartheta$	$A_1B_3C_4$	$A_1B_2C_1$		
$\sin\vartheta$	$A_3B_3C_4$	$A_3B_2C_1$		
$-\cos\vartheta$	$A_4B_3C_4$	$A_4B_2C_1$	$A_2B_3C_4$	$A_2B_2C_1$

Table 3. Eight situations of uninformative when $\phi_{\rm C}=0$ is chosen.

Inner product $8\langle\alpha\|\widetilde{\alpha}\rangle$	Outcomes of Bell-state measurement			
1	$A_3B_4C_4$	$A_3B_1C_1$		
-1	$A_1B_4C_4$	$A_1B_1C_1$		
0	$A_4B_4C_4$	$A_4B_1C_1$	$A_2B_4C_4$	$A_2B_1C_1$

Table 4. Four informative cases if $\phi_{\rm C}\neq 0$.

Inner product $8\langle\alpha\|\widetilde{\alpha}\rangle$	Outcomes of Bell-state measurement
$\cos\phi_{\rm C}-i\sin\vartheta\sin\phi_{\rm C}$	$A_3B_4C_4$
$-\cos\phi_{\rm C}+i\sin\vartheta\sin\phi_{\rm C}$	$A_1B_4C_4$
$i\cos\vartheta\sin\phi_{\rm C}$	$A_2B_4C_4$ $\quad$ $A_4B_4C_4$

Table 5. Four uninformative cases remained even if $\phi_{\rm C}\neq 0$.

Inner product $8\langle\alpha\|\widetilde{\alpha}\rangle$	Outcomes of Bell-state measurement
1	$A_3B_1C_1$
-1	$A_1B_1C_1$
0	$A_4B_1C_1$ $\quad$ $A_2B_1C_1$

It is now instructive to illustrate by some concrete examples. If the measurement outcome reads either A1B4C2 or A1B1C3 that Alice is able to know via classical communications with Bob and Charlie, Alice picks up (see Table 1) the following relation:

$$\sin\vartheta\sin\theta_{\rm C}+\cos\vartheta\cos\theta_{\rm C}\cos\varphi=8\langle\alpha|\widetilde{\alpha}\rangle, \tag{8}$$

where the magnitude on the right-hand side is determined by the final measurement, and Charlie's location parameter $\theta_{\rm C}$ is given on the left-hand side. As there are two unknown parameters ϑ and φ to be solved, Alice needs one more equation. It is then necessary to make another teleportation. Suppose that the outcome of the second teleportation is A3B1C2, which means

$$\sin\vartheta\cos\theta_{\rm C}-\cos\vartheta\sin\theta_{\rm C}\cos\varphi=8\langle\alpha|\widetilde{\alpha}'\rangle, \tag{9}$$

where $|\widetilde{\alpha}'\rangle$ denotes the state that Alice received in the second cyclic teleportation operation, and hence the right-

hand side of the above equation is determined. Therefore, Eq. (10) together with Eq. (11) provides Alice with the magnitudes ϑ and φ that identify her location. Clearly, this strategy is valid for those situations listed in Tables 1, 2, and 4. If the measurement outcomes are the situations shown in Table 5, Alice will be unable to determine her location. Thus, further experiment measurements need to be carried out. Now we can see that Tables 1, 2, and 4 contain informative measurement outcomes from which Alice is able to determine her location. Additionally, the situations in Table 2 do not contain Charlie's parameter, which means Alice can determine her location without knowing Charlie's precise position parameter $\theta_{\rm C}$. However, Table 5 contains uninformative outcomes from which she is not able to determine her location.

Table 6. Fidelity calculated for the sixty-four situations.

Outcomes	$A_iB_jC_k$	$64\,F(\theta'_{\rm A},\phi'_{\rm A},\Delta)$
$A_1B_1C_1$ $A_1B_4C_4$	$A_3B_1C_1$ $A_3B_4C_4$	$1-2\Delta$
$A_2B_1C_1$ $A_2B_4C_4$	$A_4B_1C_1$ $A_4B_4C_4$	2Δ
$A_1B_2C_1$ $A_1B_3C_4$	$A_3B_2C_1$ $A_3B_3C_4$	$\sin^2\theta'_{\rm A}+2\Delta\cos(2\theta_{\rm A'})$
$A_2B_2C_1$ $A_2B_3C_4$	$A_4B_2C_1$ $A_4B_3C_4$	$\cos^2\theta'_{\rm A}-2\Delta\cos(2\theta_{\rm A'})$
$A_1B_3C_1$ $A_1B_2C_4$	$A_3B_3C_1$ $A_3B_2C_4$	$\cos^2\theta'_{\rm A}\cos^2\phi'_{\rm A}+2\Delta[1-2\cos^2\theta'_{\rm A}\cos^2\phi'_{\rm A}]$
$A_2B_3C_1$ $A_2B_2C_4$	$A_4B_3C_1$ $A_4B_2C_4$	$\sin^2\theta'_{\rm A}\cos^2\phi'_{\rm A}+\sin^2\phi'_{\rm A}+2\Delta[-2\sin^2\theta'_{\rm A}\cos^2\phi'_{\rm A}+\cos(2\phi'_{\rm A})]$
$A_1B_4C_1$ $A_1B_1C_4$	$A_3B_4C_1$ $A_3B_1C_4$	$\cos^2\theta'_{\rm A}\sin^2\phi'_{\rm A}+2\Delta(1-2\cos^2\theta'_{\rm A}\sin^2\phi'_{\rm A})$
$A_2B_4C_1$ $A_2B_1C_4$	$A_4B_4C_1$ $A_4B_1C_4$	$\sin^2\theta'_{\rm A}\sin^2\phi'_{\rm A}+\cos^2\phi'_{\rm A}+2\Delta[-2\sin^2\theta'_{\rm A}\sin^2\phi'_{\rm A}-\cos(2\phi'_{\rm A})]$
$A_1B_1C_2$ $A_1B_4C_3$	$A_3B_1C_2$ $A_3B_4C_3$	$\sin^2\theta'_{\rm A}\cos^2\theta_{\rm C}+\cos^2\theta'_{\rm A}\sin^2\theta_{\rm C}\cos^2\phi'_{\rm A}-\frac{1}{2}\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}$ $+2\Delta[1-2\sin^2\theta'_{\rm A}\cos^2\theta_{\rm C}-2\cos^2\theta'_{\rm A}\sin^2\theta_{\rm C}\cos^2\phi'_{\rm A}+\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}]$
$A_2B_1C_2$ $A_2B_4C_3$	$A_4B_1C_2$ $A_4B_4C_3$	$\cos^2\theta'_{\rm A}\cos^2\theta_{\rm C}+\sin^2\theta'_{\rm A}\sin^2\theta_{\rm C}\cos^2\phi'_{\rm A}+\sin^2\theta_{\rm C}\sin^2\phi'_{\rm A}+\frac{1}{2}\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}$ $+2\Delta[1-2\cos^2\theta'_{\rm A}\cos^2\theta_{\rm C}-2\sin^2\theta'_{\rm A}\sin^2\theta_{\rm C}\cos^2\phi'_{\rm A}-2\sin^2\theta_{\rm C}\sin^2\phi'_{\rm A}-\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}]$
$A_1B_2C_2$ $A_1B_3C_3$	$A_3B_2C_2$ $A_3B_3C_3$	$\cos^2\theta_{\rm C}+\cos^2\theta'_{\rm A}\sin^2\theta_{\rm C}\sin^2\phi'_{\rm A}$ $+2\Delta[-2\cos^2\theta'_{\rm A}\sin^2\theta_{\rm C}\sin^2\phi'_{\rm A}-\cos(2\theta_{\rm C})]$
$A_2B_2C_2$ $A_2B_3C_3$	$A_4B_2C_2$ $A_4B_3C_3$	$\sin^2\theta'_{\rm A}\sin^2\theta_{\rm C}\sin^2\phi'_{\rm A}+\sin^2\theta_{\rm C}\cos^2\phi'_{\rm A}$ $+2\Delta(1-2\sin^2\theta'_{\rm A}\sin^2\theta_{\rm C}\sin^2\phi'_{\rm A}-2\sin^2\theta_{\rm C}\cos^2\phi'_{\rm A})$
$A_1B_3C_2$ $A_1B_2C_3$	$A_3B_3C_2$ $A_3B_2C_3$	$\sin^2\theta_{\rm C}+\cos^2\theta'_{\rm A}\cos^2\theta_{\rm C}\sin^2\phi'_{\rm A}$ $+2\Delta[-2\cos^2\theta'_{\rm A}\cos^2\theta_{\rm C}\sin^2\phi'_{\rm A}+\cos(2\theta_{\rm C})]$
$A_2B_3C_2$ $A_2B_2C_3$	$A_4B_3C_2$ $A_4B_2C_3$	$\sin^2\theta'_{\rm A}\cos^2\theta_{\rm C}\sin^2\phi'_{\rm A}+\cos^2\theta_{\rm C}\cos^2\phi'_{\rm A}$ $+2\Delta(1-2\sin^2\theta'_{\rm A}\cos^2\theta_{\rm C}\sin^2\phi'_{\rm A}-2\cos^2\theta_{\rm C}\cos^2\phi'_{\rm A})$
$A_1B_4C_2$ $A_1B_1C_3$	$A_3B_4C_2$ $A_3B_1C_3$	$\sin^2\theta'_{\rm A}\sin^2\theta_{\rm C}+\cos^2\theta'_{\rm A}\cos^2\theta_{\rm C}\cos^2\phi'_{\rm A}+\frac{1}{2}\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}$ $+2\Delta[1-2\sin^2\theta'_{\rm A}\sin^2\theta_{\rm C}-2\cos^2\theta'_{\rm A}\cos^2\theta_{\rm C}\cos^2\phi'_{\rm A}-\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}]$
$A_2B_4C_2$ $A_2B_1C_3$	$A_4B_4C_2$ $A_4B_1C_3$	$\cos^2\theta'_{\rm A}\sin^2\theta_{\rm C}+\sin^2\theta'_{\rm A}\cos^2\theta_{\rm C}\cos^2\phi'_{\rm A}+\cos^2\theta_{\rm C}\sin^2\phi'_{\rm A}-\frac{1}{2}\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}$ $+2\Delta[1-2\cos^2\theta'_{\rm A}\sin^2\theta_{\rm C}-2\sin^2\theta'_{\rm A}\cos^2\theta_{\rm C}\cos^2\phi'_{\rm A}-2\cos^2\theta_{\rm C}\sin^2\phi'_{\rm A}+\sin(2\theta'_{\rm A})\sin(2\theta_{\rm C})\cos\phi'_{\rm A}]$

Until now, our calculations and discussions have been carried out in terms of pure state formulism, which deals with a very ideal situation. In a practical system, noise[26–28] is always inevitable. Thus the aforementioned EPR pairs (pure states) shared by Alice, Bob, and Charlie are actually mixed states; namely,

$$\rho_{\rm A}=(1-\Delta')|\Phi^1_{\rm A}\rangle\langle\Phi^1_{\rm A}|+\frac{\Delta'}{3}\sum_{i=2}^{4}|\Phi^i_{\rm A}\rangle\langle\Phi^i_{\rm A}|, \tag{10a}$$

$$\rho_{\rm B}=(1-\Delta'')|\Phi^1_{\rm B}\rangle\langle\Phi^1_{\rm B}|+\frac{\Delta''}{3}\sum_{i=2}^{4}|\Phi^i_{\rm B}\rangle\langle\Phi^i_{\rm B}|, \tag{10b}$$

$$\rho_{\rm C}=(1-\Delta''')|\Phi^1_{\rm C}\rangle\langle\Phi^1_{\rm C}|+\frac{\Delta'''}{3}\sum_{i=2}^{4}|\Phi^i_{\rm C}\rangle\langle\Phi^i_{\rm C}|, \tag{10c}$$

where the three small parameters Δ's are relevant to the influence of the noises in the channels during quantum teleportation, and $|\Phi^j\rangle$ $(j=1,\,2,\,3,\,4)$ denote the four Bell bases.

Let $\rho=|\alpha\rangle\langle\alpha|$ denote the initial state prepared by Alice, the total density matrix of the whole seven particles is given by $\rho^{\rm tot}=\rho\otimes\rho_{\rm A}\otimes\rho_{\rm B}\otimes\rho_{\rm C}$. After Bell-state measurements successively by Alice, Bob, and Charlie, Alice will have some certain state represented by a reduced density matrix:

$$\widetilde{\rho}={\rm tr}_{\rm ABC}(M^{ijk}\rho^{\rm tot}). \tag{11}$$

Here ${\rm tr}_{\rm ABC}$ refers to the partial trace over the subspace where their measurements take place, and

$$M^{ijk}=\Pi^i_{\rm A}\otimes\Pi^j_{\rm B}\otimes\Pi^k_{\rm C}\otimes I,$$

in which $\Pi^i_L=|\Phi^i_L\rangle\langle\Phi^i_L|$ for $L={\rm A},\,{\rm B},\,{\rm C}$, and I is the

2×2 unit matrix. As there are four possible outcomes in the measurement for each of them, Alice will obtain 64 results of wave-package collapse. In the algebraic calculation, we need to carry out the projection process expressed by Eq. (11) one by one (i.e., for distinct i, j, k). Thus, Alice will have 64 possible expressions of the reduced density matrix.

As an experimental observable, it is worthwhile to evaluate the fidelity between the initial state ρ and the final state $\widetilde{\rho}$ after a cyclic teleportation,

$$F(\rho, \widetilde{\rho}) = \left[\mathrm{tr}\left(\sqrt{\rho^{1/2}\,\widetilde{\rho}\,\rho^{1/2}}\right)\right]^2, \tag{12}$$

of which the expression is in general dependent on the location parameters of Alice (θ'_{A}, ϕ'_{A}), Bob ($\theta_{\mathrm{B}} = 0$, $\phi_{\mathrm{B}} = 0$), and Charlie (θ_{C}, $\phi_{\mathrm{C}} = 0$), and also on the noise parameter Δ. Here we adopt prime in the polar and azimuthal angles of Alice to address the fact that the exact magnitudes to be solved are affected by the strength of noises. After long and complicated algebraic calculations, we obtain fidelities for all 64 situations one by one and list them accordingly in Table 6, where $\Delta = (\Delta' + \Delta'' + \Delta''')/3 - 4(\Delta'\Delta'' + \Delta'\Delta''' + \Delta''\Delta''')/9 + 16\Delta'\Delta''\Delta'''/27$ and the label $A_iB_jC_k$ specifies different outcomes of Bell-state measurement. There are eight cases where the fidelity is irrelevant to the location parameters (see first few lines in Table 6), which does not give any information about Alice's location but can help her estimate the magnitude Δ caused by noises. In the other cases (the other lines in Table 6), the obtained expression contains Alice's location parameters.

Table 7. Formulae for solving location and estimating accuracy.

Outcomes	$A_iB_jC_k$	Measured fidelity $64f(\vartheta,\varphi)$	Accuracy estimation $g(\delta\vartheta,\delta\varphi,\Delta)=0$
$A_1B_2C_1$	$A_3B_2C_1$	$\sin^2\vartheta$	$\delta\vartheta\sin(2\vartheta) = -2\Delta\cos(2\vartheta)$
$A_1B_3C_4$	$A_3B_3C_4$		
$A_2B_2C_1$	$A_4B_2C_1$	$\cos^2\vartheta$	
$A_2B_3C_4$	$A_4B_3C_4$		
$A_1B_3C_1$	$A_3B_3C_1$	$\cos^2\vartheta\cos^2\varphi$	$\delta\vartheta\sin(2\vartheta)\cos^2\varphi + \delta\varphi\cos^2\vartheta\sin(2\varphi) = \Delta(2-4\cos^2\vartheta\cos^2\varphi)$
$A_1B_2C_4$	$A_3B_2C_4$		
$A_2B_3C_1$	$A_4B_3C_1$	$\sin^2\vartheta\cos^2\varphi + \sin^2\varphi$	
$A_2B_2C_4$	$A_4B_2C_4$		
$A_1B_4C_1$	$A_3B_4C_1$	$\cos^2\vartheta\sin^2\varphi$	$\delta\vartheta\sin(2\vartheta)\sin^2\varphi - \delta\varphi\cos^2\vartheta\sin(2\varphi) = \Delta(2-4\cos^2\vartheta\sin^2\varphi)$
$A_1B_1C_4$	$A_3B_1C_4$		
$A_2B_4C_1$	$A_4B_4C_1$	$\sin^2\vartheta\sin^2\varphi + \cos^2\varphi$	
$A_2B_1C_4$	$A_4B_1C_4$		
$A_1B_1C_2$	$A_3B_1C_2$	$\sin^2\vartheta\cos^2\theta_{\mathrm{C}} + \cos^2\vartheta\sin^2\theta_{\mathrm{C}}\cos^2\varphi$	$\delta\vartheta[-\sin(2\vartheta)\cos^2\theta_{\mathrm{C}} + \sin(2\vartheta)\sin^2\theta_{\mathrm{C}}\cos^2\varphi + \cos(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi]$
$A_1B_4C_3$	$A_3B_4C_3$	$-\frac{1}{2}\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi$	$+\delta\varphi[\cos^2\vartheta\sin^2\theta_{\mathrm{C}}\sin(2\varphi) - \frac{1}{2}\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\sin\varphi]$
$A_2B_1C_2$	$A_4B_1C_2$	$\cos^2\vartheta\cos^2\theta_{\mathrm{C}} + \sin^2\vartheta\sin^2\theta_{\mathrm{C}}\cos^2\varphi$	$= \Delta[2-4\sin^2\vartheta\cos^2\theta_{\mathrm{C}} - 4\cos^2\vartheta\sin^2\theta_{\mathrm{C}}\cos^2\varphi$
$A_2B_4C_3$	$A_4B_4C_3$	$+\sin^2\theta_{\mathrm{C}}\sin^2\varphi + \frac{1}{2}\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi$	$+2\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi]$
$A_1B_2C_2$	$A_3B_2C_2$	$\cos^2\theta_{\mathrm{C}} + \cos^2\vartheta\sin^2\theta_{\mathrm{C}}\sin^2\varphi$	$\delta\vartheta\sin(2\vartheta)\sin^2\theta_{\mathrm{C}}\sin^2\varphi$
$A_1B_3C_3$	$A_3B_3C_3$		$-\delta\varphi\cos^2\vartheta\sin^2\theta_{\mathrm{C}}\sin(2\varphi)$
$A_2B_2C_2$	$A_4B_2C_2$	$\sin^2\vartheta\sin^2\theta_{\mathrm{C}}\sin^2\varphi + \sin^2\theta_{\mathrm{C}}\cos^2\varphi$	$= \Delta(2-4\cos^2\vartheta\sin^2\theta_{\mathrm{C}}\sin^2\varphi - 4\cos^2\theta_{\mathrm{C}})$
$A_2B_3C_3$	$A_4B_3C_3$		
$A_1B_3C_2$	$A_3B_3C_2$	$\sin^2\theta_{\mathrm{C}} + \cos^2\vartheta\cos^2\theta_{\mathrm{C}}\sin^2\varphi$	$\delta\vartheta\sin(2\vartheta)\cos^2\theta_{\mathrm{C}}\sin^2\varphi$
$A_1B_2C_3$	$A_3B_2C_3$		$-\delta\varphi\cos^2\vartheta\cos^2\theta_{\mathrm{C}}\sin(2\varphi)$
$A_2B_3C_2$	$A_4B_3C_2$	$\sin^2\vartheta\cos^2\theta_{\mathrm{C}}\sin^2\varphi + \cos^2\theta_{\mathrm{C}}\cos^2\varphi$	$= \Delta(2-4\cos^2\vartheta\cos^2\theta_{\mathrm{C}}\sin^2\varphi - 4\sin^2\theta_{\mathrm{C}})$
$A_2B_2C_3$	$A_4B_2C_3$		
$A_1B_4C_2$	$A_3B_4C_2$	$\sin^2\vartheta\sin^2\theta_{\mathrm{C}} + \cos^2\vartheta\cos^2\theta_{\mathrm{C}}\cos^2\varphi$	$\delta\vartheta[-\sin(2\vartheta)\sin^2\theta_{\mathrm{C}} + \sin(2\vartheta)\cos^2\theta_{\mathrm{C}}\cos^2\varphi - \cos(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi]$
$A_1B_1C_3$	$A_3B_1C_3$	$+\frac{1}{2}\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi$	$+\delta\varphi[\cos^2\vartheta\cos^2\theta_{\mathrm{C}}\sin(2\varphi) + \frac{1}{2}\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\sin\varphi]$
$A_2B_4C_2$	$A_4B_4C_2$	$\cos^2\vartheta\sin^2\theta_{\mathrm{C}} + \sin^2\vartheta\cos^2\theta_{\mathrm{C}}\cos^2\varphi$	$= \Delta[2-4\sin^2\vartheta\sin^2\theta_{\mathrm{C}} - 4\cos^2\vartheta\cos^2\theta_{\mathrm{C}}\cos^2\varphi$
$A_2B_1C_3$	$A_4B_1C_3$	$+\cos^2\theta_{\mathrm{C}}\sin^2\varphi - \frac{1}{2}\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi$	$-2\sin(2\vartheta)\sin(2\theta_{\mathrm{C}})\cos\varphi]$

Table 8. More formulae if $\phi_{\mathrm{C}} \neq 0$ is adopted.

Outcomes	$A_iB_jC_k$	Measured fidelity $64f(\vartheta,\varphi)$	Accuracy estimation $g(\delta\vartheta,\delta\varphi,\Delta)=0$
$A_1B_4C_4$	$A_3B_4C_4$	$\sin^2\vartheta\sin^2\phi_{\mathrm{C}} + \cos^2\phi_{\mathrm{C}}$	$\delta\vartheta\sin(2\vartheta)\sin^2\phi_{\mathrm{C}} = 2\Delta(1-2\cos^2\vartheta\sin^2\phi_{\mathrm{C}})$
$A_2B_4C_4$	$A_4B_4C_4$	$\cos^2\vartheta\sin^2\phi_{\mathrm{C}}$	

Considering that the Δ caused by noise during quantum teleportation is a small quantity and introducing angle derivations $\delta\vartheta$ and $\delta\varphi$ to measure the uncertainty of Alice's location parameters that she solved, i.e., $\theta'_{\mathrm{A}} = \vartheta + \delta\vartheta$, $\phi'_{\mathrm{A}} = \varphi + \delta\varphi$, we can separate the expression of fidelity as

$$F(\theta'_{\mathrm{A}}, \phi'_{\mathrm{A}}, \Delta) = f(\vartheta,\varphi) + g(\delta\vartheta,\delta\varphi,\Delta). \tag{13}$$

Then the cancelation of first-order terms in the derived fidelity in Table 6, $g(\delta\vartheta,\delta\varphi,\Delta)=0$, helps Alice to estimate such a uncertainty. The zeroth order term together with the measured fidelity f_{expt}, i.e.,

$$f(\vartheta,\varphi) = f_{\mathrm{expt}}, \tag{14}$$

enables experimentalists to solve Alice's location parameters. Thus, Alice is able to determine her relative position by measuring the fidelity and solving the relevant

equations that can be selected from Table 7 according to messages from Bob and Charlie through classical channels. Details of the formulae for both fidelity and accuracy estimation are listed in Table 7. Considering nonzero ϕ_C, we also recalculate the four more informative cases of Table 4 and obtain additional formulae in Table 8.

It is worthwhile to approximatively estimate the success probability of the aforementioned protocol on the basis of Table 7. One can reasonably take the occurrence probability of each situation there as 1/64. For a single performance of this protocol, the probabilities of attaining class I, a trivial formula without any angle parameters, or class Ⅱ, a formula containing ϑ without φ, are $8/64 = 1/8$. When performing the protocol twice, the failure probability is $19/64 = 0.297$, which implies that the occurrence of either the first or the second performance is in class I, or both performances are in class I or in class Ⅱ, or both in the other classes but with the same formula. Thus, the success probability of performing the protocol twice is about 70.3 percent. Similarly, the success probability of performing the protocol three times is estimated to be 97.4 percent. The dependence of fidelity on the location parameters given in the second column of Table 7 was obtained for zero ϕ_C, in which there are always four situations with the same formula. If one considers nonzero ϕ_C, the degeneracy of four-to-one correspondence will split into two groups with two-to-one correspondence. Again, the success probability for $\phi_C \neq 0$ is estimated to be 82.0 percent for performing the protocol twice and to be 99.4 percent for performing the protocol three times.

In quantum communication, the receiver has to wait for classical signals to deal with quantum signals. In order to avoid this obstacle of quantum memory, various proposals were attempted,[24,25,29] where a series of challenges at various levels of difficulty occur when researchers devise methods to effectively manipulate remote parties sharing highly entangled photons. Although the all-photonic scheme can remove quantum memories at the intermediate repeater nodes, quantum memory at the end nodes is still necessary if the receiver requires a quantum output state.[25] However, for the purposes of determining the relative position by the aforementioned scheme, one can compare the states before and after the teleportation without waiting. Thus the technological obstacle of quantum memory makes no sense. The role of the classical signals is only helping Alice to resolve her location information correctly (because there are 64 situations, Alice needs information via the classical channel to choose the right formula to calculate).

Quantum mechanics is a mathematical framework for the construction of physical theories. It has been an indispensable part of science and applied with great success to almost every aspect of nature, including nuclear fusion in stars, the structure of atoms, elementary particles, superconductors, the structure of DNA, etc. Quantum information and quantum computation[30,31] are regarded as the second revolution of quantum mechanics, where the Bell states are regarded as a resource for quantum communication. There is still a long way to go before achieving an universal quantum computer. Quantum simulation[32–35] is proposed as intermediary steps on the lengthy path forward. One can perceive that our global positioning scheme is a terrific application of quantum teleportation beyond conventional quantum communications. Additionally, unlike the conventional GPS that requires ultra-highly accurate timers so that the distances between the user and neighboring satellites can be determined as precisely as possible, the present scheme via quantum teleportation does not rely on timers. As a rough magnitude estimation, for example, a location-resolution of 100 meters on the Earth requires the time-accuracy to be $\delta t \approx 3.336 \times 10^{-10}$ s in conventional GPS, while it requires the noise parameter $\Delta \approx 1.573 \times 10^{-5}$ in the present teleportation scheme. As the noise parameter Δ affects the angle resolution in our teleportation scheme, while the accuracy of the timer determines the distance resolution in the GPS scheme, if these are applied to the moon, the resolution will become 27 meters in the teleportation scheme but it remains 100 meters in the conventional GPS scheme. This is undoubtedly a remarkable application of quantum mechanics. Like the theoretical discovery[11] and subsequent experimental realization[13,14] of quantum teleportation, as a first step, it is worthwhile for experimentalists to set up an experiment in a laboratory by introducing certain "angle parameters" to demonstrate our proposal. Note that present protocol is a prototype. The entanglement is sensitive to the environment, which causes either spin mixing or amplitude damping. We only consider the former case and do not deal with the latter in the present study. In order to enhance the practical feasibility of present scheme, further investigations by means of the tolerant entanglement distribution approach[26,31] will be necessary.

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