A Hierarchy in Majorana Non-Abelian Tests and Hidden Variable Models

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The recent progress of the Majorana experiments paves a way for the future tests of non-Abelian braiding statistics and topologically protected quantum information processing. However, a deficient design in those tests could be very dangerous and reach false-positive conclusions. A careful theoretical analysis is necessary so as to develop loophole-free tests. We introduce a series of classical hidden variable models to capture certain key properties of Majorana system: non-locality, topologically non-triviality, and quantum interference. Those models could help us to classify the Majorana properties and to set up the boundaries and limitations of Majorana non-Abelian tests: fusion tests, braiding tests and test set with joint measurements. We find a hierarchy among those Majorana tests with increasing experimental complexity.

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The building blocks of topological quantum computation[1–3] are non-Abelian anyons, which are proposed to show novel non-Abelian braiding statistics.[3–6] A simple type of non-Abelian anyon, i.e., Majorana zero mode (MZM), was theoretically proposed[7–15] in many physically realizable systems. The latest advancements in experimental studies[16–37] have substantially contributed to our understanding, although there remains contentious interpretation surrounding the observation of Majorana resonances. The gradual, experiment-based step-by-step construction of Majorana devices[38,39] is essential; however, these developments present a promising platform for examining non-Abelian braiding statistics and actual quantum information processing. The most popular schemes proposed for validating non-Abelian statistics are braiding tests and fusion tests.[3,40–44] Moreover, joint-measurement, C-NOT gate, non-trivial T-gate test and Bell non-locality test[45,46] are believed to be necessary for quantum information processing. Here, we focus on relatively simple properties: Majorana non-local behaviors, topologically non-triviality, and quantum interference and entanglement in the near future. Among them, the realistic experimental schemes for controlling Majorana systems were theoretically proposed;[40,44] and some key proposals have already been tested in the laboratory for Majorana quantum information processing.[47–54]

In the hidden variable theories, the probabilities appear when we measure physical quantities due to the lack of the knowledge or statistical approximation in the underlying complex deterministic theory,[55–65] which reproduces certain behaviors in quantum mechanics. The experimental activities in the Bell inequality test eventually rule out local hidden variable explanations in several physical systems.[57,66–77] However, this is not sufficient to say that the behaviors observed in other systems, e.g., Majorana platforms, are of quantum mechanics. Furthermore, the hidden variable theory provides some significant viewpoints for investigating in quantum information process and fundamental aspects of the micro world.[78–84] For our purpose, if we want to validate Majorana behaviors seriously for future quantum information applications, it is reasonable to treat the system as a black-box and perform the test subsequently. In that sense, potential hidden variable theories need to be carefully considered and excluded. Otherwise, a deficient test design may reach a positive but incorrect conclusion. For example, braiding and measurements in Majorana systems can generate topologically protected Clifford gates including entanglement gates, which can also be realized in certain hidden variable theories.[64] Therefore, it is natural to ask whether we can find certain hidden variable theories such that each theory can capture certain properties of Majorana systems but can not capture other tests' results; and the hope is to set up the limitations and boundaries of each Majorana test by using these theories.

In this work, we propose a couple of classical hidden variable (HV) models, which are designed to capture and/or distinguish certain key quantum mechanical properties of Majorana systems: simplest Majorana non-local behaviors, topologically non-triviality and quantum interference. We would show what properties can be seen in a particular test; and examine if the test results can be captured by both the theories, which help us to set up the boundaries and limitations of the corresponding test. Based on this philosophy, we show: (1) fusion tests only capture certain Majorana non-local behaviors but cannot

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capture others; (2) braiding tests show non-Abelian statistics indicating topologically non-triviality, but not necessarily quantum interference; (3) test set with joint measurements is helpful and necessary to capture quantum interference in Majorana systems. Finally, we conclude that there exists a hierarchy among the Majorana non-Abelian tests: fusion test, braiding test, and test set with joint measurements.

Theoretical Framework. Now, we introduce a couple of HV models which are inspired from famous Spekkens' model.[64]

Classical HV Theory I. We assume that: (1) we have incomplete knowledge about the system states for some unknown complexity, (2) we get an outcome and observe incomplete knowledge after specific operations called measurements, (3) we cannot get access to their hidden complexity.

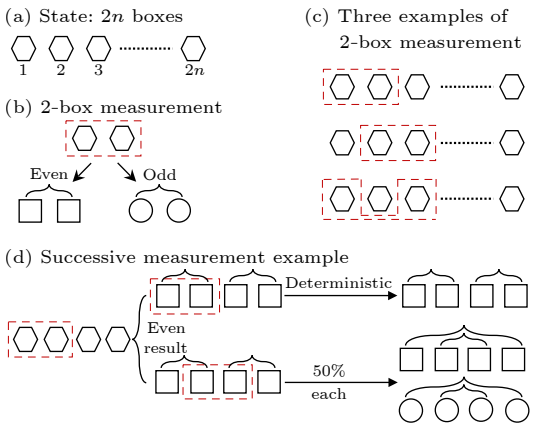


Fig. 1. (a)–(c) The HV model I, where the number from 1 to $2n$ labels the position of boxes from left to right. (d) A four-box system (with total parity even) for fusion. If we get even parity result after box-1,2 measurement, and continue to apply box-1,2 measurement, the state will be unchanged. However, if we apply a box-2,3 measurement in the second step, we will have two possible outcomes with equal probability.

(i) *State Formalism.* We consider a classical theory with a box representation, where each box corresponds to an MZM as shown in Fig. 1; and the box can be either filled or empty. We assume that we can only know the parity of any (pair of) two boxes after measurement. For example, we measure the combination of the first and second boxes and then get the result even. We can only know that the two boxes are either both filled or empty, but cannot determine the exact state of each box. This uncertainty is due to the unobserved classical complexity corresponding to the hidden variables.

(ii) *Measurement.* We show three kinds of 2-box measurements (box 1,2, box 2,3, and box 1,3) in Fig. 1. Each measurement returns two-box parity: even or odd. We also assume that the parity of the all-box system is fixed similarly to the Majorana systems. After box-1,2 measurement as an example, we could either obtain an even-parity result $\overbrace{\square\square}$ with a pair of square symbols or obtain an odd parity result $\overbrace{\bigcirc\bigcirc}$ with a pair of circle symbols; we use a bracket to label post-measurement box-pair called "connections". Considering a four-box (with total parity even) example shown in Fig. 1(d), box-1,2 measurement will reach either an even-even state: $\overbrace{\square\square}\,\overbrace{\square\square}$, or an odd-odd state: $\overbrace{\bigcirc\bigcirc}\,\overbrace{\bigcirc\bigcirc}$ with equal probability. If we repeat the previous measurement, we will get the same result; if the measurement is different from the previous one, we will get an uncertain result.

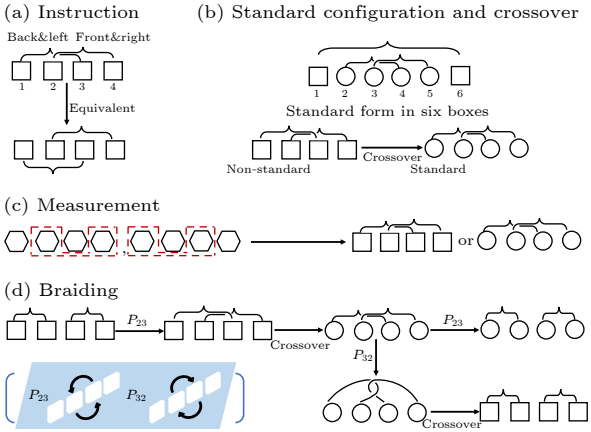


Fig. 2. The HV model Ⅱ. (a) Two equivalent forms of state description: the lower means the back. (b) The standard configuration of a six-box state: the connection of box 1,6 is both the rightmost and frontmost as definition, and connection of box 3,5 is relative right and front to box 2,4. For non-standard configurations, we can apply crossover along with a parity change. (c) Both box-1,3 and box-2,4 measurements lead to standard configurations. (d) Braiding procedures P_{23}^2 and $P_{23}P_{32}$. The braiding is confined in the plane perpendicular to the paper, or in the light-blue plane shown in the bracket.

Classical HV Theory Ⅱ: To include topology, we take into account the relative positions of connections if two connections have overlaps as shown in Fig. 2(a): (1) the front-back order (front connections are on the top of back ones) of the connections has a non-trivial consequence; (2) we also define the right-left order of connections, where the connection is on the right as long as one box in the pair is on the right of all boxes in other box pairs.

(i) *Standard States.* For the order of connections discussed above, we can choose any order. However, for the convenience of discussion, we define a standard configuration with fixed order: the right connection is the front connection [e.g. in the upper part of Fig. 2(b)]. For non-standard cases, we need to apply crossovers (exchange positions of two overlapping connections) to reach a standard configuration. Each crossover will induce a parity change [an example shown in Fig. 2(b)]. In the text, we sometimes use an up-down bracket to describe an equivalent front-back position, e.g., in Fig. 2(a).

(ii) *Measurement.* We only consider measurements in standard configurations, i.e., "standard measurement rule". For an unidentified state, measurements [e.g., both box-2,4 and box-1,3 measurements in Fig. 2(c)] always result in standard configurations. On the other hand, if we know the connections of boxes, we will only apply the

standard rule for the standard cases. If we are in a non-standard form, we will need to apply crossovers to reach a standard one.

(iii) *Braiding.* In our theory, braiding corresponds to the position exchange of boxes in the plane shown in Fig. 2(d). Considering different directions of braiding, the exchange of box n_1 and box n_2 is labeled by $P_{n_1 n_2}$ for anticlockwise exchange if $n_1 < n_2$ (for clockwise exchange if $n_1 > n_2$). In an example shown in Fig. 2(d), we apply two paths of braiding operations on the same initial state $\overbrace{\square\square}\,\overbrace{\square\square}$. The first path contains two successive P_{23}, where the first P_{23} takes the state to a non-standard form and we apply a crossover to reach the standard one, then applying the second P_{23} to get $\overbrace{\bigcirc\bigcirc}\,\overbrace{\bigcirc\bigcirc}$. In the second path, the first part is the same while we apply P_{32} in the next step and we get a non-standard form with a knot. To reach a standard configuration, we apply a crossover to remove the knot and to obtain the form $\overbrace{\square\square}\,\overbrace{\square\square}$.

Majorana Non-Abelian Tests. Let us now consider a variety of Majorana tests, and check if their key properties, e.g., Majorana non-local behaviors, topologically non-triviality, and quantum interference, can be captured by our classical HV theories I and Ⅱ.

(I) *Fusion Test.* The non-Abelian anyon system follows special fusion processes which describe the outcomes after anyon combinations.[3] Majorana zero modes belong to the Ising non-Abelian anyon model, which includes three types of anyons: the vacuum I, non-Abelian anyon σ (i.e., Majorana), and the fermion ψ. A pair of σ combines to fuse into either a vacuum or a fermion: $\sigma \times \sigma \to I + \psi$. With more anyons, we have multiple ways to fuse them together, where the quantum states describing fusion transformation can be written as

$$|a, b \to i\rangle|i, c \to d\rangle = \sum_j (F^d_{abc})^i_j |b, c \to j\rangle|a, j \to d\rangle, \quad (1)$$

where $|a, b \to i\rangle|i, c \to d\rangle$ indicates a state with particular fusion channel with anyons a and b first fusing to i, and then i and c fusing to d. The matrix $(F^d_{abc})^i_j$ describes the transformation under different fusion channels. For Ising anyons,

$$F^{\sigma}_{\sigma\sigma\sigma} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Now, let us first focus on our HV theory I. We have

$$\bigcirc\bigcirc\bigcirc\bigcirc \to \overbrace{\square\square}\,\overbrace{\square\square} \to (\overbrace{\square\,\overbrace{\square\square}\,\square} + \overbrace{\bigcirc\,\overbrace{\bigcirc\bigcirc}\,\bigcirc})/2, \quad (2)$$

$$\bigcirc\bigcirc\bigcirc\bigcirc \to \overbrace{\bigcirc\bigcirc}\,\overbrace{\bigcirc\bigcirc} \to (\overbrace{\square\,\overbrace{\square\square}\,\square} + \overbrace{\bigcirc\,\overbrace{\bigcirc\bigcirc}\,\bigcirc})/2. \quad (3)$$

Here, the first arrow in Eqs. (2) and (3) indicates the fusion (measurement) of box 1,2, which yields either fixed even parity (vacuum I) or odd parity (fermion ψ); the following box-2,3 measurement (second arrow) can have two possibilities: box 2 and box 3 first fuse into even parity or odd parity. We see that the measurements in the HV model are analogous to the fusion, and reach the same measurement statistics as those in the Ising anyon model described by the above F matrix. Considering a proposed fusion test in Ref. [41], the topological superconducting device can realize two different 4-MZM fusion routes: (1) First create two pairs of MZMs and then fuse in the same pair as their creation. (2) First create two pairs of MZMs and then fuse in the different pairs. These two fusion routes give different measurement statistics. As shown in Fig. 3, it is simple to check that the corresponding fusion procedures in our HV model generate the same measurement outcome statistics as those in Majorana cases. Our HV model can fully capture the measurement outcome after fusion-only processes in Majorana systems (see part I in the Supplemental Information (SI)[85] for more examples). This HV theory I does not include any topological consideration. Therefore, there is no doubt that the fusion tests can only capture certain Majorana non-locality behaviors but not including topological non-triviality.

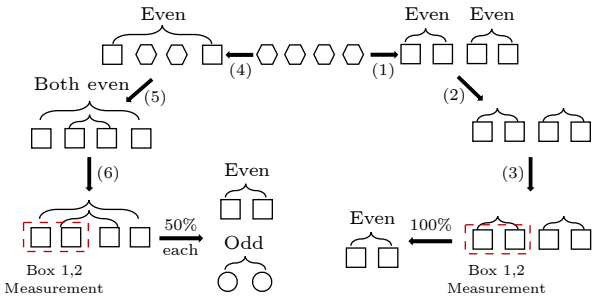


Fig. 3. Fusion process of our HV model reproducing the Majorana fusion processes in Ref. [41]. In the right path steps 1–3: the 4-box state splits into two pairs, both 2-box measurements obtain even, and then the measurement in the same box pair (box 1,2) obtains deterministic even parity result. In the left path steps 4–6: the 4-box state splits into two different pairs (box 1,4 and box 2,3) with even parity after measurement, and then the box-1,2 measurement gives us an equal probability of even or odd result.

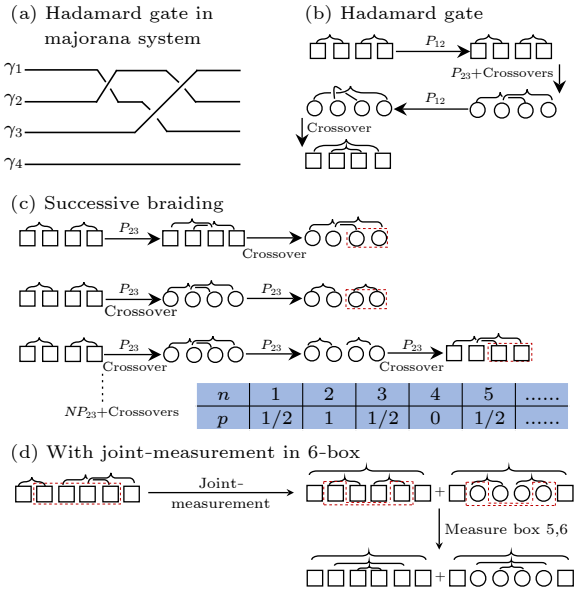
(Ⅱ) *Braiding and Topology.* Non-Abelian anyons show nontrivial behaviors under the braiding process, and the exchange operation of anyons a and b can be captured by the operator $B_{\rm ab}$, where these braiding operations can be written as $B_{ij} \propto 1 + \gamma_i\gamma_j$ for Majorana modes.[3]

Let us consider whether our HV theory I can capture braiding relations. For example, braiding box 2,3 of initial state $\overbrace{\square\square}\,\overbrace{\square\square}$ requires the exchange of boxes in different box pairs, as in different fusion channels of the Majorana theory. After this braiding, we can reach $\overbrace{\square\,\overbrace{\square\square}\,\square}$ (no connection order is considered here), then the box-1,3 measurement (corresponding to σ_y measurement in the quantum case[85]) results in even parity. However, the σ_y measurement of quantum state $B_{23}|0\rangle$ reaches odd. Therefor, we fail to capture the braiding process without topological consideration. Now let us look at HV theory Ⅱ. Considering the the same process on the initial state $\overbrace{\square\square}\,\overbrace{\square\square}$, we reach the state $\overbrace{\bigcirc\bigcirc}\,\overbrace{\bigcirc\bigcirc}$ as shown in the first path in Fig. 2(d), which is the same as the quantum case.

We further examine the Hadamard gates which can

be realized using a sequence of Majorana braidings $H = B_{12}B_{23}B_{12}$ in the quantum anyon model as shown in Fig. 4(a). For our HV theory Ⅱ in Fig. 4(b), we also successively braid box 1,2 (P_{12}), box 2,3 (P_{23}) and box 1,2 (P_{12}) and then apply a crossover to get a standard form. We can simply show any measurement here to give the same results as those in the Majorana Hadamard gate. For the complete cases please see part Ⅱ of the SI.[85]

Let us consider the "successive braiding", i.e., a non-Abelian braiding test[40] with four Majorana modes $\gamma_1, \gamma_2, \gamma_3, \gamma_4$, where both γ_1, γ_2 and γ_3, γ_4 pair are initialized in even parity. We apply multiple braiding procedures (n times) of γ_2 and γ_3 in an anti-clockwise way. We focus on the probability to obtain the odd result in the $(\gamma_3\gamma_4)$ parity measurement at different n's. Braiding once, the parity of γ_3, γ_4 has 50% chance to be odd in measurement. After two successive braidings, we get the definite odd parity; after three successive braidings, we will get an equal probability of both results; after four successive braidings, we get the definite even parity. As the number n increases, the probability will repeat this pattern with period 4. In our HV theory Ⅱ, as shown in Fig. 4(c), we get the same measurement consequence as those in the quantum case.



n	1	2	3	4	5	.....
p	1/2	1	1/2	0	1/2	.....

Fig. 4. (a) Hadamard gate in Majorana-based quantum computation. (b) Hadamard gate in our HV theory Ⅱ wherein the second step and last step have been applied in crossovers to change into a standard configuration. (c) Example of non-Abelian braiding in the four boxes case in our HV theory Ⅱ. Suppose that in the initial state, box 3,4 are in even parity. The table shows the possibility p to obtain the odd parity in box-3,4 measurement, where the label n describes the number of successive braiding procedures of box 2 and box 3.

In either the Majorana model or our HV theory Ⅱ, the objects (MZMs or boxes) describing a specific fusion channel are all paired. Therefore, the braiding of two objects involves at most two pairs that are of four MZMs or boxes. Considering the braiding of two boxes among $2n$ boxes, this procedure only changes the topology of two connections (of two box-pairs) involved in the braiding, but can not change the relative positions between these two connections and the other n-2 connections. Therefore, any single braiding step in more box cases involves only four relevant boxes. Examples for braiding in more-MZM systems can be found in part Ⅱ of the SI.[85] In summary, the braidings can be well described in our HV theory Ⅱ with topology considerations; and this theory goes beyond the fusion tests and shows the unique topological-nontrivial property of braiding operations.

(Ⅲ) *Joint-Measurement and Quantum Interference.* Let us assume that there are two Majorana qubits, where each one is encoded into four MZMs; and the single-qubit Pauli measurements can be realized by fermion-parity measurement of different Majorana pairs within each qubit.[3] The joint-measurement can be realized by the fermion-parity measurement of four MZMs, in which two MZMs are from one qubit and the other two are from the other qubit.[3] For joint-measurement ZZ, the state $\frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ will collapse onto $(|00\rangle + |11\rangle)/\sqrt{2}$ if the measurement outcome is even, and thus generate entanglement between the two qubits. In our HV theory Ⅱ with two "qubits" (eight boxes), we can measure the four-box joint-parity, in which two boxes are from the first qubit and the other two are from the second qubit. For example, if the box 1,4 and box 2,3 are initially even for both qubits, the joint-parity measurement of box 1,2 in the first qubit and box 1,2 in the second qubit gives the following projection:

(4)

Here, we assume that the measurement outcome is even, and the plus sign indicates two classical uncertain states with equal probability. This is an analog to joint-measurement ZZ in the quantum theory.

Meanwhile, for another example with 6 MZMs in the initial state $|0_{12}0_{35}0_{46}\rangle$, we first joint-measure MZM $\gamma_2\gamma_3\gamma_4\gamma_5$ and assume even result, we get $(|0_{16}0_{23}0_{45}\rangle + |0_{16}1_{23}1_{45}\rangle)/\sqrt{2}$. Then the measurement of MZM $\gamma_2\gamma_5$ will reach a definite even result $|0_{16}0_{25}0_{34}\rangle$ due to the cancelation of quantum interference. However, as shown in Fig. 4(d), the HV theory Ⅱ fails to capture this process and reaches $(\square\overbrace{\square\square\ \square\square}\square + \square\overbrace{\bigcirc\bigcirc\ \bigcirc\bigcirc}\square)/2$. We can look at another example in the Majorana system, i.e., the CNOT gate of two-MZM qubit which involves both joint-measurement and two-box measurement. We show in the part Ⅲ of the SI[85] that our HV theory Ⅱ can not capture CNOT gate process. It is clear that any HV theory cannot capture the complete signature of quantum interference. Therefore, we conclude that, to reveal certain signatures of quantum interference, the combination of joint-measurement and two-box measurement are helpful and necessary.

Discussions. There are some other tests for more properties of Majorana systems. (1) Braiding and measurement

only cannot generate T-gate, which however can be generated by preparing a noisy magic state ancilla along with state distillation.[86] T-gate test along with other Clifford operations specifies quantum property which could generate arbitrary quantum states. (2) Bell nonlocality tests clearly complete the Majorana system validation but require a fully universal quantum gate-set including T-gate to break Bell bound. However, the braiding and measurement only cannot break the Bell bound.[87–89] In the future, we need either to fabricate more sophisticated devices along with complicated experimental procedures or to propose novel and simple test schemes.

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