Induced Fission-Like Process of Hadronic Molecular States

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We predict a new physical phenomenon, induced fission-like process and chain reaction of hadronic molecular states. As a molecular state, if induced by a D meson, the $X(3872)$ can split into $D\bar{D}$ final state which is forbidden due to the spin-parity conservation. The breeding of the D meson of the reaction, such as $D^0X(3872) \to D^0\bar{D}^0D^0$, makes the chain reaction of $X(3872)$ matter possible. We estimate the cross section of the D meson induced fission-like process of $X(3872)$ into two D mesons. With very small D^0 beam momentum of 1 eV, the total cross section reaches an order of 1000 b, and decreases rapidly with the increasing beam momentum. With the transition of D^* meson in molecular states to a D meson, the $X(3872)$ can release large energy, which is acquired by the final mesons. The momentum distributions of the final D mesons are analyzed. In the laboratory frame, the spectator D meson in molecular state concentrates in the low momentum area. The energy from the transition from D^* to D meson is mainly acquired by two scattered D mesons. The results suggest that the D meson environment will lead to the induced fission-like process and chain reaction of the $X(3827)$. Such a phenomenon can be extended to other hadronic molecular states.

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$X(3872)$ is the first observed hadronic molecule candidate with a mass very close to the total mass of a ground state and an excited charm meson.[1] Hence, it is suggested to be a hadronic molecular state composed of two (anti)charm mesons.[2] Its wave function can be written as[3]

$$|X\rangle = \frac{1}{\sqrt{2}}(|\bar{D}^{*0}D^0\rangle - |D^{*0}\bar{D}^0\rangle). \tag{1}$$

$X(3872)$ carries the spin parity $J^P = 1^+$, which cannot decay into a $\bar{D}^0D^0$ pair due to the conservation of spin parity. In fact, if we do not consider the small decay probabilities of the $\bar{D}^{*0}$ and D^0 mesons, this hadronic molecular state should be stable, similar to a deuteron.

In nuclear physics, if induced by a nucleon, a nucleus can split into two or lighter nuclei by recombination of nucleons, even the spontaneous fission is forbidden.[4] If more neutrons can be produced from the neutron-induced nuclear fission, that is, a chain reaction happens. Hence, it is interesting to see whether or not the $X(3872)$ can decay into a $\bar{D}^0D^0$ pair if induced by an additional D^0 meson. Since the molecular state is composed of two hadrons, the same type of fission of heavy nucleus is impossible for molecular state. However, the kinds of constituent hadrons of molecular states are much richer than nucleus, which contains only nucleons.[5–7] Such a property will lead to new type of induced fission of molecular state compared with nucleus.

In Ref. [8], we studied the nucleon-induced fission-like process of the T_{cc}^+. With interaction of a nucleon, the forbidden decay of T_{cc}^+ to two D mesons is allowed. Such a phenomenon can also be expected in the case of $X(3872)$. Furthermore, if we use the D meson to attack the $X(3872)$, the produced D mesons can be taken as the inducing particles in the sequence reaction, which is more analogous to the nuclear fission and chain reaction. Hence, in this Letter, we study decay of $X(3872)$ induced by an additional D^0 meson with more D mesons produced. The explicit scheme is shown on the upper left of Fig. 1. $X(3872)$ can be either the $\bar{D}^{*0}D^0$ molecule or the $D^{*0}\bar{D}^0$ molecule with equal probability. That is, it is the superposition of the two molecular states that can be understood and treated analogously, and hereafter, we take the first term $\bar{D}^{*0}D^0$ as representative.

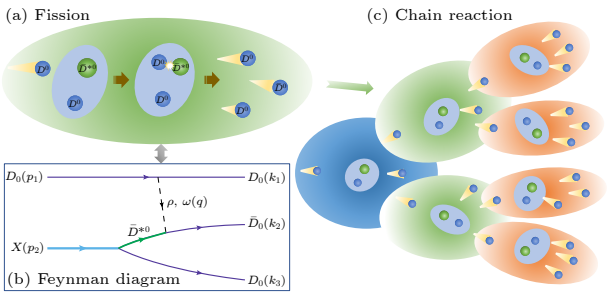

Fig. 1. Schematic depiction of the induced fission-like process and chain reaction of $X(3872)$. (a) The induced fission-like process of $X(3872)$ with an incoming D^0 meson. The blue and green balls denote D^0 and $\bar{D}^{*0}$ mesons, respectively. The collision occurs between the incoming D^0 meson and $\bar{D}^{*0}$ meson in the $X(3872)$, which is denoted by a small oval. (b) The Feynman diagram corresponding to the induced fission-like process in (a) with ρ/ω exchange. (c) The chain reaction of $X(3872)$ matter. The large oval means an induced fission-like process in (a). The overlap of the ovals and the meson therein means that the final meson of previous fission-like becomes the incoming meson of next fission-like process.

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An incoming D^0 meson is used to bombard the molecular state $X(3872)$, which produces an intermediate state as $D^0\bar{D}^{*0}D^0$ that can decay into three ground mesons as

$$D^0 + X(3872) \to D^0\bar{D}^{*0}D^0 \to 2D^0 + \bar{D}^0. \tag{2}$$

In the reaction, the incoming D^0 meson is analogous to the neutron in nuclear fission. However, in the current reaction, the D^{*0} meson in $X(3872)$ transmutes to a D^0 meson, which is different from the nuclear fission where only recombination of constituents happens. We define such a reaction as the induced fission-like process of a molecular state. The chain reaction is important to make nuclear fission proceed continuous. If the induced fission-like process occurs in $X(3872)$ as shown in the right part of Fig. 1, the produced D^0 mesons will induce the fission-like process of more molecular states. Thus, the chain reaction is also possible for hadronic molecular states.

Cross Sections. The probability of induced fission-like process can be estimated by the cross section σ. In the induced fission-like process of $X(3872)$, the incoming D^0 meson strikes on the $\bar{D}^{*0}$ meson in $X(3872)$ while the interior D^0 meson is taken as a spectator, and then one $\bar{D}^0$ and two D^0 mesons are produced. This can be described by a Feynman diagram within field theory, as shown on the bottom left of Fig. 1. From the diagram, the $\bar{D}^{*0}$ meson in $X(3872)$ exchanges a ρ/ω meson with the incoming D^0 meson and transforms into a $\bar{D}^0$ meson. For the meson-induced fission-like process of a molecular state it is logical to use the rest frame of the molecular state, that is, the so-called laboratory frame. In this reference frame, the cross section for the reaction $D^0 + X \to 2D^0 + \bar{D}^0$ is expressed as

$$d\sigma = \frac{1}{4[(p_1\cdot p_2)^2 - m_1^2m_2^2]^{1/2}}\frac{1}{3}\sum_\lambda |\mathcal{M}_\lambda|^2 d\Phi_3\frac{1}{2}, \tag{3}$$

where $p_{1,2}$ and $m_{1,2}$ represent the momenta and masses of incoming D^0 meson or $X(3872)$. The phase space $d\Phi_3$ is produced with the help of GENEV code in FAWL as $R_3 = (2\pi)^5 d\Phi_3 = \prod_i^3 \frac{d^3k_i}{2E_i}\delta^4(\sum_i^n k_i - P)$ with k_i and E_i being the momentum and energy of final particle i. The mechanism of the fission-like process reaction can be described by an amplitude $\mathcal{M}_\lambda$ with λ being the helicity of $X(3872)$. For the first term of the wave function in Eq. (1), the amplitude can be written as

$$\begin{aligned}\mathcal{M}_\lambda = \Big[\sum_{\lambda_{\bar{D}^{*0}}} &\mathcal{A}_{\lambda,\lambda_{\bar{D}^{*0}}}(X\to\bar{D}^{*0}D^0)\mathcal{A}_{\lambda_{\bar{D}^{*0}}} \\ &\cdot(\bar{D}^{*0}D^0\to\bar{D}^0D^0)\Big]\Big/\Big[p^2 - m_{D^*}^2\Big],\end{aligned} \tag{4}$$

where the different helicities for intermediate $\bar{D}^{*0}$ meson $\lambda_{\bar{D}^{*0}}$ should be summed up and the helicities will be omitted if unnecessary.

In the literature,[9] the $X(3872)$ splitting into $\bar{D}^{*0}D^0$ can be related to the scattering of $\bar{D}^{*0}D^0$. The coupling of the molecular state to its constituents can be related to binding energy.[10] Hence, the amplitude for $X(3872)$ splitting into $\bar{D}^{*0}D^0$ is determined by the scattering length a as[9]

$$\mathcal{A}_{\lambda,\lambda_{\bar{D}^{*0}}}(X\to\bar{D}^{*0}D^0) = \sqrt{\frac{16\pi m_X m_{D^*}m_D}{\mu^2 a}}\epsilon_{X\lambda}\cdot\epsilon_{\lambda_{\bar{D}^{*0}}}, \tag{5}$$

where $m_{X,D^*,D}$ is the mass of $X(3872)$, the constituent $\bar{D}^{*0}$, or D^0. The ϵ_X and ϵ are the polarized vectors for $X(3872)$ and $\bar{D}^{*0}$. Scattering length $a = 1/\sqrt{2\mu E_{\rm B}}$ with the reduced mass reading $\mu = m_D m_{D^*}/(m_D + m_{D^*})$ and $E_{\rm B}$ being the binding energy.

The propagator of $\bar{D}^{*0}$ in laboratory frame, where the $X(3872)$ is static, can be written as

$$\frac{1}{p^2 - m_{D^*}^2} = \frac{1}{(m_X - E_3 - E_{D^*})(m_X - E_3 + E_{D^*})}. \tag{6}$$

Before being struck, the $X(3872)$ is static and the binding energy is very small, which suggest that the momenta of the constituent mesons are small. The energy of two constituent mesons can be safely approximated as $E_3 = m_D^2 + \boldsymbol{k}_3^2/2m_D$ and $E_{D^*} = m_{D^*}^2 + \boldsymbol{k}_3^2/2m_{D^*}$. As in Ref. [11], the amplitudes for $X(3872)$ splitting with the propagator of D^* meson can be expressed with wave function of $X(3872)$ as

$$\frac{\mathcal{A}(X\to\bar{D}^{*0}D^0)}{p^2 - m_{D^*}^2} \simeq -\frac{\sqrt{8m_X m_{D^*}m_D}}{m_X - m_D + m_{D^*}}\psi(\boldsymbol{k}_3)\epsilon_X\cdot\epsilon^*, \tag{7}$$

where the wave function is

$$\psi(\boldsymbol{k}) = \sqrt{\frac{8\pi}{a}}\frac{1}{\boldsymbol{k}^2 + 1/a^2}, \tag{8}$$

with normalization $\int d^3k(2\pi)^{-3}|\psi(\boldsymbol{k})|^2 = 1$. Such a wave function is consistent with the wave function adopted by Voloshin in coordinate space.[12]

The energy-releasing transition of the $\bar{D}^{*0}$ meson to the $\bar{D}^0$ meson is induced by the incoming D^0 meson, which involves the inelastic scattering $\bar{D}^{*0}D^0 \to D^0\bar{D}^0$ through vector exchange, as shown in Fig. 1. To depict the scattering, the following Lagrangians under the heavy quark and chiral symmetries are adopted,[13]

$$\begin{aligned}\mathcal{L}_{P^*PV} =& -i\lambda g_V\varepsilon_{\lambda\alpha\beta\mu}(D^{*0\mu\dagger}\overleftrightarrow{\partial}^\lambda D^0 + D^{0\dagger}\overleftrightarrow{\partial}^\lambda D^{*0\mu}) \\ &\cdot\partial^\alpha V^\beta - i\lambda g_V\varepsilon_{\lambda\alpha\beta\mu}(\bar{D}^{*0\mu\dagger}\overleftrightarrow{\partial}^\lambda\bar{D}^0 \\ &+ \bar{D}^{0\dagger}\overleftrightarrow{\partial}^\lambda\bar{D}^{*0\mu})\partial^\alpha V^\beta, \\ \mathcal{L}_{PP\mathbb{V}} =& -i\frac{\beta g_V}{2}D^{0\dagger}\overleftrightarrow{\partial}^\mu D^0 V^\mu + i\frac{\beta g_V}{2}\bar{D}^{0\dagger}\overleftrightarrow{\partial}^\mu\bar{D}^0 V^\mu,\end{aligned} \tag{9}$$

where $V = \rho^0$ or ω, and the parameters involved here were determined in the literature as $\beta = 0.9$, $\lambda = 0.56\,\mathrm{GeV}^{-1}$, and $g_V = 5.9$.[13,14]

Applying standard Feynman rules, the amplitude for the inelastic scattering $\bar{D}^{*0}D^0 \to \bar{D}^0D^0$ can be written as

$$\begin{aligned}&\mathcal{A}_{\lambda_{\bar{D}^{*0}}}(\bar{D}^{*0}D^0\to\bar{D}^0D^0) \\ &= 2\sqrt{2}\lambda\beta g_V^2\varepsilon_{\lambda\alpha\beta\mu}p_1^\lambda k_1^\alpha k_2^\beta\epsilon^\mu P(q^2),\end{aligned} \tag{10}$$

with the propagator of exchanged ρ and ω mesons

$$P(q^2) = \sum_{i=\rho,\omega}\frac{1}{q^2 - m_i^2}\,\frac{m_i^2 - \Lambda^2}{q^2 - \Lambda^2}, \tag{11}$$

with a standard cutoff $\Lambda = 1\,\mathrm{GeV}$.

With the above amplitudes for splitting of $X(3872)$ and inelastic scattering $\bar{D}^{*0}D^0 \to \bar{D}^0D^0$, the amplitudes for total reaction $D^0 + X \to 2D^0 + \bar{D}^0$ can be written as

$$\begin{aligned}\mathcal{M} =& \frac{-8\lambda\beta g_V^2\sqrt{m_X m_{D^*}m_D}}{m_X - m_D + m_{D^*}}P(q^2) \\ &\cdot[\psi(\boldsymbol{k}_3)\varepsilon_{\lambda\alpha\beta\mu}p_1^\lambda k_1^\alpha k_2^\beta\epsilon_X^\mu + (2\leftrightarrow 3)].\end{aligned} \tag{12}$$

The $(2 \leftrightarrow 3)$ term is an exchange of the momenta for final particles 2 and 3, which is for the second term $D^{*0}\bar{D}^0$ in the wave function in Eq. (2).

Numerical Results. The binding energy is an important metric of a hadronic molecular state. Though there is a very small suggested value of $X(3872)$ listed in the PDG,[15] we will vary the binding energy from 0.05 to 50 MeV. With such a variation, more properties of the molecular can be unveiled and they are also helpful to show the behavior of other molecular states with different binding energies. In Fig. 2, for a binding energy of approximately 0.1 MeV, the cross section is smaller.

Considering that the radius of a D or D^* meson is smaller than 1 fm, such a binding energy means a large radius, approximately 14 fm, which makes the probability of collision of the incoming D^0 with D^{*0} in $X(3872)$ very small. With increasing binding energy, the hadronic molecular state becomes more compact, and the cross section increases. If the binding energy is larger than 10 MeV, which corresponds to a radius of approximately 1 fm on the same order as the force range of ρ/ω exchange, then the cross section decreases again due to the small size of the molecular state.

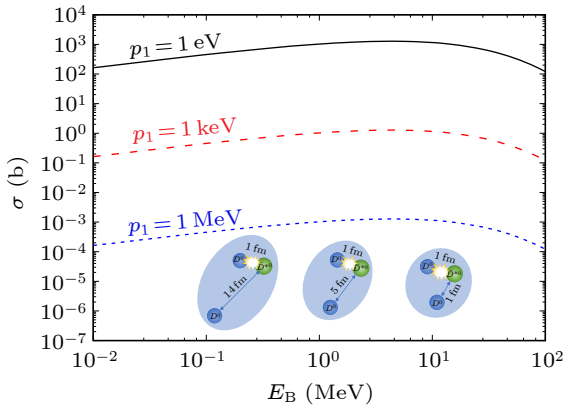

Fig. 2. Cross section of the $D^0 + X \to 2D^0 + \bar{D}^0$ reaction as a function of the binding energy. The results for three choices of the momentum of the incoming D^0 meson $p_1 = 1$ eV, 1 keV, and 1 MeV, are presented by solid (black), dashed (red), and dotted (blue) curves. The three subfigures show the radii of the molecular states with binding energies of 0.1, 1, and 10 MeV. The distance 1 fm between collision D^0 and $\bar{D}^{*0}$ mesons (two upper mesons) represent the interaction range of ρ/ω exchange.

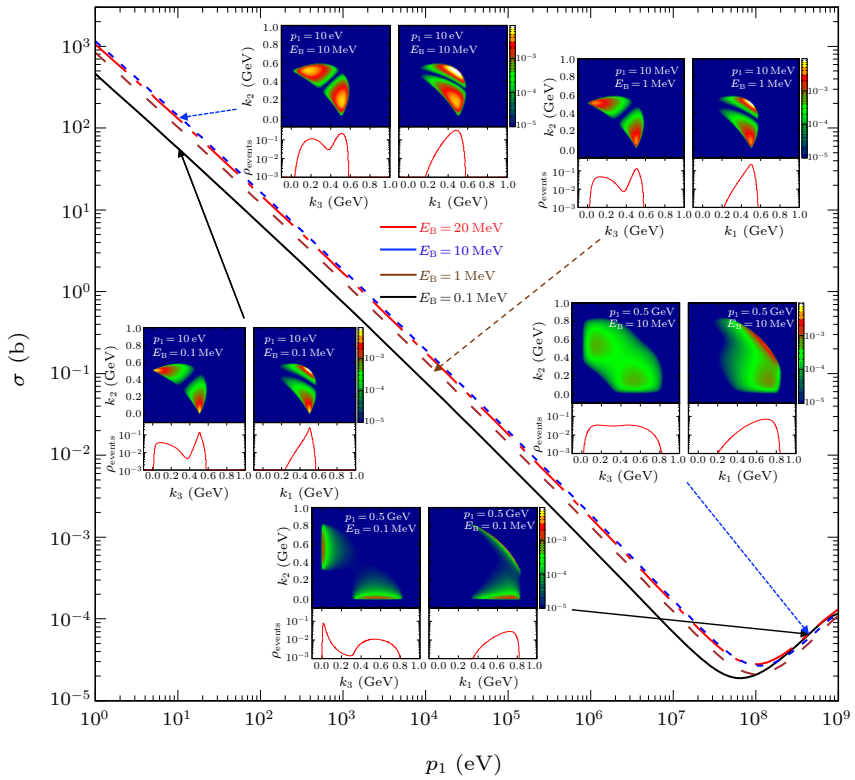

Fig. 3. Cross section σ of the $D^0 + X \to 2D^0 + \bar{D}^0$ reaction as a function of the momentum of the incoming D^0 meson p_1, and the momentum distributions of final particles. The solid (black), dashed (brown), dotted (blue), and dash-dotted (red) curves are for the results with four binding energies $E_B = 0.1$, 1, 10, and 20 MeV, respectively. The subfigures show the momentum distributions of final particles. For each example choice of p_1 and E_B, the figures represent the k_3–k_2 (left) and k_1–k_2 (right) planes, showing the momentum $k_i = |\boldsymbol{k}_i|$ of the final meson i. The colorbox means the ratio of event number in a bin of 0.002 GeV × 0.002 GeV to the total number of events. The two lower panels show the ratio of event number in a bin of 0.002 GeV to the total number of events, ρ_{events}, against k_3 (left) and k_1 (right) for particles 3 and 1 as shown in the Feynman diagram in Fig. 1. The results here include both component of $X(3827)$ as shown in Eqs. (1) and (12). The present results are obtained with 10^9 simulation.

In Fig. 3, the cross section with the variation in the momentum of the incoming D^0 meson p_1 and the momentum distribution of final particles are presented. The largest cross section is found at a small incoming momentum p_1. With increasing incoming momentum, the cross section decreases almost linearly and reaches a minimum at approximately 100 MeV. This is reasonable because the faster incoming D^0 meson has a shorter time to interact, which makes the probability of reaction smaller.

It is important to study the momentum distribution of three final mesons. In Fig. 3, the distributions are separated into two parts, which correspond to two terms of the wave function in Eq. (1) and the amplitudes in Eq. (12), and the results for k_3–k_2 verifies the symmetry of particles 2 and 3. With a slow incoming D^0 meson, e.g., a momentum of 10 eV, more events with k_3 about zero and k_2 about 0.5 GeV, or symmetrically k_2 about zero and k_3 about 0.5 GeV, can be observed. It is suggested that the $\bar{D}^0$ meson off the struck $\bar{D}^{*0}$ meson acquires large momentum about 0.5 GeV while the spectator meson, i.e., the pseudoscalar D^0 meson in the $X(3872)$ has a relatively small momentum after induced fission. If the incoming energy increases to 0.5 GeV and keeps the binding energy as 0.1 MeV, the events of the spectator D^0 meson will concentrate in a small momentum range, exhibited as a sharp peak in the momentum distribution spectrum of k_3. This suggests that with large incoming momentum the spectator meson in $X(3872)$ is almost unaffected and maintains a very small momentum as in hadronic molecular state. The D^0 meson off the struck $\bar{D}^{*0}$ meson has a wide distribution around 0.6 GeV. The two terms of wave function in Eq. (1) result in the two peaks in the distribution of the momentum of the D^0 or $\bar{D}^0$ meson off the $X(3872)$. The incoming D^0 meson (particle 1 as shown in Feynman diagram) also acquires a larger momentum k_1 about 0.5 GeV with large dispersion.

In addition to the $\bar{D}^0$ meson-induced fission-like process, a $\bar{D}^0$ meson can also induce the fission-like process of $X(3872)$ as follows:

$$\bar{D}^0 + X(3872) \to \bar{D}^0 \bar{D}^{*0} D^0 \to D^0 + 2\bar{D}^0. \tag{13}$$

Both D^0 and $\bar{D}^0$ mesons can play the role of neutrons in nuclear fission and continuously induce chain reactions. Moreover, "cross" chain reactions can also occur; for example, the $\bar{D}^0$ meson-induced fission-like process produces one D^0 meson, which further induces another kind of fission-like process. The fission-like process of $X(3872)$ may show phenomena other than those of nuclear fission, and these phenomena provide various views and will help us understand the mechanisms of fission-like process more deeply.

In summary, we have predicted an interesting phenomenon, possible fission-like process and chain reaction of hadron molecular states. Since only two constituents exist in the molecular state, the standard type of fission of nuclei is impossible. We note that the kinds of hadrons in the molecular states are richer than the nuclei where only nucleon involved. For example, the $X(3872)$ considered in the present work, the transition of the D^* meson to D meson make the fission-like process possible. With the explicit calculation, such a reaction exhibits a behavior very analogous to the nuclear fission especially the rapid increase of the cross section with small incoming momentum. Moreover, the breeding of the D meson of the reaction, such as $D^0 X(3872) \to D^0 \bar{D}^0 D^0$, makes the chain reaction of a molecular-state matter possible. There can be induced fission-like process with other hadronic molecular states. Here, we list three typical reactions in the strange, charmed, and bottom sectors,

$$\begin{aligned} &K + f_1(1285) \to K\bar{K}^* K \to 2K + \bar{K}, \\ &D + T_{cc}(3875) \to DD^* D \to 3D, \\ &B + Z_b(10610) \to B\bar{B}^* B \to 2B + \bar{B}. \end{aligned}$$

Due to the short lifetime of the molecular states, such reactions should not be observed directly in near future. However, it may exhibit its effect in some scenes. Taking $X(3872)$ as an example, Quark-gluon-plasma will be produced through nuclear collisions in which there will be a large number of charm mesons.[16] Charmed mesons can form many $X(3872)$ particles through strong interaction, and then further react with the remaining charmed D mesons to make the proposed fission-like process of hadron molecule happen.

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