Two-Qubit Geometric Gates Based on Ground-State Blockade of Rydberg Atoms

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We achieve the robust nonadiabatic holonomic two-qubit controlled gate in one step based on the ground-state blockade mechanism between two Rydberg atoms. By using the Rydberg-blockade effect and the Raman transition mechanism, we can produce the blockade effect of double occupation of the corresponding ground state, i.e., ground-state blockade, to encode the computational subspace into the ground state, thus effectively avoiding the spontaneous emission of the excited Rydberg state. On the other hand, the feature of geometric quantum computation independent of the evolutionary details makes the scheme robust to control errors. In this way, the controlled quantum gate constructed by our scheme not only greatly reduces the gate infidelity caused by spontaneous emission but is also robust to control errors.

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In recent years, quantum computation has attracted increasing attention.[1–3] Compared with classical computation, quantum computation has higher computation speed and can deal with many intractable problems for classical computation. One key method in realizing quantum computation is to design and manufacture a set of universal quantum gates with high fidelities and strong robustness in one step. Unlike step-by-step schemes, the one-step scheme does not require individual site addressing, which not only simplifies the complexity of the experiment but also increases its feasibility. Thus, a one-step implementation of high fidelity and robust quantum gates has served as the current mainstream blueprint for serious experimental efforts.

Rydberg atoms are considered as a good candidate for quantum computation due to their stable long lifetime Rydberg states and controlled Rydberg-mediated interaction between Rydberg atoms.[4–7] When an atom is excited to a high-energy Rydberg state, the strong Rydberg-mediated interaction can inhibit the excitation of its surrounding atoms to the Rydberg state,[4,8,9] which is known as the Rydberg blockade regime and has been experimentally confirmed[10,11] and applied to construct quantum logic gates.[12–16] In general, Rydberg atoms have been extensively applied to quantum computation and quantum simulation.[16–19]

However, the realization of high fidelity and robust quantum gates still faces the following two challenges: (i) control errors caused by imprecise operation of quantum systems, (ii) decoherence caused by unavoidable interactions between the system and environment. To reduce the control errors, geometric quantum computation[20] based on geometric phase[21–24] is proposed. Geometric phase has nothing to do with the details of evolution, but only depends on the evolution path, so geometric quantum computation is robust to control errors. The early adiabatic geometric quantum computation[25–28] required the quantum system to evolve for a long time, which makes the gate vulnerable to the influence of the environment and causes the decoherence. To resolve this problem, nonadiabatic geometric quantum computation[29–32] was put forward and experimentally implemented in different quantum systems such as superconducting,[33] nitrogen-vacancy center in diamond,[34,35] and nuclear magnetic resonance.[36,37]

Previous schemes based on the excited-state blockade of Rydberg atoms cause the decoherence of system due to the spontaneous emission from the excited Rydberg state,[4,38–42] which adversely affects the performance of the quantum gate schemes. In order to reduce the influence of decoherence, we consider to replace the excited-state blockade of Rydberg by the ground-state blockade of atoms. In this Letter, we construct a set of universal nontrivial two-qubit nonadiabatic geometric quantum gates in the two-atom system by using the Rydberg blockade mechanism to dress the ground states. The system consists of two identical three-level Rydberg atoms, each containing two ground states and one Rydberg excited state, where the ground states in each atom are coupled by a microwave-frequency field and one of the ground states is excited to the Rydberg state by a laser. In this system, the excitation process of atoms from ground state to Rydberg state will be inhibited, resulting in the suppressed double

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occupation of $|rr\rangle$, and the laser couples the ground $|00\rangle$ state to the entangled state $|B\rangle = \sin\frac{\phi}{2}|01\rangle - \cos\frac{\phi}{2}|10\rangle$ [see the following definition in Eq. (6)].[43] By using the nonadiabatic holonomic quantum computation (NHQC),[41] we then achieve a gate operation scheme which is robust not only to the environmental errors caused by the spontaneous emission from excited-states but also to control errors.

Two-qubit Gate. Considering two three-level Rydberg atoms, as illustrated in Fig. 1, each atom consists of two ground states $|0\rangle$ and $|1\rangle$, and an excited Rydberg state $|r\rangle$, where the microwave-frequency field with different Rabi frequencies resonantly couples the ground states $|0\rangle$ and $|1\rangle$ of the atom, meanwhile the ground state $|1\rangle$ is excited to the Rydberg state $|r\rangle$ by a dressing laser pulse. In the interaction picture, the Hamiltonian of the system can be written as ($\hbar = 1$)

$$H_I = \sum_{i=1}^{2} \Omega_i |1\rangle_i\langle 0| + \Omega_r |r\rangle_i\langle 1| e^{-i\Delta_r t} + U|rr\rangle\langle rr| + \text{H.c.} \tag{1}$$

with U denoting the Rydberg-mediated interaction when both atoms are excited to the Rydberg states.

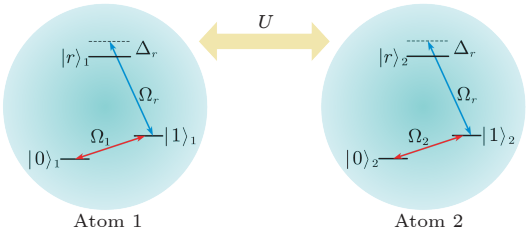

Fig. 1. The diagrammatic sketch of two interacting Rydberg atoms where the ground states $|0\rangle_i$ and $|1\rangle_i$ are resonantly coupled by a microwave field with the Rabi frequency Ω_i, where $i = 1, 2$ denote the ith atom, and $|1\rangle_i$ is coupled to the Rydberg state $|r\rangle_i$ by a laser with the Rabi frequency Ω_r and detuning Δ_r, respectively.

In the large detuning limits $\Delta_r \gg \Omega_r$, through the standard second order perturbation theory, we obtain an effective Hamiltonian as

$$H_{\text{eff}} = \sum_{i=1}^{2} \Omega_i |1\rangle_i\langle 0| + \frac{\Omega_r^2}{\Delta_r}|1\rangle_i\langle 1| + \frac{2\Omega_r^2}{\Delta_r}|11\rangle\langle rr| + \text{H.c.} + (U - 2\Delta_r + \frac{2\Omega_r^2}{\Delta_r})|rr\rangle\langle rr|. \tag{2}$$

The second term causes unwanted shifts to our system, which can be canceled via introducing other ancillary levels. We can eliminate the last term by making $U = 2\Delta_r - 2\Omega_r^2/\Delta_r$. Now the above Hamiltonian can be rewritten in a concise form

$$H_{\text{eff}} = \sum_{i=1}^{2} \Omega_i |1\rangle_i\langle 0| + \lambda|11\rangle\langle rr| + \text{H.c.}, \tag{3}$$

where $\lambda = 2\Omega_r^2/\Delta_r$. The first term in the above equation can be written as

$$\begin{aligned} H_\alpha &= \Omega_1|1\rangle_1\langle 0| \otimes I_2 + \Omega_2 I_1 \otimes |1\rangle_2\langle 0| \\ &= (\Omega_1|10\rangle + \Omega_2|01\rangle)\langle 00| + |11\rangle(\Omega_1\langle 01| + \Omega_2\langle 10|). \end{aligned} \tag{4}$$

Defining two parameters $\Omega \equiv \sqrt{\Omega_1^2 + \Omega_2^2}, \phi \equiv -2\arctan\Omega_2/\Omega_1$, and $|B\rangle = \sin\frac{\phi}{2}|01\rangle - \cos\frac{\phi}{2}|10\rangle$, we can rewrite H_α as $H_\alpha = \Omega|B\rangle\langle 00| + \Omega|11\rangle\langle B|$. Thus, H_{eff} can be rewritten as

$$H_{\text{eff}} = \Omega|B\rangle\langle 00| + \Omega|11\rangle\langle B| + \lambda|11\rangle\langle rr| + \text{H.c.} \tag{5}$$

In the limits of $\lambda \gg \Omega$, the second term in the above equation can be ignored. For the high-frequency oscillation term of the third term, we can choose appropriate value of λ to make $\lambda t = 2n\pi$ with n being an integer to eliminate it. Thus an approximated ground-state blockade Hamiltonian is obtained,

$$H_{\text{eff}} = \Omega(|B\rangle\langle 00| + |00\rangle\langle B|). \tag{6}$$

Equation (6) shows that the effective Hamiltonian no longer contains Rydberg state $|r\rangle$ and the evolution of the system will only take place in the ground states $|0\rangle$ and $|1\rangle$, thus, encoding the quantum bits in the ground states for quantum computation will be a robustness way.

In Ref. [41], one of the computational basis $|rr\rangle$ would keep invariant in the rotation frame but would evolve in the original frame due to the Rydberg–Rydberg interaction. This can be addressed well by choosing the evolution time t match with Rydberg interaction strength V to make $Vt = 2n\pi$ with n being a positive integer. Similarly, in our scheme, based on Eq. (6) the state $|11\rangle$ also does not evolve in the rotation frame, while would evolve in the original frame. To address this issue, one can choose an appropriate value of λ to make $\lambda t = 2n\pi$ with n being an integer.

According to the form of H_{eff} in Eq. (6), we can construct a nontrivial two-qubit nonadiabatic geometric gate. Choosing the computational space as $S = \text{Span}\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, it can be divided into three subspaces $S_1 = \text{Span}\{|00\rangle\}$, $S_2 = \text{Span}\{|01\rangle, |10\rangle\}$ and $S_3 = \text{Span}\{|11\rangle\}$, where S_2 can be rewritten as $S_2 = \text{Span}\{|B\rangle, |D\rangle\}$ with $|D\rangle = \cos\frac{\phi}{2}|01\rangle + \sin\frac{\phi}{2}|10\rangle$ denoting a dark state of the effective Hamiltonian, decoupled from other states. The evolution operator of the system is then obtained as

$$\begin{aligned} U(t) &= e^{-i\int_0^t H_{\text{eff}}(t')dt'} \\ &= \cos\alpha_t(|B\rangle\langle B| + |00\rangle\langle 00|) - i\sin\alpha_t(|B\rangle\langle 00| \\ &\quad + |00\rangle\langle B|) + |D\rangle\langle D| + |11\rangle\langle 11|, \end{aligned} \tag{7}$$

with the cumulative phase $\alpha_t = \int_0^t \Omega(t')dt'$.

Making the evolution period τ satisfy $\alpha_\tau = \pi$, in the computational space S, the evolution operator can be expressed as

$$U(\tau) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & \sin\phi & -\cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

which provides a nontrivial two-qubit gate. During the evolution, the dynamic phase is always zero, satisfying the

parallel transport condition:

$$\begin{aligned}
&\langle 00|U^{\dagger}(t)H_{\rm eff}(t)U(t)|00\rangle = 0,\\
&\langle 10|U^{\dagger}(t)H_{\rm eff}(t)U(t)|10\rangle = 0,\\
&\langle 01|U^{\dagger}(t)H_{\rm eff}(t)U(t)|01\rangle = 0,\\
&\langle 11|U^{\dagger}(t)H_{\rm eff}(t)U(t)|11\rangle = 0.
\end{aligned} \tag{9}$$

Therefore, the operator $U(\tau)$ is a two-qubit geometric gate.

Numerical Simulations. In the following, we obtain the performance of two-qubit gates through numerical simulation. We first select the Rydberg atom which can experimentally realize the scheme. Then, we set ϕ to take two special values $\frac{\pi}{2}$ and $\frac{\pi}{4}$ to obtain controlled-NOT gate and controlled-Hadamard gate, respectively. Next, we compare the dynamic process of the original Hamiltonian and the effective Hamiltonian by numerical simulation to verify the feasibility of the scheme. Finally, we show the superiority of this scheme by comparing it with the traditional scheme of encoding qubits in Rydberg states.

In our model, we choose ^{133}Cs atom as the three-level Rydberg atom whose relevant energy level structure is illustrated in Fig. 2(a). The two ground states are chosen as $|0\rangle \equiv |6S_{1/2}, F = 4, m_F = 0\rangle$ and $|1\rangle \equiv |6S_{1/2}, F = 3, m_F = 0\rangle$,[44] and the Rydberg state is chosen as $|r\rangle \equiv |64P_{3/2}\rangle$ with a radiative decaying rate $\gamma_r/2\pi = 2\,\text{kHz}$ and a dephasing rate $\gamma_p/2\pi = 1\,\text{kHz}$.[45] The ground states $|0\rangle_i$ and $|1\rangle_i$ $(i = 1, 2)$ of two atoms are coupled by two microwave-frequency fields with Rabi frequencies $\Omega_1/2\pi = -20\cos(\phi/2)\,\text{kHz}$ and $\Omega_2/2\pi = 20\sin(\phi/2)\,\text{kHz}$, respectively. Meanwhile, we utilize a Rydberg dressing laser at 319 nm to directly drive the single-photon transitions from $|1\rangle$ to $|r\rangle$ with the Rabi-frequency $\Omega_r/2\pi = 9.1\,\text{MHz}$ and detuning $\Delta_r/2\pi = 180\,\text{MHz}$. On the other hand, we set the Rydberg interaction strength as $U/2\pi = (2\Delta_r - 2\Omega_c^2/\Delta_r)/2\pi = 359.08\,\text{MHz}$. Finally, we set the evolution period $\tau = \pi/\Omega = 50\,\mu\text{s}$ to satisfy the condition $\alpha_\tau = \pi$.

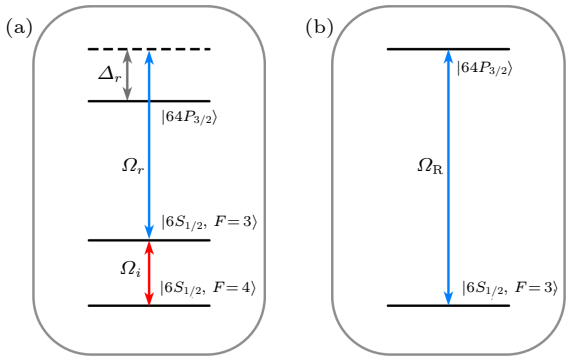

Fig. 2. Diagrammatic sketch of the Rydberg atom with the relevant energy level structure of the ^{133}Cs atom to realize our two-qubit gate (a) and to realize the comparison scheme (b).

Firstly, we numerically simulate the dynamics of the average fidelities of the two two-qubit gates by using original Hamiltonian $H_{\rm I}$ and effective Hamiltonian $H_{\rm eff}$ in Fig. 3, where the results show that the average fidelities for each of them, driven by $H_{\rm I}$ and $H_{\rm eff}$, respectively, are almost equivalent. Therefore, the effective Hamiltonian can approximately replace the original Hamiltonian under the large detuning condition. Thus, our scheme is feasible. Moreover, the average fidelities of the controlled-NOT gate and controlled-Hadamard gate can reach 0.9981 and 0.9983, respectively.

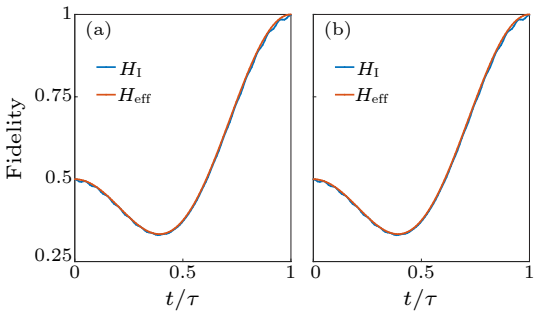

Fig. 3. Dynamics of the average fidelity for the controlled-NOT gate (a) and controlled-Hadamard gate (b) driven by original Hamiltonian $H_{\rm I}$ (blue curve) and effective Hamiltonian $H_{\rm eff}$ (red curve), respectively. The parameters of numerical calculation is adopted from the ^{133}Cs atom.

As a comparison scheme, we use the same atom system but with the qubits directly encoded in the ground state $|g\rangle$ and Rydberg state $|r\rangle$, and the computational space is taken as $S = \text{Span}\{|gg\rangle, |gr\rangle, |rg\rangle, |rr\rangle\}$. For each of these two atoms, as shown in the energy level structure of Fig. 2(b), the transition from $|g\rangle_i$ to $|r\rangle_i$ is driven by a resonant laser pulse with Rabi frequency $\Omega_{{\rm R}i}$ $(i = 1, 2)$. Making $\Omega_{\rm R1} = -\cos(\phi/2)\Omega_{\rm R}$, $\Omega_{\rm R2} = \sin(\phi/2)\Omega_{\rm R}$ with Rabi frequency $\Omega_{\rm R}/2\pi = 1.5\,\text{MHz}$ and Rydberg interaction $U/2\pi = 359.08\,\text{MHz}$, the same evolution operator and two-qubit logic gate, provided in Eq. (8), can be obtained when the evolution period τ and Rabi frequency $\Omega_{\rm R}$ satisfy the same relation as our scheme, i.e., the cumulative phase $\alpha_\tau = \pi$.

In practical quantum systems, the fidelity of quantum gate will decrease due to the decoherence caused by decay and dephasing of atoms, and the control errors of laser's and microwave's parameters. When the decaying rate γ_r and dephasing rate γ_p are taken into account, the average fidelities of the two schemes are shown in Table 1 such that our scheme owns the higher average fidelities than the traditional scheme.

Table 1. The average fidelities of the gates of the two schemes.

	C-NOT gate	C-Hadamard gate
Our scheme	0.9955	0.9955
Contrast scheme	0.9947	0.9950

In practice, due to the instability of laser, the Rabi frequency Ω will produce a certain error provided that we assume the amplitudes of the driven pulse Ω to vary in the range of $(1 + \alpha)\Omega$ with the error fraction $\alpha \in [-0.1, 0.1]$. As shown in Figs. 4(a) and 4(b), the variation of the average fidelities for these two schemes under the same Rabi error is obtained by numerical simulation. It can be seen that in our scheme, when the Rabi frequency Ω_r of the dressed laser changes, the fidelities of the gates show periodic oscillation and always stay at a high level which demonstrate the highly robust to the errors of Rabi fre-

quency Ω_r. Therefore, the numerical simulation shows that compared with the previous scheme, our scheme retains its advantages on the fidelity, meanwhile, reduces the influence of dissipation on the system, and further improves the robustness of the two-qubit quantum gates. In addition, the uncertainty of atomic position will cause an error in Rydberg interaction strength U, which also affect the fidelity of the gate. We consider this effect through numerical simulation and the results are shown in Fig. 4(c). Although Fig. 4(c) shows that the disturbance of U has a certain impact on the fidelity of our scheme, we can focus both lasers onto the atom array in counterpropagating configuration to minimize the disturbance of U.[13]

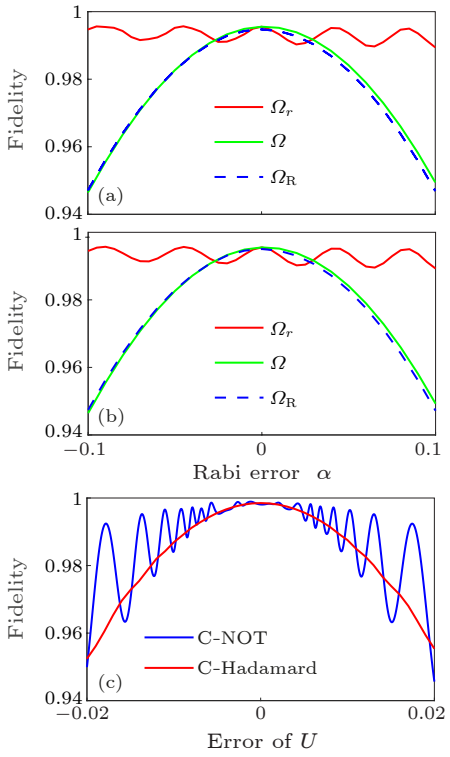

Fig. 4. The change of the average fidelities of the controlled-NOT gate (a) and controlled-Hadamard gate (b) with the Rabi error α in the Rabi frequency, where Ω_r (red line) and Ω (green line) are the Rabi frequency in our scheme, and Ω_R (blue line) is the Rabi frequency in the comparison scheme. (c) The change of the average fidelities of the controlled-NOT gate (blue line) and controlled-Hadamard gate (red line) with the error of Rydberg interaction strength U.

Discussion. In our scheme, we implement the two-qubit nonadiabatic geometric quantum logic gate in ground state of the Rydberg atom. The scheme is robustness against control errors due to the feature of geometric quantum computation. In addition, our scheme also has other features.

Firstly, the scheme would generate mechanical effect if there exist spontaneous decay and dephasing of Rydberg states of both the atoms. In contrast to Ref. [41] by locking the evolution of atoms in the ground state for quantum computation, it greatly reduces the adverse effect, caused by the Rydberg state decay process, on the quantum gates.

Second, in the previous scheme, the computational space is $S = \text{Span}\{|gg\rangle, |gr\rangle, |rg\rangle, |rr\rangle\}$. However, due to Rydberg blockade, it is difficult for two atoms to be excited to Rydberg state at the same time, so the state $|rr\rangle$ used to encode the qubits cannot appears. In our scheme, the qubits are encoded in the ground states, which makes the corresponding encoded state $|11\rangle$ to be experimentally implemented. Therefore, we have modified the quantum gates on the basis of the previous scheme to further improve the experimental performance of the quantum gates.

Finally, we point out that although we use two Rydberg atoms for quantum computation in this study, our scheme can be further extended to multiple Rydberg atoms for multi-bit quantum computation.

In conclusion, we propose a scheme for nonadiabatic geometric quantum computation of quantum bits encoded in the ground states of Rydberg atoms. We implement this scheme by dressing the ground state with the excited Rydberg state, which locks the atoms in the ground state. Our scheme further improves the performance of quantum gates while retaining the advantages of nonadiabatic geometric quantum computation and Rydberg atom. Therefore, our scheme is expected to provide a more promising alternative to Rydberg quantum gates.

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