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Semi-Measurement-Device-Independent Quantum State Tomography

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As a fundamental tool in the quantum information field, quantum state tomography can be used to reconstruct any unknown state. Generally, it needs a tomographically complete set of measurements, such that all measurements are fully characterized. Here, we propose a semi-measurement-device-independent quantum state tomography protocol, which only needs one characterized measurement and a trusted ancillary system. Furthermore, we perform corresponding experiments using linear optics. Our results show that the average state fidelity is as high as 0.973, verifying the effectiveness of the scheme.

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Quantum information science can more efficiently achieve information processing tasks than the classical analog.[1,2] To benchmark the performance of the processing devices used in quantum information, completely and precisely characterizing the dynamics of the device is fundamental. Quantum state tomography is a fundamental task to reconstruct an unknown state of a quantum system using the statistical outcomes of several measurements on identical copies.[3] It has been extensively developed as a standard method,[4–9] widely used in both discrete[10–16] and continuous[17] variable quantum systems.

The procedure of quantum state tomography consists of the measurements of the system and data processing. Without *a prior*, the measurement set should be informationally complete, allowing the expansion on the operator bases. Furthermore, general data processing is an implementation of the theory of operator expansions. Therefore, the experimental validity of quantum tomography relies on the characterized and trusted set of informationally complete measurements. In other words, quantum state tomography is a device-dependent state certification protocol.

To relax the assumptions on measurement devices in the traditional quantum state tomography, we propose a protocol for the semi-measurement-device-independent quantum state tomography. Contrary to requirements on all fully characterized measurements of the informationally complete set, our protocol needs only one set of trusted ancillary states and one trusted joint measurement. Next, we perform a photonic demonstration with single photons from spontaneous parametric down-conversion (SPDC) processes and a Hong–Ou–Mandel (HOM) interference.[18–21]

The goal of state tomography is to reconstruct the unknown state of a system using the statistical data collected from a tomographically complete set of measurements. Let ρ denote the density matrix of an unknown state for a d-dimensional quantum system, satisfying

$$\rho^\dagger = \rho, \quad \rho \geq 0, \quad \mathrm{tr}(\rho) = 1, \quad \mathrm{tr}(\rho^2) \leq 1. \tag{1}$$

In traditional quantum state tomography, a tomographically complete set of measurements $\{E_k\}_{k=1}^K$ is considered. For each E_k, N identical systems are measured independently and n_k of them click.[10] When N is large, the expected value is $n_k = Np_k(\rho)$, where $p_k(\rho) = \mathrm{tr}(\rho E_k)$ by referring Born's rule of quantum mechanics. Suppose that the noise in practical detection follows a Gaussian distribution. Then, the joint probability of obtaining $\{n_k\}_{k=1}^K$ conditioned on the unknown state ρ is[10]

$$P(\{n_k\}|\rho) = \frac{1}{\mathcal{N}} \prod_{k=1}^{K} \exp\Big[-\frac{[n_k - \bar{n}_k(\rho)]^2}{2\sigma_k^2}\Big], \tag{2}$$

where $\mathcal{N}$ is a normalization constant, and σ_k is the standard deviation for the k-th measurement E_k, approximately estimated as $\sqrt{n_k}$. The likelihood function is chosen as the logarithm of the joint probability,[10]

$$\mathcal{L}(\{n_k\}|\rho) = -\sum_k \frac{[n_k - \bar{n}_k(\rho)]^2}{2n_k}. \tag{3}$$

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Now, the estimated state operator $\rho_{\rm est}$ can be deduced by maximizing the likelihood function $\mathcal{L}$,

$$\rho_{\rm est} = \arg\max_{\rho} \mathcal{L}(\{n_k\}|\rho). \tag{4}$$

Without any prior information on the state, the choice of the measurement set should be tomographically complete. Practically, it consists of $K(\geq d^2-1)$ projectors, then $\{E_k = |\psi_k\rangle\langle\psi_k|\}_{k=1}^{K}$. For qubits, the projector set widely used in experiments is $\{|0\rangle, |1\rangle, |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)\}$.

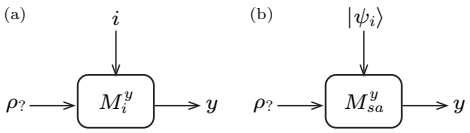

Fig. 1. (a) The quantum state tomography with characterized measurement devices. The device performs the measurement E_k according to the classical input k. (b) The quantum state tomography with trusted inputs. The device performs only one characterized joint measurement M_{sa} on the system and trusted ancilla, prepared in $|\phi_k\rangle_a$ using the classical input k.

The above process assumes the use of perfectly characterized and trusted measurement devices for a tomographically complete set. However, in practical experiments, it is more convenient and feasible to have trusted sources than trusted measurement devices. By assuming that the dimension of the target system is bounded, the tomography process can be modified to work in a measurement-device-independent scenario, which only needs a set of trusted quantum inputs $\{|\phi_k\rangle_a\}$ and one trusted joint measurement M_{sa}. The observed probability for a system in ρ_s (to be estimated) and an ancilla in $|\phi_k\rangle_a$ can be written as $p_k(\rho) = {\rm tr}(\rho_s \tilde{E}_k^s)$, where $\tilde{E}_k^s$ is an effective measurement on the system,

$$\tilde{E}_k^s = {\rm tr}_a(\boldsymbol{I}_s \otimes |\phi_k\rangle_a\langle\phi_k| M_{sa}). \tag{5}$$

Suppose that the joint measurement is a projector on the maximally entangled state $|\Phi\rangle_{sa} = \frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}|i\rangle_s \otimes |i\rangle_a$, the effective projector for the ancilla input $|\phi_k\rangle_a$ is $\tilde{E}_k^s = \frac{1}{d}|\phi^*\rangle_s\langle\phi^*|$, where $|\phi^*\rangle$ is the complex conjugate of $|\phi\rangle$. If the trusted input set of the ancilla $\{|\phi_i\rangle\}$ is tomographically complete, then the unknown system ρ can be reconstructed. The advantage of the revised protocol is that only one well-characterized measurement is required.

The proposed scheme can be obtained using the polarization degree of freedom of single photons generated using the spontaneous parametric down-conversion process. Here, the arbitrary qubit state in $\{H, V\}$ polarization is prepared using a half-wavelength plate (HWP) and a quarter-wavelength plate (QWP). Although the arbitrary two-qubit gate for photons can be obtained with a fixed probability, the most convenient joint measurement is the singlet projection $\{|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle\otimes|V\rangle - |V\rangle\otimes|H\rangle)\}$ via an HOM interference.[18,20,21]

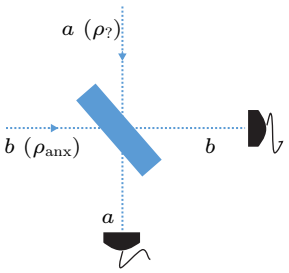

Fig. 2. Joint measurement for two photons with polarization degree of freedom based on the Hong–Ou–Mandel interference using a balanced beam splitter.

The HOM interference is one of the simplest quantum optical phenomena, which is distinct from the classical ones. When both input photons are indistinguishable in properties, such as polarization, spectral mode, and temporal mode, the beam splitter with reflectivity η can be modeled as a unitary transformation from two input spatial modes into two output spatial modes, for both polarization modes (see Fig. 2), where the action in the annihilation operators is written as

$$\begin{aligned} \hat{a}_{\rm out} &= \sqrt{\eta}\hat{a}_{\rm in} + \sqrt{1-\eta}\hat{b}_{\rm in}, \\ \hat{b}_{\rm out} &= \sqrt{1-\eta}\hat{a}_{\rm in} - \sqrt{\eta}\hat{b}_{\rm in}. \end{aligned} \tag{6}$$

If there is only one photon in each input mode, it is easy to verify that only one photon is detected in each output mode if and only if the state of the two input photons is in the single case $|\Psi^-\rangle$, i.e., the two-fold coincident count claims a successful projection on the singlet state $|\Psi^-\rangle\langle\Psi^-|$ for $\eta = \frac{1}{2}$.

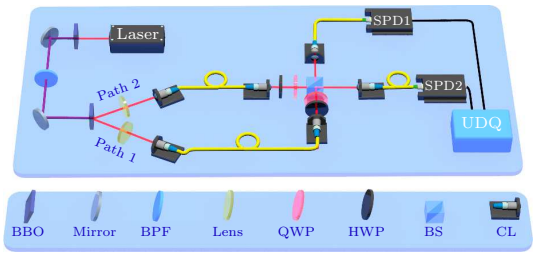

Fig. 3. Schematic setup of the semi-device-independent state tomography. Here, an SPDC source is used to generate the photon pairs, one (path 1) as the system to be estimated, and the other (path 2) as the ancilla to be prepared as the trusted input. BBO: β-barium borate, BPF: band-pass filter, HWP: half-wavelength plate, QWP: quarter-wavelength plate, BS: beam-splitter, CL: collimation lens; SPD: single-photon avalanche diode, UDQ: multichannel logic.

Figure 3 shows the photonic experimental setup of the semi-device-independent state tomography. A β-

barium borate (BBO) crystal cut for beam-like type-Ⅱ phase-matching is pumped using a pulsed laser with a central wavelength of 390 nm. Two photons, both having a central wavelength of 780 nm, are generated from the degenerate spontaneous parametric down-conversion process and coupled into single-mode fibers, respectively. Before being injected into the beam splitter (BS), the signal photon in path 1, is collimated and prepared in an arbitrary state with a pair of waveplates (HWP and QWP), whereas the idler, as the trusted ancilla in path 2 is collimated and prepared in any state from tomographically complete set $\{|\phi_k\rangle\}$ with another pair of waveplates. The photon is detected in each output path using a single-photon avalanche diode (SPD, Exlite: SPCM-AQRH-14-FC) with an interference filter of 3 nm bandwidth to erase the distinguishability in the spectrum. No information on polarization is extracted or disturbed after BS. For the ancilla in $|\phi_k\rangle$, the successful coincident count implies an effective measurement $\tilde{E}_k = |\phi_k^*\rangle\langle\phi_k^*|$. Here, the ancilla photon is prepared in three mutually unbiased bases, $\{|H/V\rangle, |D/A\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle, |L/R\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm i|V\rangle)\}$, which is tomographically complete. Therefore, the set of effective measurements $\{\tilde{E}_k\}$ is also tomographically complete. The two-fold coincidence count rate is about 2 kHz.

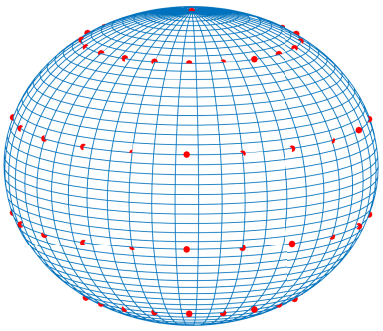

Fig. 4. The pure states are chosen from the Bloch sphere (red dots) to be estimated.

The performance of the photonic semi-measurement-device-independent tomography can be quantified by the fidelity $\mathcal{F}$ between the theoretical and reconstructed states, ρ_0 and ρ, respectively,[19]

$$\mathcal{F}(\rho_{\text{est}}, \rho_0) = \left(\text{tr}\sqrt{\sqrt{\rho_0}\rho_{\text{est}}\sqrt{\rho_0}}\right)^2. \tag{7}$$

Here, the system to be estimated is prepared in a pure state, $\rho_0 = |\psi\rangle\langle\psi|$ for $|\psi(\theta,\varphi)\rangle = \cos\frac{\theta}{2}|H\rangle + \sin\frac{\theta}{2}e^{i\varphi}|V\rangle$, with $\theta(\in[0,\pi])$ and $\varphi(\in[0,2\pi])$ being the angles of corresponding vectors on the Bloch sphere (see Fig. 4). We select different pairs of angles, $\theta = \frac{j\pi}{6}$ for $j \in \{1,2,\ldots,5\}$ and $\varphi = \frac{l\pi}{10}$ for $l \in \{0,1,\ldots,19\}$. In addition, two additional states $|H\rangle$ and $|V\rangle$ correspond to $\theta = 0, \pi$. The experimental results are summarized in Fig. 5. Figures 5(a) and 5(b) correspond to the state fidelity and state purity, respectively. For the 102 target states, the results show that the average fidelity and purity are 0.973 ± 0.010 and 0.961 ± 0.016, respectively. Note that some electronically controlled rotators are implemented to change the angles of all HWPs and QWPs, where systematic errors exist and cause fluctuations in calculating both state fidelity and purity.

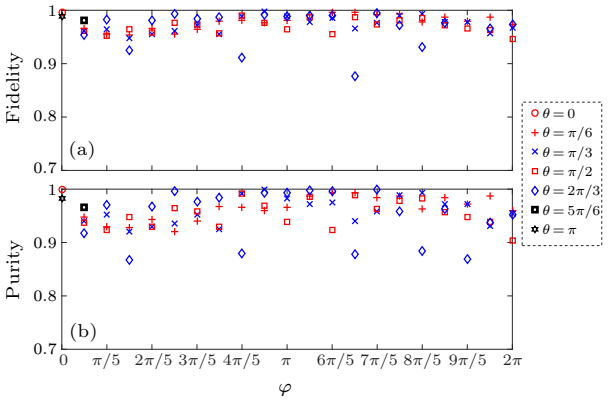

Fig. 5. (a) Fidelity between the theoretical states (red dots in Fig. 4) and the estimated states via the state tomography using trusted inputs and one characterized joint measurement. (b) Purity of the estimated results.

The traditional quantum state tomography is based on trusted measurement devices and requires full device characterization for all measurements. To relax the assumptions on measurement devices, we propose a semi-measurement-device-independent protocol for quantum state tomography. This protocol only requires one set of trusted ancillary states and one trusted measurement. As a result, this increases system security. Furthermore, experiments are conducted using the signal photon from SPDC as the estimated system, the idler photon as the trusted ancilla, and the HOM interference as the trusted measurement. The unknown states are reconstructed with the tomographically complete set of ancilla states by maximizing the likelihood function. In principle, our proposed scheme can be generalized to other protocols, such as the Bayesian quantum and quantum process tomographies.[22] Therefore, our scheme provides a valuable tool for the practical implementation of quantum information and quantum optics.

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References

[1] Arute F, Arya K, Babbush R *et al.* 2019 *Nature* **574** 505
[2] Zhong H S, Wang H, Deng Y H *et al.* 2020 *Science* **370** 1460
[3] Paris M and Rehacek J 2004 *Quantum State Estimation* (Berlin: Springer)
[4] Jones K R W 1994 *Phys. Rev. A* **50** 3682
[5] D'Ariano G M, Macchiavello C and Paris M G A 1994 *Phys. Rev. A* **50** 4298
[6] Dunn T J, Walmsley I A and Mukamel S 1995 *Phys. Rev. Lett.* **74** 884
[7] Leonhardt U, Paul H and D'Ariano G M 1995 *Phys. Rev. A* **52** 4899
[8] Raymer M G, Beck M and McAlister D F 1994 *Phys. Rev. Lett.* **72** 1137
[9] Leonhardt U and Raymer M G 1996 *Phys. Rev. Lett.* **76** 1985
[10] James D F V, Kwiat P G, Munro W J and White A G 2001 *Phys. Rev. A* **64** 052312
[11] Thew R T, Nemoto K, White A G and Munro W J 2002 *Phys. Rev. A* **66** 012303
[12] O'Brien J, Pryde G, White A *et al.* 2003 *Nature* **426** 264
[13] Li J, Liu T J, Wang S, Jebarathinam C and Wang Q 2019 *Opt. Express* **27** 13559
[14] Sun K, Ye X J, Xiao Y *et al.* 2018 *npj Quantum Inf.* **4** 12
[15] Xiao Y, Ye X J, Sun K, Li C F and Guo G C 2017 *Phys. Rev. Lett.* **118** 140404
[16] Sun K, Xu J S, Ye X J *et al.* 2014 *Phys. Rev. Lett.* **113** 140402
[17] Lvovsky A I and Raymer M G 2009 *Rev. Mod. Phys.* **81** 299
[18] Wang S, Liu C X, Li J *et al.* 2019 *Sci. Rep.* **9** 3854
[19] Liu T J, Wang C Y, Li J, Wang Q 2017 *Epl* **119** 14002
[20] Ou Z Y, Rhee J K and Wang L J 1999 *Phys. Rev. A* **60** 593
[21] Jin R B *et al.* 2015 *Opt. Express* **23** 28836
[22] Huszár F and Houlsby H M T 2012 *Phys. Rev. A* **85** 052120