Approach the Gell-Mann–Okubo Formula with Machine Learning

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Machine learning is a novel and powerful technology and has been widely used in various science topics. We demonstrate a machine-learning-based approach built by a set of general metrics and rules inspired by physics. Taking advantages of physical constraints, such as dimension identity, symmetry and generalization, we succeed to approach the Gell-Mann–Okubo formula using a technique of symbolic regression. This approach can effectively find explicit solutions among user-defined observables, and can be extensively applied to studying exotic hadron spectrum.

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Spectroscopy is a complex art, but interesting and helpful, to shed light on the underlying physics. It has exhibited its power in cosmology, molecular physics, atomic physics, particle physics, and so on. For instance, in cosmology, it can tell how an object like a black hole, neutron star, or active galaxy is producing light, how it moves, and even the elements of the interested object. In atomic physics, the Rydberg formula of the spectrum of hydrogen leads to the quantum mechanics interpretation. In particle physics, Gell-Mann and Okubo developed eightfold way classification schemes for bloomy hadrons uncovered by new experimental techniques in 1960s.[1–3] This discovery also indirectly led to the establishment of the quark model, which stated that baryon and meson were made of three quarks and quark-antiquark, respectively. This picture was challenged until the first observation of exotic candidate $X(3872)$ in 2003. Subsequently, numerous exotic candidates emerged in various high energy accelerators.[4–13] Particle physicists keep trying to discover a mass formula for describing the observed exotic candidates and predicting the new ones.

Obviously, these works require physicists to develop an excellent insight and a rich imagination after a long professional training. Very recently, machine learning has been widely used in various topics,[14,15] which has exhibited great success and power in solving specific application problems. However, it is still at infancy stage in discovering general scientific laws[16] from experimental data. Recent activities have been made with different approaches, for example, neural network in finding conserved quantities in gravitation[17] and the equation of a primordial state of matter in high-energy heavy-ion collisions,[18] or symbolic regressions in extracting physical parameters from complex data sets[19] and unknown functions from the Feynman Lectures on Physics.[20]

In this work, we demonstrate a physics-inspired machine-learning framework consisting of a set of general rules and metrics to discover explicit connections from many observables, even to discover hidden physical laws and to explain more complex systems without too much prior knowledge of particle physics. More specifically, we establish a machine-learning framework to find the Gell-Mann–Okubo formula from baryon decuplet and octet. Unlike purely mathematical approaches, this approach takes advantages of physical constraints, such as dimension identity, symmetry and generalization, to effectively find proper solutions.

Framework. The primitive framework is described as follows:

1. Input data: The observable $\boldsymbol{x} \in R^m$ and the target $\boldsymbol{y} \in R$ can be a scalar or a vector, for example, the motion speed, a position, and economic indices of the society. Data in the observation space can be collected from experiments or generated with simulations. Unlike massive data used in deep learning, a few data should be enough. In our case, the decuplet and octet baryon masses are the input data.

2. Model evolution:[21] Technically, we use a chain tree to construct an evolutionary model f as illustrated in Fig. 1, in which green (blue) nodes represents operators (observables). Here, f represents an analytical formula consisting of a set of operators and observables predefined as listed in Table 1, and p free parameters. Any operator must carry two observables, one of them can be unfolded into a sub tree until reaching the tail. A model is expressed by sequentially folding nodes of a chain tree from right to left. For instance, Fig. 1 (top) represents $f(Y, I) = p_1 + p_2 Y$. Here, p_1 and p_2 are free parameters, Y and I are observables. Clearly one operator and two observables can only construct simplest

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models. Complicated models stem from evolution of simple models by forking the tail node or mutating any node for many times following an idea of the evolutionary algorithm.[21] Figure 1 (bottom) represents another model $f(Y, I) = p_1 + p_2(Y + p_4 I)$ after two times evolution.

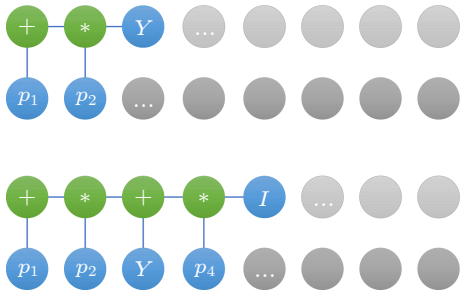

Fig. 1. A formula is unfolded from left to right into a tree consisting of a set of operators and a set of observables. This formula is calculated by folding the tree from right to left.

Table 1. Predefined operators and observables, which are arbitrary in principle.

Operators	Descriptions	Observable	Descriptions
$+$	Addition	M	Hadron mass
$-$	Subtraction	Y	Hypercharge
$\times$	Multiplication	I	Isospin
$\div$	Division	p	Free parameters

3. Physics filter: The challenging task is to construct all models, because available models in mathematical space are infinite. Initial enumeration generates all possible models by mutating the operator (observable) with the rest ones given the depth of a chain tree. However, physics filters can strongly guide how to remove meaningless models. For example, dimension identity requires any two observables belonging to the "addition" operator of having identical dimension. Or, on the other hand, observables with different dimensions only allow for multiplication or division. Obviously meaningless formulas Y/Y or $Y - Y$ can be easily filtered out. These rules are referred to as physical filters.
4. Metrics: They measure the goodness of a model fit to data. A typical metric is $\chi^2 = \sum_{j=1}^{N}(f_p(x_j) - y_j)^2$, which measures the goodness of model fitting to data. Minimizing a series of metrics achieves a reasonable solution after many iterations with the numerical gradient descent method given a learning rate. This technique is called symbolic regression.[22] Symbolic regressions not only find models but also solve them, while traditional regressions only solve a model for a given data. The return values of metrics are used to guide the evolution direction. For examples, models like p_1/Y or p_1/I will be kicked out at the beginning if data favors $p_1 Y$. Generally speaking, traditional approaches will stop here, but this approach will further guide the evolution of current model.
5. Generalization: the generalization proceeds along two directions, the model and the data. For the former case, we need an invariant model. For example, the gravitation law works for all planets in the solar system. For the latter case, we need a model that can work well for all data at different domains. For example, relativity theory works for motion objects of both low velocity and high velocity. Generalization possibly brings new knowledge as discussed in next step.
6. Inference: In the case of holding the form of current model invariant, refinement of current model with updated data can figure out connections among free parameters. In the case of the expanding current model, new evolution of the current model can tell hidden concepts. For example, suppose a model $F = p_1/r^2$ to be learned from the trajectory data of Earth, a new model $F = p_2/r^2$ can be obtained from the trajectory data of Mars. Generalization results in a better model $F = p_{12} m_i/r^2$ and infers a planet-related concept m_i ($i =$ Earth, Mars) which actually represents a dimensionless mass normalized by the mass of Earth.[23]
7. Output: Current model will be stored in model database for future prediction, unification and expansion.

In a word, this framework combines the basic ideas of physics with guidelines of machine learning to build a reason system, by which hidden equations or new physics concepts can be inferred.

Time went back in 1960s, newly discovered hadrons led to Enrico Fermi said "Young man, if I could remember the names of these particles, I would have been a botanist." A classification of schemes for hadrons became as natural as what Mendeleev did for chemical elements. Starting from one of fundamental concepts of symmetries, Gell-Mann and Okubo have actually studied the relation of hadron mass, hypercharge and isospin of baryon decuplet and octet. They obtained a formula

$$M = a + bY + c\left[I(I+1) - \frac{1}{4}Y^2\right], \tag{1}$$

with prior knowledge of particle physics. Parameters a, b, c are extracted from measured hadron masses for a given irreducible representation. From Eq. (1), one can see that hadrons' mass M can be determined by their quantum numbers, i.e., hypercharge Y and isospin I, which stem from the underlying dynamics. Thus, we use the machine-learning-based approach to discover the Gell-Mann–Okubo formula from the $SU(3)$ baryon decuplet and octet.

Results and Discussions. With the above framework, we show an example in approaching the Gell-Mann–Okubo formula (1) from baryon spectrum. As the concepts of isospin[24] and hypercharge[25] have been well established before the Gell-Mann–Okubo formula, we can directly start from observable $\boldsymbol{x} = (Y, I)$ and the target observable $\boldsymbol{y} = (M)$ listed in Table 2. A predefined set of the above observables $\{M,\ Y,\ I\}$ and operators $\{+,\ -,\ \times,\ \div\}$ as listed in Table 1 are used to build models $f_C(Y, I)$, where

C represents free parameters. An evolution means that a model $f_C(Y, I)$ changes into a different form $f'_{C'}(Y, I)$. Parameters belonging to the model $f_C(Y, I)$ are solved with the gradient descent method after N times iteration,

$$\chi^2 = \sum_k^N \frac{(f_C(Y,I) - M)_k^2}{\epsilon_k^2}, \tag{2}$$

$$C_j^{i+1} = C_j^i - \alpha \cdot \frac{\partial \chi^2}{\partial C_j^i}, \tag{3}$$

where α is the learning rate, ϵ_k is the kth particle's mass error, C_j^i is the jth parameter for the ith iteration, C_j^{i+1} is the updated value for the $(i+1)$th iteration, and $\frac{\partial \chi^2}{\partial C_j}$ is the gradient of χ^2 with respect to C_j.

Table 2. The dataset of decuplet ($J^P = \frac{3}{2}^+$) and octet ($J^P = \frac{1}{2}^+$) baryons. The masses in the first column are taken from Ref. [26]. The central values are the isospin averaged ones. Their uncertainties are set to cover all the masses within the same isospin multiplet instead of the experimental values.

Y	I	$J^P = \frac{1}{2}^+$	$J^P = \frac{3}{2}^+$	Mass (MeV/c^2)
1	1/2	n, p		939 ± 1
0	0	Λ		1116 ± 1
0	1	$\Sigma^-, \Sigma^0, \Sigma^+$		1193 ± 4
−1	1/2	Ξ^-, Ξ^0		1318 ± 3
1	3/2		$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	1232 ± 2
0	1		$\Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}$	1385 ± 3
−1	1/2		Ξ^{*-}, Ξ^{*0}	1533 ± 2
−2	0		Ω^-	1672 ± 1

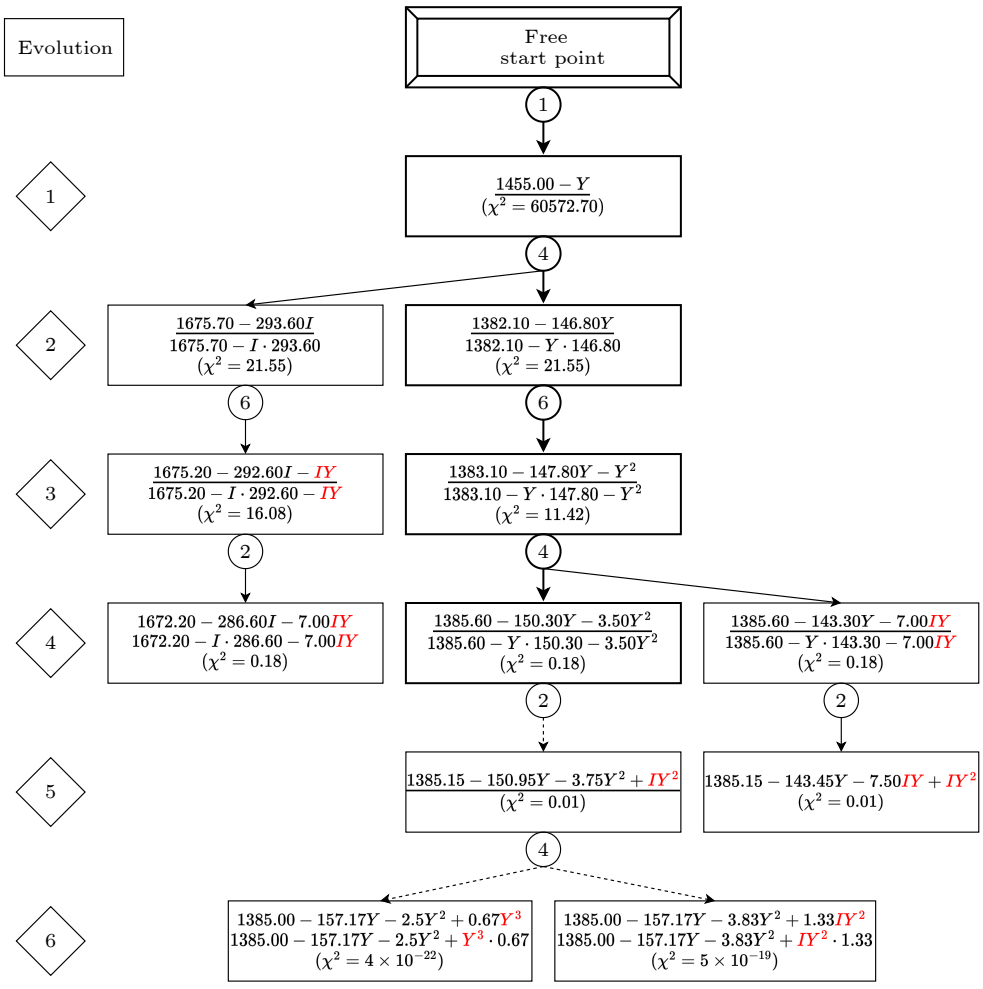

Fig. 2. Evolution results with baryon decuplet. The first column represents the times of evolution. The equations in the rectangle represent the evolutionary results of the generation, where the underlined equations are those needed to evolve, and the numbers in the circles below the rectangle represent the number of evolutionary results of the next generation. In addition, the bold and dashed lines represent the best and ineffective evolutionary processes, respectively.

As the Gell-Mann–Okubo formula works for ground hadrons, i.e., baryon decuplet, baryon octet and meson octet, in the $SU(3)$ flavor symmetry, baryon decuplet and baryon octet are used in our framework. The reason for neglecting meson octet is that its relation between I and Y is the same as that of the baryon octet, which would not provide further information about the Gell-Mann–Okubo formula. We firstly test data of the $SU(3)$ flavor symmetric decuplet. At the beginning, enumeration of the simplest models consisting of one operator and two ob-

servables yields $\{C_1Y, C_1I, C_1+Y, C_1+I, C_1/Y, C_1/I, C_1-Y, C_1-I, I+Y, I-Y, IY, I/Y\ldots\}$, among which the model $\{C_1-Y\}$ is favorite. Next, it evolves to either one branch $A:\{C_1-C_2I\}$ (the left column in Fig. 2) or the other branch $B:\{C_1-C_2Y\}$ (the middle column in Fig. 2). Evolution of branch A reaches a solution $\{C_1-IY+C_2I\}$ with a reasonable χ^2 value as listed in Table 3. However, this solution is rejected since the IY item is prohibited as isospin and hypercharge are different degrees of freedom. In 1932, Dirac[24] proposed isospin to represent the symmetry in isospin space in analogy with spin. In 1956, Gell-Mann[25] summarized the Gell-Mann–Nishijima formula to propose the hypercharge concept as the sum of baryon number and strangeness number. They also found that those two were additive conservation quantities in strong interaction, which mean that they should also be added in the hadron mass formula. Thus the crossed term IY item is prohibited. Evolution of branch B reaches a solution $\{C_1-C_2Y-C_3Y^2\}$ and another solution $C:\{C_1-C_2Y-C_3IY\}$ (the right column in Fig. 2). Note that, more evolution of solution B raises items like IY^2 or Y^3 and the corresponding χ^2 value becomes extremely small, which indicates a termination sign of over-fitting. As the result, we obtain an expression for baryon decuplet,

$$M_{\text{decuplet}} = C_1 - C_2Y - C_3Y^2. \tag{4}$$

Table 3. Evolution results for baryon decuplet. The first and second columns represent the times of evolution and the corresponding χ^2, respectively. The right column is the evolution result for each evolution time with the explicit values of all C_is. A learning rate $\alpha = 0.01$ is used. Branches A, B, C correspond to the left, middle and right columns of Fig. 2, respectively. The bold formula is accepted, while formulas with underlined items are rejected by physics constraints.

Branch A		
Evolution	χ^2	Model
1	60572.70	$1455.00 - Y$
2	21.55	$1675.70 - 293.60I$
3	16.08	$1675.20 - 292.60I - \underline{IY}$
4	0.18	$1672.20 - 286.60I - 7.00\underline{IY}$
Branch B		
Evolution	χ^2	Model
1	60572.70	$1455.00 - Y$
2	21.55	$1382.10 - 146.80Y$
3	11.42	$1383.10 - 147.80Y - Y^2$
4	0.18	$\mathbf{1385.60 - 150.30Y - 3.50Y^2}$
5	0.01	$1385.15 - 150.95Y - 3.75Y^2 + \underline{IY^2}$
6a	5×10^{-19}	$1385.00 - 151.17Y - 3.83Y^2 + 1.33\underline{IY^2}$
6b	4×10^{-22}	$1385.00 - 151.17Y - 2.50Y^2 + 0.67\underline{Y^3}$
Branch C		
Evolution	χ^2	Model
1	60572.70	$1455.00 - Y$
2	21.55	$1382.10 - 146.80Y$
3	11.42	$1383.10 - 147.80Y - Y^2$
4	0.18	$1385.60 - 143.30Y - 7.00\underline{IY}$
5	0.01	$1385.15 - 143.45Y - 7.50\underline{IY} + \underline{IY^2}$

We perform a new evolution using baryon octet, and list the results in Table 4 with repeating aforementioned procedure. Note that the χ^2's of models 7, 8a, 8b and 8c are the same, because the data is very limited. In total, there are only 4 groups of (I, Y), which result in these models to be exactly the same in the cases. Models 7 and 8a are of the same form. Models 8b and 8c cannot be acceptable due to the I^2Y term. As a result, we consider model 7 as the expression for baryon octet,

$$M_{\text{octet}} = C_a - C_bY + C_cI^3. \tag{5}$$

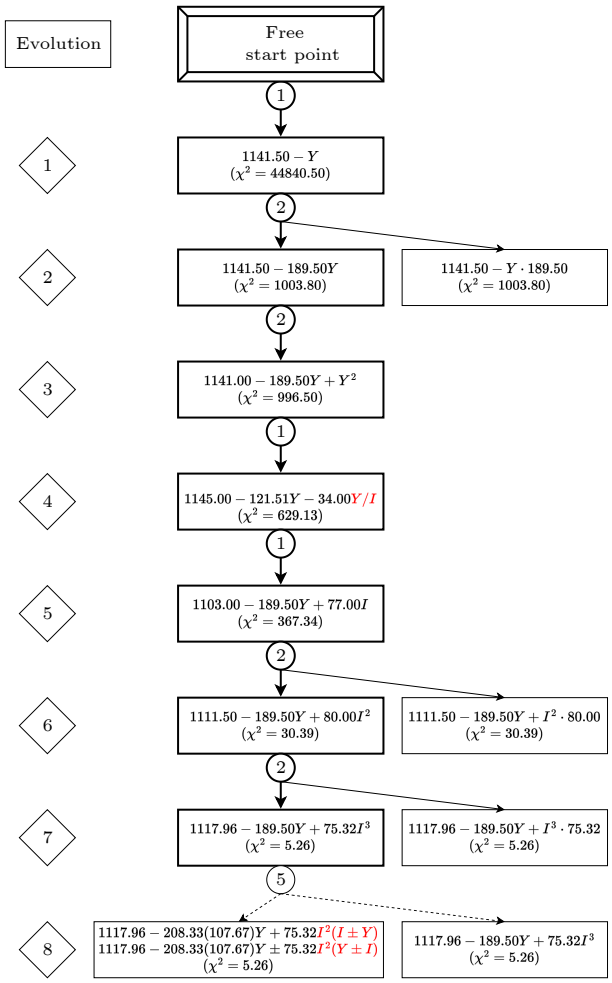

Fig. 3. Evolution results with baryon octet. The labels are analogous to those in Fig. 2.

Table 4. Evolution results for baryon octet. The caption is analogous to that of Table 3.

Evolution	χ^2	Model
1	44840.50	$1141.50 - Y$
2	1003.80	$1141.50 - 189.50Y$
3	996.50	$1141.00 - 189.50Y + Y^2$
4	629.13	$1145.00 - 121.51Y - 34.00\underline{Y/I}$
5	367.34	$1103.00 - 189.50Y + 77.00I$
6	30.39	$1111.50 - 189.50Y + 80.00I^2$
7	5.26	$\mathbf{1117.96 - 189.50Y + 75.32I^3}$
8a	5.26	$1117.96 - 189.50Y + 75.32I^3$
8b	5.26	$1117.96 - 208.33Y + 75.32\underline{I^2(I+Y)}$
8c	5.26	$1117.96 - 170.67Y + 75.32\underline{I^2(I-Y)}$

A further evolution required by generalization is possi-

ble to establish a universal equation of incorporating both Eqs. (4) and (5) using all data of baryon decuplet and octet. This evolution starts from Eq. (4), and is simultaneously guided by two metrics χ^2_8 and χ^2_{10}, i.e., the χ^2 defined in Eq. (2) for baryon octet and decuplet. The first metric χ^2_8 makes sure a model work well for baryon octet, and the second metric χ^2_{10} makes sure a copy of current model work well for baryon decuplet. Note that the form of current model is hold while its parameters are free. Evolution results are listed in Table 5. A generalized model for both baryon decuplet and octet can be taken as either

$$M = C_1 - C_2 Y - C_3 Y^2 + C_4 I \tag{6}$$

or

$$M = C_1 - C_2 Y - C_3 Y^2 + C_4 [I(I+1)]. \tag{7}$$

In both cases, the χ^2 values are reasonable. Furthermore, without the unknown $SU(3)$ flavory symmetry breaking mechanism, for instance, the electromagnetic interaction, particles within the same isospin multiplet should have the same mass. The value of the mass should be the eigen value of the operator $\hat{I}^2$, i.e., $I(I+1)$. Thus, the formula $C_1 - C_2 Y - C_3 Y^2 + C_4[I(I+1)]$ can be accepted by physics. The number of the parameters is consistent with that in the octet model,[27] where the four free parameters are the reduced matrix elements of the irreducible representations of one singlet, two octets and one 27-plet from the group theory point of view. The Gell-Mann–Okubo mass formula, i.e., Eq. (1), is obtained based on the assumption that the reduced matrix element of the 27-plet is zero. Thus the number of the free parameters is reduced to three. In our framework, this relation can also be seen by the relations between Y and I hidden in baryon decuplet and octet after evolution as listed in Table 6.

$$Y = 2I - 2, \text{ for baryon decuplet,} \tag{8}$$

$$Y^2 = 4I - 4I^2, \text{ for baryon octet.} \tag{9}$$

Table 5. Evolution results after applying the generalization to two models for baryon decuplet and octet. The first and second columns are the evolution time and the corresponding χ^2. The further evolution starts from the expression of baryon decuplet, i.e., Eq. (4). The third and forth columns are the explicit results for baryon decuplet and octet, respectively. The bold expressions are the final results. The terms underlined are rejected by physical constraints.

Evolution	$\chi^2_8 + \chi^2_{10}$	Decuplet	Octet
1	0.2055520	$803.16 - 441.52Y - 3.50Y^2 + 582.44I$	$1115.84 - 189.50Y - 25.95Y^2 + 77.24I$
2	0.1800060	$511.97 - 587.12Y - 3.50Y^2 + 873.63I$	$1116.00 - 189.50Y - 26.00Y^2 + 77.00I$
3	0.1799985	$\mathbf{689.54 - 672.34Y - 90.51Y^2 + 348.03I(I+1)}$	$\mathbf{1116.00 - 189.50Y - 16.38Y^2 + 38.50I(I+1)}$
4	0.1208117	$1381.94 - 153.19Y - 4.12Y^2 + 2.48\underline{[I(Y^2+1)]}$	$1116.00 - 189.50Y - 64.50Y^2 + 77.00\underline{[I(Y^2+1)]}$
5	1×10^{-10}	$1384.33 - 151.50Y - 3.83Y^2 + 0.67I + 1.33\underline{IY^2}$	$1116.00 - 189.50Y - 103.00Y^2 + 77.00I + 154.00\underline{IY^2}$

Table 6. Evolution results between Y and I for baryon decuplet and octet. Note that Y and I have no errors. A learning rate $\alpha = 0.01$ is used. The first and second columns are for the evolution time and the corresponding χ^2. The last column is the evolution result. The bold formulae are final results.

Baryon decuplet		
Evolution	χ^2	Model
1	1.25	$I - 1.25$
2	5×10^{-16}	$\mathbf{2.00I - 2.00}$
Baryon octet: Branch A		
Evolution	χ^2	Model
1	1.00	$0.50 + 0.50$
2	0.56	$0.375 + I - I^2$
Baryon octet: Branch B		
Evolution	χ^2	Model
1	1.00	0.71×0.71
2	1×10^{-15}	$\mathbf{4.00(I - I^2)}$

Substituting Eq. (9) to Eq. (7), one obtains

$$M_{\text{octet}} = C_1 - C_2 Y + (C_4 - 4C_3)I + (C_4 + 4C_3)I^2. \tag{10}$$

When $C_4 = 4C_3$, the above formula comes back to the mass formula (5) evolution from baryon octet. Along the same line, one can substitute Eq. (8) to Eq. (7) and obtain

$$M_{\text{decuplet}} = C_1 + 2C_4 - \Big(C_2 - \frac{3C_4}{2}\Big)Y + \Big(\frac{C_4}{4} - C_3\Big)Y^2. \tag{11}$$

Analogously, when $C_4 = 4C_3$, the above formula comes back to the expression $C_1 + 2C_4 - (C_2 - \frac{3C_4}{2})Y$. Although the expression does not come back to Eq. (4) as we expected, the vanishing Y^2 term can be obviously seen by the final result in Table 3, i.e., the coefficient of Y^2 is about two orders smaller than the others. As discussed above, to obtain an universal mass formula for both baryon decuplet and octet, $C_4 = 4C_3$ is required, which leads Eq. (7) to the Gell-Mann–Okubo mass formula (1). Note that the Gell-Mann–Okubo formula is obtained from physical considerations, combining with the data. As a result, the Gell-Mann–Okubo formula cannot give the least χ^2. We use machine learning to approach the Gell-Mann–Okubo formula.

Summary and Outlook. Machine learning is a novel and powerful technology for commerce and industry. It has been widely and successfully used in various science topics.[14,15] However, its application in hadron physics, especially theoretical hadron physics, is still on its early stage. Analogous to the role of periodic table of elements in chemistry, hadron spectroscopy can also imply the underlying dynamics. As a result, we start from hadron spectroscopy, more specifically baryon decuplet and the baryon octet. A physics-inspired machine learning approach is constructed in this work to approach the Gell-Mann–Okubo formula. This approach can also be extended to study the mass formula for exotic hadrons discovered and intensely discussed recently years.

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