Demonstration of Einstein–Podolsky–Rosen Steering with Multiple Observers via Sequential Measurements

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(Received 31 August 2022; accepted manuscript online 18 October 2022)

Quantum correlation is a fundamental property that distinguishes quantum systems from classical ones, and is also a fragile resource under projective measurements. Recently, it has been shown that a subsystem in entangled pairs can share nonlocalities with multiple observers in sequence. Here we present a new steering scenario where both subsystems are accessible by multiple observers. Simulation results show that the two qubits in a singlet state can be simultaneously steered separately by two sequential observers.

DOI: 10.1088/0256-307X/39/11/110301

The correlation between entangled distant quantum systems can be beyond any classical system and plays an important role in both fundamental quantum physics and quantum information science. The quantum correlation is also a powerful resource in quantum information processes, which is widely applied in many different applications such as teleportations, quantum key distributions, and quantum computations. Generally, quantum correlations can be described as a hierarchy with three inequivalent concepts, quantum entanglement,[1] Einstein–Podolsky–Rosen (EPR) steering,[2] and Bell nonlocality.[3] For bipartite systems, the weakest quantum correlation, e.g., entanglement, means that the state density cannot be decomposed into any separable ones. Bell nonlocality, i.e., the strongest quantum correlation, allows the violation of Bell inequalities based on the joint probability distribution of local measurements by two spatially separated players. As the intermediate form, EPR steering, which was first introduced by Schrödinger, and then formally defined by Wiseman *et al.*,[4,5] can be verified if one of the two spatially separated parties can steer the subsystem on the other player's side via local measurements. For the certification of these correlations,[6] the violation of Bell inequalities can verify nonlocality without any assumption of the measurement devices on both sides, which is considered as a device-independent scenario.[7] For EPR steering, the measurement devices on the one side need to be fully characterized, which is also called the one-side device-independent scenario, or the semi-device-independent scenario.[8]

Usually, quantum correlations are investigated for n particles with entanglements, and each of them is accessible by one and only one space-separated observer for projective measurements.[9] Some studies have used different degrees of freedom of parties. In Ref. [10], the steerability of three entangled qubits is demonstrated through two photons with the polarization and the spatial degree of freedoms. However, it is interesting to consider whether sequentially measuring the same subsystem by multiple observers can share different quantum correlations. Generally, for a sharp measurement, a system would collapse into one of the eigenstates of the projector, whence the system would be separable from other systems. However, an unsharp or a weak measurement can preserve some entanglement in the post-measurement state. Here the weak measurement refers to the measurement with intermediate coupling strengths between the system and the probe. In contrast to strong projective measurements, weak measurements can be less destructive and also remain some original features of the measured system.[11] It has been proven to be a good method for signal amplification,[12] state tomography,[13] and nonlocality verification. For a pair of entangled particles, it is possible to reserve some quantum correlation after measurements with intermediate strengths.[14] Silva *et al.*[15] showed that multiple observers can share the nonlocality on a pair of entangled qubits by using optimal weak measurements in a Bell scenario. Shneoy *et al.*[16] theoretically investigated the EPR steering with multiple observers. It was found that with the increase of the system dimensions, the number of Bobs that can steer Alice will be $N_{\mathrm{Bob}} \sim d/\log d$, as long as with appropriate measurement strengths. Here d represents the system dimension. For the case of qubits, i.e., $d = 2$, each Bob knows his respective position in the sequence, as he performs a measurement with a well-chosen strength η_i, the maximal sequence of Bobs that can steer Alice is five. Later, Choi *et al.*[17] experimentally demonstrated it by accomplishing the sequential steering including three Bobs and one Alice.

To date, most discussions of quantum correlations for multiple observers with a single entangled qubit pair have the focus that one qubit is accessible by several Bobs se-

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quentially, while the other one is accessible by one Alice. Recently, Cheng *et al.*[18] found that a two-qubit system cannot generate the Bell nonlocality between multiple independent observers on each side. In this study, we exploit the EPR steerability of a maximally entangled qubit pair, where each qubit is accessible by multiple observers sequentially. We find that on each side, the qubit can be steered by two remote independent observers. It is worth mentioning a similar scene in revealing of hidden nonlocalities,[19,20] with both qubits being locally filtered before the final sharp measurements. The local filter on each qubit can be considered as a special positive operator-valued measurement (POVM), where only one output is delivered to the successive observer, while the weak measurements above are one type of POVM with two outputs, both of which are accessible for all successive observers.

Considering a pair of quantum subsystems, see Fig. 1, A and B are shared by two spatially separated observers, Alice and Bob, where each subsystem is accessible by one and only one observer. Generally said, the steering describes the ability that one observer, Alice (Bob), via the measurement on A (B), can nonlocally steer the other's subsystem B (A).

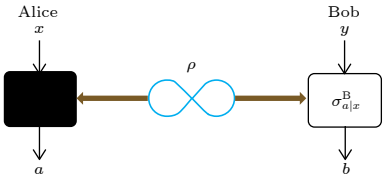

Fig. 1. A general scenario of the quantum steering. Alice and Bob are two spatially separated observers. The measurement device of Alice can be represented by a black box, with the classical input x (choice of measurements) and the output a. Bob performs the state tomography and records his own measurement choice y and the result b.

To be specific, the quantum steering can be described as a scenario where two spatially separated players, Alice and Bob, share a bipartite quantum state $\rho_{\rm AB}$. Alice implements a measurement with the inputs x on the subsystem A and gets the output a, here both x and a are classical variables. Bob can fully characterize the state in his subsystem with trusted devices. Conditioned on Alice's declaration of (x, a) with the probability $p(a|x)$, Bob would found his subsystem B collapsed into the state $\tilde{\sigma}^{\rm B}_{a|x}$ (unnormalized):[21]

$$\tilde{\sigma}^{\rm B}_{a|x} = {\rm Tr}_{\rm A}(M_{a|x} \otimes I_{\rm B} \cdot \rho_{\rm AB}). \tag{1}$$

The normalized state is $\sigma^{\rm B}_{a|x} = \tilde{\sigma}^{\rm B}_{ax}/p$, where $p = {\rm Tr}(\tilde{\sigma}^{\rm B}_{a|x})$. The set of Bob's conditioned states for all measurement choices and Alice's results forms an assemblage. The assemblage will obey a local hidden state (LHS) model if each state from the assemblage can be written as

$$\sigma^{\rm B}_{a|x} = \int f(\lambda) P_{\rm A}(a|x,\lambda)\sigma_\lambda {\rm d}\lambda, \tag{2}$$

where λ is a random variable with the distribution $f(\lambda)$ representing Alice's state. Then Alice cannot steer Bob's subsystem as long as the state conditioned on (x, a) can be chosen from the pre-existing ensemble $\{\sigma_\lambda\}$ with the distribution $f(\lambda)$ by Alice's declaration probability $P_{\rm A}(a\,|\,x,\lambda)$.[16] Otherwise, Bob can be convinced that his subsystem can be steered by Alice remotely. Steerability could be one-directional, where only one player (Alice) can steer the subsystem accessible by the other one (Bob).[22] Here, we focus on the bidirectional steering scenario and show that Alices and Bobs can all steer the remote subsystem, due to the existence of sequential and weak measurements.

The EPR steerability can be verified by the violation of linear steering inequalities,[20] just like the Bell inequalities for the Bell nonlocality. In the scenario of a two-qubit system, Alice declares her result $a_k \in \{-1, +1\}$ for each choice of the measurement $k \in \{1, 2, \ldots, n\}$, and Bob makes the corresponding dichotomic measurements $\hat{B}_k$ on his subsystem. The inequality reads

$$S_n = \frac{1}{n}\sum_{k=1}^{n}\langle a_k \hat{B}_k\rangle \le C_n. \tag{3}$$

C_n is the upper bound of inequalities allowed by the LHS model,

$$C_n = \max_{\{a_k\}}\Big\{\lambda_{\max}\Big(\frac{1}{n}\sum_{k=1}^{n} a_k \hat{B}_k\Big)\Big\}, \tag{4}$$

where $\lambda_{\max}(G)$ is the maximum eigenvalue of the operator G. To verify the steerability, both players can perform dichotomic quantum measurements, $\{\hat{A}_k\}$ and $\{\hat{B}_k\}$ on the qubits A and B respectively, and they can calculate the expected value of the steerability parameter S_n from the statistics. If the inequality is violated (observed S_n exceeds C_n), Alice can steer Bob.

Conversely, Bob declares his result $b_k \in \{+1, -1\}$ and Alice makes her corresponding measurements $\hat{A}_k$. The steerability on Alice's subsystem by Bob can be verified if the following inequality is violated,

$$S'_n = \frac{1}{n}\sum_{k=1}^{n}\langle b_k \hat{A}_k\rangle \le C'_n, \tag{5}$$

where

$$C'_n = \max_{\{b_k\}}\Big\{\lambda_{\max}\Big(\frac{1}{n}\sum_{k=1}^{n} b_k \hat{A}_k\Big)\Big\}. \tag{6}$$

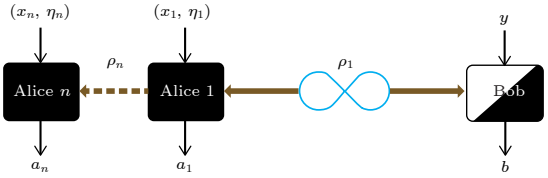

Fig. 2. The scenario of 1 vs n nonlocality (steering) verification. Here ρ is the shared state of two subsystems, one for all n Alices and the other only for Bob. Each of Alice$_1$ ... Alice$_{n-1}$ implements a weak measurement with strength $\eta_1 \ldots \eta_{n-1}$, respectively; Alice$_n$ and Bob implement strong measurements. Bob, half black and half white, represents two modes under different conditions. In the Bell-nonlocality scenario, the black box denotes no preassumption on the measurement, which is device-independent. However, in the steering scenario, the white box allows Bob to perform the trusted quantum measurement or the state tomography, which is device-dependent.

In most studies on multi-observer nonlocalities with single entangled pairs, one subsystem is measured sequentially by different observers, while the other is accessible

by only one observer. Here we call it the 1 vs n nonlocality. The model is shown in Fig. 2. For the multiple steering protocol, it is the one-directional steering on the same subsystems. In Ref. [17], it was shown that for optimal weak measurements, the number of Alices that can steer Bob is found to be $N_{\text{Alice}} \sim d/\log d$. For a qubit system, $d = 2$, the maximal number of Alices is 5.

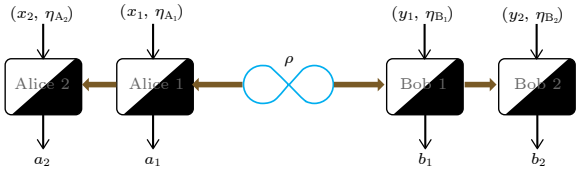

Fig. 3. A new type of quantum networks. Two subsystems share the state ρ, where Alice$_1$ and Bob$_1$ implement weak measurements and the parameter strengths are η_{A_1} and η_{B_1}, respectively. Alice$_2$ and Bob$_2$ implement strong measurements. The box with black and white shows that the observer can steer others and meanwhile can also be steered by others.

In our protocol, the maximally entangled two subsystems A and B are sequentially accessible by two observers, (Alice$_1$, Alice$_2$) and (Bob$_1$, Bob$_2$), respectively. On the one side with subsystem A, Alice$_1$ randomly selects her input $k \in \{1, 2, \ldots, n\}$, performs a weak measurement $A_1(k, \eta_A)$ with the sharpness $\eta_A \in [0, 1])$, keeps the outcome a_1, and delivers the post-measurement subsystem A to Alice$_2$. Here, Alice$_2$ has no knowledge of the measurement choice or the outcome of Alice$_1$. Then with a random choice of $j \in \{1, 2, \ldots, n\}$, Alice$_2$ performs her measurement $A_2(j)$. Since there is no further observer of A, the measurements by Alice$_2$ will always be sharp ones. The process is similar to the other side with subsystem B, where Bob$_1$ performs weak measurements and Bob$_2$ performs the sharp ones (see Fig. 3).

All the dichotomic measurements implemented by the four players can be represented by positive operator-valued measurements (POVMs) with two Kraus operators:

$$\mathrm{K}_{\pm|k} = \frac{1}{\sqrt{2}}\Big[\sqrt{1 \pm \eta} \cdot \Big(\frac{I + \boldsymbol{m}_k \cdot \boldsymbol{\sigma}}{2}\Big) + \sqrt{1 \mp \eta} \cdot \Big(\frac{I - \boldsymbol{m}_k \cdot \boldsymbol{\sigma}}{2}\Big)\Big], \tag{7}$$

where $k \in \{1, 2, \ldots, n\}$ is the choice of measurement, $\pm$ are the outcomes of dichotomic measurements, and η is the strength of the measurement for Alice's or Bob's. The observable of POVM can be written as [17]

$$\hat{\mathrm{E}}_k = M_{+|k} - M_{-|k} = \eta\, \boldsymbol{m}_k \cdot \boldsymbol{\sigma}, \tag{8}$$

where

$$M_{\pm|k} = K^{\dagger}_{\pm|k} \cdot K_{\pm|k}. \tag{9}$$

Here $\boldsymbol{\sigma}$ represents the vector of Pauli operators $(\sigma_x, \sigma_y, \sigma_z)$, and $\boldsymbol{m_k}$ denotes a unit vector on Bloch sphere. In other words, the set of all observables can be represented by n vectors $\{m_k\}_{k\in\{1,2,\ldots,n\}}$ and the measurement strength is η_i, for each player. The measurement is sharp for $\eta_i = 1$. For $\eta_i = 0$, both Kraus operators are proportional to the identity, where the system is unchanged and no information can be inferred from the measurement.

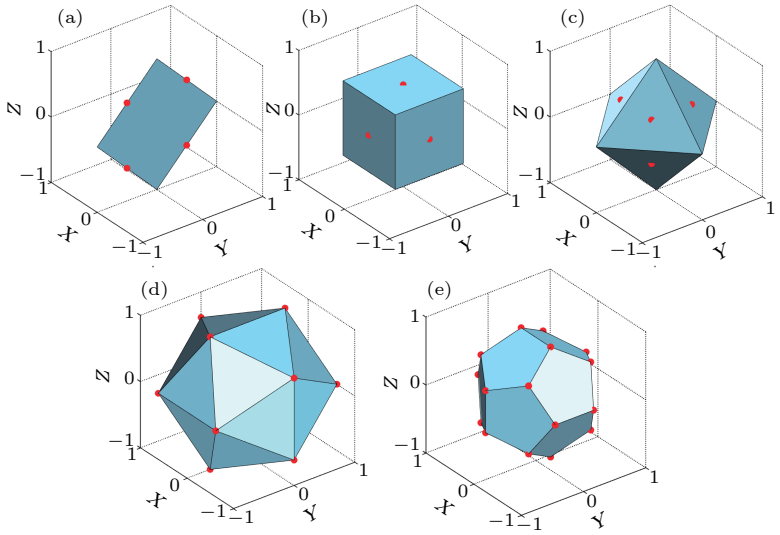

Fig. 4. The bullet symbols show the orientations of pure states in optimal cheating ensembles for two-qubit states. [23] From (a) to (e), we can get different pairs of coordinates as $n = 2$, 3, 4, 6, 10.

To verify the steerability, the choice of the measurement set should be optimized. Saunders *et al.* [24] showed that, for the singlet state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, the optimal measurements can be represented by the vectors $\{m_k, -m_k\}$, which is related to the Platonic solid depending on n. Here n represents the number of measurements allowed for each player. As shown in Fig. 4 and Table 1, the measurement settings correspond to different solids in the Bloch space, e.g., octahedron ($n = 3$), cube ($n = 4$), icosahedron ($n = 6$), and dodecahedron ($n = 10$). Except for the case of $n = 2$, here the four vectors form a square in a plane. Antipodal pairs of faces centered or vertices of a Platonic solid define the measurement vectors $\{m_k\}$. For $n = 2$, 3, 4, the vectors are from the origin of coordinate to the face centers; while for $n = 6$, 10, the vectors are from the origin of coordinate to the vertices of solid. [23] The LHS bound C_n changes with the number of measurements allowed, e.g., $C_2 = 1/\sqrt{2} \approx 0.7071$,

$C_3 = C_4 = 1/\sqrt{3} \approx 0.5773$, $C_6 \approx 0.5393$, $C_{10} \approx 0.5236$. The lower the bound is, the more the steering phenomena are robust to the noise.

Now, we consider the steering scenario. The two qubits in the state $\rho^{i'j'}$ is delivered to a pair of observers, A to Alice$_i$ and B to Bob$_j$, respectively. Alice$_i$ performs the measurement according to the random input k with the Kraus operators $\{K^{\eta_A}_{+|k}, K^{\eta_A}_{-|k}\}$ and gets an output $a \in \{+,-\}$. Bob$_j$ performs the measurement according to the random input l with the Kraus operators $\{K^{\eta_B}_{+|l}, K^{\eta_B}_{-|l}\}$ and gets an output $b \in \{+,-\}$. The (unnormalized) conditional post-measurement two-qubit state could be expressed as

$$\rho^{ij}_{a|k,b|l} = K^{\eta_A}_{a|k} \otimes K^{\eta_B}_{b|l} \rho^{i'j'} K^{\eta_A\dagger}_{a|k} \otimes K^{\eta_B\dagger}_{b|l}, \tag{10}$$

with the conditional probability $p(a,b|k,l) = \mathrm{Tr}(\rho^{ij}_{a|k,b|l})$. Then the steerability parameter for Alice$_i$ and Bob$_j$ can be written as

$$S^{ij}_n = \frac{1}{n}\sum_{k=1}^{n} \sum_{a,b\in\{+,-\}} (-1)^{a+b} p(a,b|k,k). \tag{11}$$

Table 1. Measurement directions for different settings [$a = (1+\sqrt{5})/2$].

Measurement setting n	(X, Y, Z)
2	(1,0,1),(1,0,−1)
3	(1,0,0),(0,1,0),(0,0,1)
4	(1,1,1),(1,−1,−1),(1,1,−1),(1,−1,1)
6	(0,1,a),(0,1,−a),(1,a,0),(1,−a,0),(a,0,1),(a,0,−1)
10	(0,1/a,a),(0,1/a,−a), (1/a,a,0),(1/a,−a,0) (a,0,1/a),(a,0,−1/a),(1,1,1),(1,−1,−1),(1,1,−1),(1,−1,1)

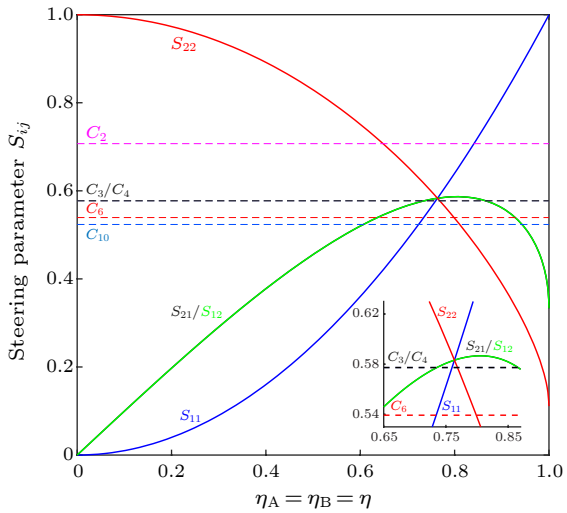

Fig. 5. Theoretical predictions of the EPR steering correlation S_n (S_{ij}) changing with the sharpness η, where we set $\eta_A = \eta_B = \eta$, and $\eta \in (0,1)$. The blue solid curve represents S_{11} (Alice$_1$ and Bob$_1$) in theoretical predictions, black/green for S_{21}/S_{12} (Alice$_2$ and Bob$_1$/Alice$_1$ and Bob$_2$), and red for S_{22} (Alice$_2$ and Bob$_2$). $S_{21} = S_{12}$ for all η. The magenta dashed horizontal line represents steering bound C_2, and the black one shows C_3 (C_4), red dashed line represents C_6, blue dashed line represents C_{10}.

The average post-measurement state after Alice$_i$'s and Bob$_j$'s measurements is

$$\rho^{ij} = \frac{1}{n^2}\sum_{k,l=1}^{n} \sum_{a,b\in\{+,-\}} \rho^{ij}_{a|k,b|l}, \tag{12}$$

which can be delivered to subsequential observers for further investigations. In our multi-observer steering scenario, the initial state is in the singlet state, $\rho^{00} = |\Psi^-\rangle\langle\Psi^-|$. If $S^{ij}_n > C_n$, the bidirectional steerability can be declared between Alice$_i$ and Bob$_j$ from the state $\rho^{i'j'}$, where $i' = i-1$ and $j' = j-1$, and the post-measurement state is ρ^{ij}. To investigate the steerability between Alice$_i$ and an observer after Bob$_j$, Alice$_i$ must postpone her measurement. Equivalently, she can perform the weak measurements with the sharpness $\eta_a = 0$ and keeps the post-measurement subsystem A for further measurements.

For our multi-observer steering scenario, each subsystem is accessible by two players separately. Then the steering should be investigated under four observer pairs, S_{11}, S_{12}, S_{21}, and S_{22}. Let the two-qubit initially be in the singlet state, we can numerically calculate the steerability parameters.

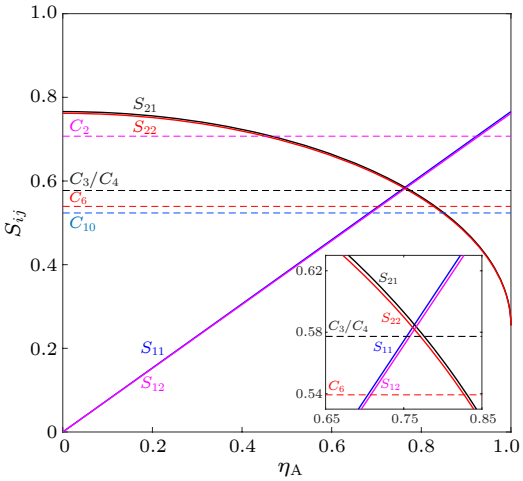

Fig. 6. The variation of S_{ij} with η_A, conditional on the fixed η_B. Only one variable affects the violations of different measurement settings n. Herein, S_{11}, blue solid line; S_{21}, black solid line; S_{12}, magenta solid line; S_{22}, red solid line; C_2, magenta dashed line; C_3/C_4 black dashed; C_6, red dashed line; C_{10}, blue dashed line.

Assume that both Alice$_1$ and Bob$_1$ perform independent measurements with the same sharpness, $\eta_A = \eta_B = \eta$. The theoretical predictions of the EPR steering correlation S_n (S_{ij}) and the violation of steering inequalities are shown in Fig. 5. The steerability parameters are calculated with a varying strength $\eta \in [0,1]$ by using the measurement set based on the Platonic solid. It is interesting that the steerability parameters are independent of n that is the size of the measurement set. Due to the symmetry between Alice's and Bob's, it is found that $S_{12} = S_{21}$. When

both Alice_1 and Bob_1 perform the sharp measurement, only the observer pair (Alice_1, Bob_1) can steer each other. When both Alice_1 and Bob_1 perform the zero-sharpness measurement, only the observer pair (Alice_2, Bob_2) can steer each other. Around $\eta \sim 0.76$, it is found that all the four observer pairs can violate against the linear steering inequalities for $n = 3, 4, 6, 10$. This means that every player can steer the state of the remote subsystem. When η increases from 0 to 1, the expected value of the steerability parameter S_{11} monotonically increases from 0 to 1 for the observer pair (Alice_1, Bob_1) while S_{22} monotonically decreases from 1 to 0 for (Alice_2, Bob_2). However, for (Alice_1, Bob_2) and (Alice_2, Bob_1), S_{12} and S_{21} increase from 0, reach the maximum value for $\eta \sim 0.8$, and then decreases. In Fig. 6, we fix the sharpness of Bob_1's measurements as $\eta_{\text{B}} = 0.766$. It is found that the steering abilities of two Bob's are very close, while they will monotonically increase/decrease for Alice_1/Alice_2 with the increase of Alice_1's measurement strength η_{A}.

In summary, we have presented a new type of steering scenario with a single pair of entangled subsystem, each of which is accessible by multiple observers. We also show that, by using weak measurements under the Platonic-solid configure, all the four independent observers, with two sequentially measuring on the subsystems separately, can steer the remote subsystem. We note that a double EPR steering scheme, proposed by Zhu *et al.*[11] independently, was demonstrated with a two-qubit entangled state and four observers. The difference is that, in the scheme of Zhu *et al.*,[11] the steerability was studied either between the first pair of observers, Alice_1 and Bob_1, or between the second pair, Alice_2 and Bob_2, while here two additional settings of steering, between Alice_1 and Bob_2, and between Alice_2 and Bob_1, can be realized in our scheme. Moreover, it is worth mentioning a similar scene in revealing of the hidden nonlocality,[19,20] while both qubits are locally filtered before the final sharp measurements. The local filter on each qubit can be considered as a special POVM, where only one output is delivered to the successive observer, while the weak measurements above are one type of POVM with two outputs, both of which are accessible for all successive observers.

Moreover, when fixing Bob_1's measurement strength, it would be more interesting to study the asymmetry of the steerability originating from the entangled state and different measurement sets. In such a scenario, it is worth studying whether more observers can steer the remote subsystem and how about the high-dimensional entanglement shared. Furthermore, recently, it is found that the sequential multi-party quantum random access code can witness the quantum channel,[25] verify the unsharp measurement,[26,27] and be generalized to characterize the correlations of quantum networks in prepare-and-measurement scenarios.[28] We believe that our steering scenario can also be implemented in similar scenarios of quantum networks.

The data sets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Acknowledgments. We gratefully acknowledge the financial support from the National Key R&D Program of China (Grant Nos. 2018YFA0306400 and 2017YFA0304100), the National Natural Science Foundation of China (Grant Nos. 12074194 and U19A2075), Leading-Edge Technology Program of Jiangsu Natural Science Foundation (Grant No. BK20192001), and the Industrial Prospect and Key Core Technology Projects of Jiangsu Provincial Key R&D Program (Grant No. BE2022071).

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