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Electromagnetic Form Factors of Λ Hyperon in the Vector Meson Dominance Model and a Possible Explanation of the Near-Threshold Enhancement of the $e^+e^- \to \Lambda\bar{\Lambda}$ Reaction

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The near-threshold $e^+e^- \to \Lambda\bar{\Lambda}$ reaction is studied with the assumption that the production mechanism is due to a near-$\Lambda\bar{\Lambda}$-threshold bound state. The cross section of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction is parameterized in terms of the electromagnetic form factors of Λ hyperon, which are obtained with the vector meson dominance model. It is shown that the contribution to the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction from a new narrow state with quantum numbers $J^{PC} = 1^{--}$ is dominant for energies very close to threshold. The mass of this new state is around 2231 MeV, which is very close to the mass threshold of $\Lambda\bar{\Lambda}$, while its width is just a few MeV. This gives a possible solution to the problem that all previous calculations seriously underestimated the near-threshold total cross section of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. We also note that the near-threshold enhancement can also be reproduced by including these well established vector resonances $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, or $\phi(2170)$ with a Flatté form for their total decay width, and a strong coupling to the $\Lambda\bar{\Lambda}$ channel.

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Electromagnetic form factors (EMFFs) are taken as an important tool for studying the electromagnetic structure of hadrons.[1] The measurements of space-like region EMFFs of proton can be carried out in elastic as well as inelastic ep scattering.[2] However, for Λ hyperon, its EMFFs at the space-like region have very hardly been experimentally measured. Instead, the electron-positron annihilation process, $e^+e^- \to \Lambda\bar{\Lambda}$, allows to study the Λ hyperon EMFFs at the time-like region.[3–11] In addition, the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction can be used to study vector mesons with light quark flavors and mass above 2 GeV,[12,13] especially for the ϕ excitations.[14,15]

Recently, the BESIII collaboration measured the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction with much improved precision. The Born cross section at the center of mass energy $\sqrt{s} = 2.2324$ GeV is determined to be $305 \pm 45^{+66}_{-36}$ pb.[4] This indicates that there is an evident threshold enhancement for the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. The observed value is larger than the previous theoretical predictions, which predicted that the total cross section of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction should be close to zero near the reaction threshold. In fact, before the new measurements of Ref. [4], there were several theoretical studies of this reaction, which proposed the final state interactions[7,9] to explain the unexpected features of the $e^+e^- \to \Lambda\bar{\Lambda}$ cross sections near threshold.

After the observations of Ref. [4] by the BESIII collaboration, the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction was investigated in Ref. [15] finding that the $\phi(2170)$ is responsible for the threshold enhancement. In Ref. [16], by using a modified vector meson dominance (VMD) model, an analysis on the EMFFs of Λ hyperon and also on the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction was performed, where those contributions from ϕ, ω, $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, and $\phi(2170)$ were taken into account. In Refs. [9,17], with the role played by the final state interactions of the baryon-anti-baryon pairs, the EMFFs of hyperons (Λ, Σ, and Ξ) were studied in the time-like region. The threshold enhancement of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction was also theoretically investigated in Refs. [9,16,17]. However, a large finite experimental value on the total cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ reaction at $\sqrt{s} = 2.2324$ GeV cannot be well reproduced.[9,16,17] Further investigations about the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction are mostly welcome.

On the other hand, the tails of vectors below threshold have to be detected as large effects in the time-like form factors of Λ hyperon and possibly as small structures in $e^+e^- \to \Lambda\bar{\Lambda}$ reaction near threshold. Indeed, as discussed in Ref. [15], the $\phi(2170)$ plays

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an important role to reproduce the threshold enhancement. However, the width of $\phi(2170)$, $\Gamma_{\phi(2170)} = 165 \pm 65\,\text{MeV}$,[18] is too wide, thus it will affect a large energy region. In addition, with the Godfrey–Isgur model, a narrow $\Lambda\bar{\Lambda}$ bound state with quantum numbers $J^{\text{PC}} = 1^{--}$ and mass around 2232 MeV was predicted,[14] and it has significant couplings to both the $\Lambda\bar{\Lambda}$ and e^+e^- channels. However, in Refs. [19,20], within the one-boson-exchange potential model, a $\Lambda\bar{\Lambda}$ bound state can also be obtained. This narrow $\Lambda\bar{\Lambda}$ bound state, if really exists, will contribute to the threshold enhancement of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction and also the EMFFs of the Λ hyperon in the time-like region.

In this Letter, we take the achievement of the vector meson dominance model and predictions of the narrow $\Lambda\bar{\Lambda}$ bound state as motivation to explore the electromagnetic form factors of the Λ hyperon in the time-like region. The EMFFs of baryons have been studied with the VMD model for the proton,[21–24] Λ hyperon,[16] Σ hyperon,[25] and charmed Λ_c^+ baryon.[26] Following Ref. [16], we revisit the EMFFs of Λ hyperon and the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction near threshold by employing the modified VMD model. In addition to the contributions from the ground ω and ϕ mesons, we also consider a new narrow vector meson with mass around 2232 MeV, as predicted in Ref. [14]. Yet, the $\phi(2170)$ resonance is not taken into account in the present work. (The contributions from $\omega(1420)$, $\omega(1650)$, and $\phi(1680)$ resonances are not considered either, since their masses are far from the reaction threshold of $e^+e^- \to \Lambda\bar{\Lambda}$, and their contributions could be absorbed into the ground ω and ϕ mesons. In addition, we refrain from including such contributions in this work because the model already contains a large number of free parameters.) Since the experimental information of it is still diverse, and the measured mass and width of $\phi(2170)$ resonance are controversial.[27–35] Indeed, there have also been different theoretical explanations for $\phi(2170)$ resonance.[36–50] In the following, the theoretical formalism of the Λ hyperon in the VMD model is shown. Then, we present our numerical results and discussions of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. A short summary is given finally.

Theoretical Formalism. First, we briefly review the vector meson dominance model to study the electromagnetic form factors of baryons with spin-1/2, and the total cross sections of $e^+e^- \to \Lambda\bar{\Lambda}$ reaction and the effective form factor of Λ.

Following Ref. [16], the electromagnetic current of Λ hyperon with spin-1/2 in terms of the Dirac form factors $F_1(Q^2)$ and Pauli form factors $F_2(Q^2)$ can be written as

$$J^\mu = \gamma^u F_1(Q^2) + \mathrm{i}\frac{\sigma^{\mu\nu}q_\nu}{2m_\Lambda}F_2(Q^2), \tag{1}$$

where F_1 and F_2 are functions of the squared momentum transfer $Q^2 = -q^2$. In the space-like region, $q^2 < 0$, while in the time-like region, $q^2 > 0$. The observed electric and magnetic form factors $G_{\text{E}}(Q^2)$ and $G_{\text{M}}(Q^2)$ can be expressed in terms of Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ by

$$G_{\text{E}}(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_\Lambda^2}F_2(Q^2), \tag{2}$$

$$G_{\text{M}}(Q^2) = F_1(Q^2) + F_2(Q^2). \tag{3}$$

In the VMD model, the virtual photon couples to Λ hyperon through vector mesons, thus the Dirac and Pauli form factors are parameterized as follows (There is no contribution from the ρ meson with isospin $I = 1$ because the isospin of Λ hyperon is zero):

$$F_1(Q^2) = g(Q^2)\Big[-\beta_\omega - \beta_\phi - \beta_x + \beta_\omega\frac{m_\omega^2}{m_\omega^2+Q^2} + \beta_\phi\frac{m_\phi^2}{m_\phi^2+Q^2} + \beta_x\frac{m_x^2}{m_x^2+Q^2}\Big], \tag{4}$$

$$F_2(Q^2) = g(Q^2)\Big[(\mu_\Lambda - \alpha_\phi - \alpha_x)\frac{m_\omega^2}{m_\omega^2+Q^2} + \alpha_\phi\frac{m_\phi^2}{m_\phi^2+Q^2} + \alpha_x\frac{m_x^2}{m_x^2+Q^2}\Big], \tag{5}$$

with $\mu_\Lambda = -0.723\hat{\mu}_\Lambda$ in natural unit, i.e., $\hat{\mu}_\Lambda = e/(2M_\Lambda)$. The $g(Q^2)$ is the Λ intrinsic form factor, and the other terms in Eqs. (4) and (5) are from the vector mesons (V) ω, ϕ, and a new introduced state, which will be discussed in the following.

The intrinsic form factor is a dipole $g(Q^2) = 1/(1+\gamma Q^2)^2$, which was well used for the proton case,[21–24] Λ case,[16] and Σ case.[25] In this work, the parameter γ in $g(Q^2)$ and the coefficients β_ω, β_ϕ, β_x, α_ϕ, α_x, m_x and Γ_x are model parameters, which will be determined by fitting them to the experimental data on the time-like electromagnetic form factors of Λ hyperon. The parameters β_ω, β_ϕ, β_x, α_x, and α_ϕ represent the products of a $V\gamma$ coupling and a $V\Lambda\Lambda$ coupling, while m_x and Γ_x are mass and total width of the new vector state included in this work. It is worth mentioning that the VMD model is valid in both space-like and time-like regions, the model parameters in both regions are usually considered to be unified, thus these parameters are real since the EMFFs of baryons in the space-like region are real.

In the time-like region we also consider the width of vector mesons to introduce the complex structure of the electromagnetic form factors of Λ hyperon.[5] For this purpose, we need to replace

$$g(Q^2) \to \frac{1}{(1-\gamma q^2)^2}, \tag{6}$$

$$\frac{m_\omega^2}{m_\omega^2+Q^2} \to \frac{m_\omega^2}{m_\omega^2 - q^2 - im_\omega\Gamma_\omega}, \tag{7}$$

$$\frac{m_\phi^2}{m_\phi^2+Q^2} \to \frac{m_\phi^2}{m_\phi^2-q^2-im_\phi\Gamma_\phi}, \tag{8}$$

$$\frac{m_x^2}{m_x^2+Q^2} \to \frac{m_x^2}{m_x^2-q^2-im_x\Gamma_x}, \tag{9}$$

where $q^2 = s$ is the invariant mass square of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. On the other hand, we take $m_\omega = 782.65\,\text{MeV}$, $\Gamma_\omega = 8.49\,\text{MeV}$, $m_\phi = 1019.461\,\text{MeV}$, and $\Gamma_\phi = 4.249\,\text{MeV}$, as quoted in the particle data group.[18]

Under the one-photon exchange approximation, the total cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ can be expressed in terms of the electric and magnetic form factors G_E and G_M of the Λ hyperon as[51]

$$\sigma_{e^+e^-\to\Lambda\bar{\Lambda}} = \frac{4\pi\alpha^2\beta}{3s^2}(s|G_\text{M}(s)|^2 + 2M_\Lambda^2|G_\text{E}(s)|^2), \tag{10}$$

where $\alpha = e^2/(4\pi) = 1/137.036$ is the fine-structure constant and $\beta = \sqrt{1-4M_\Lambda^2/s}$ is a phase-space factor.

The measurement of the total cross section in Eq. (10) at a fixed energy allows for determination of the combination of $|G_\text{E}|^2$ and $|G_\text{M}|^2$. With precise measurements of the angular distributions of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction, a separate determination of $|G_\text{E}|$ and $|G_\text{M}|$ is possible. Instead of a separation between G_E and G_M, one can easily obtain the effective form factor $G_\text{eff}(s)$ of the Λ hyperon from the total cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ annihilation process.[4,5] It is defined as

$$\begin{aligned} G_\text{eff}(s) &= \sqrt{\frac{\sigma_{e^+e^-\to\Lambda\bar{\Lambda}}}{[1+1/(2\tau)][4\pi\alpha^2\beta/(3\,s)]}} \\ &= \sqrt{\frac{2\tau|G_\text{M}(q^2)|^2 + |G_\text{E}(q^2)|^2}{1+2\tau}}, \end{aligned} \tag{11}$$

where $\tau = s/(4M_\Lambda^2)$. The effective form factor square $G_\text{eff}^2(s)$ is a linear combination of $|G_\text{E}|^2$ and $|G_\text{M}|^2$, and proportional to the square root of the total cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ reaction, which is definitely real. On the other hand, the effective form factor $G_\text{eff}(s)$ also indicates how much the experimental $e^+e^- \to \Lambda\bar{\Lambda}$ cross section differs from a point-like Λ hyperon.

Numerical Results and Discussions. In this work, following Ref. [22], we consider only the β_x term in the Dirac form factor F_1, and α_x is taken as zero (In fact, we found that only the sum of the two parameters β_x and α_x can be determined by the Λ effective form factor in the energy region of the mass threshold of $\Lambda\bar{\Lambda}$, thus we keep only the β_x term, that is, $\alpha_x = 0$, since α_x is associated to the tensor coupling in Pauli form factor F_2). Then, we perform seven-parameter (γ, β_ω, β_ϕ, β_x, α_ϕ, m_x and Γ_x) χ^2 fits to the experimental data on the effective form factor G_eff of Λ hyperon and the form factor ratio $R = |G_\text{E}/G_\text{M}|$. There are totally 18 data points. These data correspond to the center of mass energy $\sqrt{s}$ ranging from 3.08 down to 2.2324 GeV. The fitted parameters are listed in Table 1, with a reasonably small $\chi^2/\text{dof} = 0.9$.

Table 1. Values of model parameters determined in this work.

Parameter	Value	Parameter	Value
$\gamma(\text{GeV}^{-2})$	0.43 (0.48 ± 0.08)	β_ϕ	1.35
$\beta_\omega(\beta_{\omega\phi})$	-1.13 (-0.21 ± 0.14)	α_ϕ	-0.40
$\beta_x(10^{-3})$	1.50 (-3.21 ± 4.35)	Γ_x(MeV)	4.7 (4.8 ± 15.6)
m_x(MeV)	2230.9 (2229.1 ± 11.5)		

In the present work, since both the ω and ϕ are far from the mass threshold of $\Lambda\bar{\Lambda}$, the behavior of the contributions from them are similar, we have performed a new fit, in which only the ω term was considered. Thus, we have assumed that $\beta_\phi = \alpha_\phi = 0$ and $\beta_{\omega\phi} = \beta_\omega$, $m_\omega \to (m_\omega + m_\phi)/2$ and $\Gamma_\omega \to (\Gamma_\omega + \Gamma_\phi)/2$. The fitted parameters are shown in brackets in Table 1. In the case, we get errors for the fitted model parameters, however, the errors obtained from the fit are large.

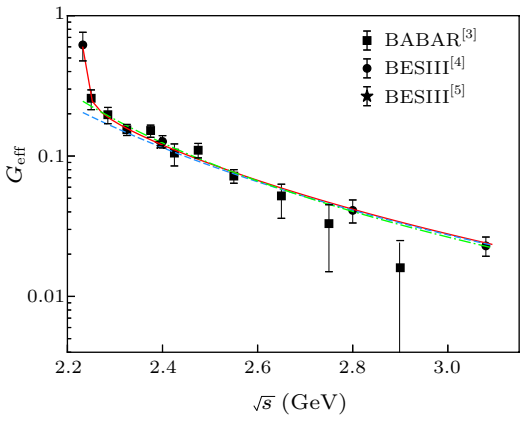

Fig. 1. The effective form factor G_eff of Λ hyperon compared with the experimental data taken from Refs. [3–5]. The red solid curve represents the total contributions from ω, ϕ and $X(2231)$, while the blue dashed curve stands for the results without the contribution from the new $X(2231)$ state. The green-dash-dotted curve stands for the fitted results with the effective form factor as in Eq. (12).

In Fig. 1 we depict the effective form factor G_eff of the Λ hyperon obtained with the fitted parameters given in Table 1 of the seven-parameter fit, as a function of $\sqrt{s}$. The experimental data points are taken from Refs. [3–5]. The red curve is the total contribution, while the blue dashed curve is the contributions from only ω and ϕ, with $\beta_x = 0$. One can see that the experimental data can be well described with the contribution from the new narrow state, especially for the first four data points close to reaction threshold.

Note that the only fifteen available data points about the effective form factor and three data points about the ratio of $R = |G_\text{E}/G_\text{M}|$ do not allow to obtain unique values for the model parameters, we introduce the Dirac and Pauli form factors. Above

$\sqrt{s} = 2.3\,\text{GeV}$, the line shape of G_{eff} is trivial and there are many solutions to describe it. Thus, it is very difficult to get the parameter errors in the fit. In fact, one can also get a good fit to the effective form factor data except the first one with the following parameterized G_{eff}:[52,53]

$$G_{\text{eff}} = C_0 g(q^2) = \frac{C_0}{(1-\gamma q^2)^2}. \tag{12}$$

The fitted parameters are $\gamma = 0.33 \pm 0.03\,\text{GeV}^{-2}$ and $C_0 = 0.10 \pm 0.03$. The fitted G_{eff} is shown in Fig. 1 with the green-dash-dotted curve containing the central values of γ and C_0. One can see that the experimental data can be well reproduced except the first point.

To get more precise information of m_x and Γ_x obtained from the seven-parameter fit, by fixing other parameters with their values as listed in Table 1, within the range of $m_x(1\pm10\%)$ and $(0, 2\Gamma_x)$, we generate random sets of the fitted parameters (m_x, Γ_x) with a Gaussian distribution. For each set of (m_x, Γ_x), we perform a χ^2 fit to the first four data points of the effective form factor. We collect these sets of the fitted parameters, such that the corresponding χ^2 are below $\chi^2_{\text{min}} + 1$, where χ^2_{min} is obtained with these parameters given in Table 1. With these collected best fitted parameters, we obtain the errors of parameters m_x and Γ_x, which are $m_x = 2230.9^{+3.4}_{-3.5}\,\text{MeV}$, and $\Gamma_x = 4.7^{+2.2}_{-4.7}\,\text{MeV}$.

One may think that a Flatté type[54] for Γ_x can improve the fitting situation, since the mass of the new state is very close to the $\Lambda\bar{\Lambda}$ threshold, and it can also couple strongly the $\Lambda\bar{\Lambda}$ channel. The Flatté form is useful for coupled-channel analysis, however, we currently have experimental information about only the $\Lambda\bar{\Lambda}$ channel. Yet, we have explored such a possibility. We take

$$\Gamma_x = \Gamma_0 + \Gamma_{\Lambda\bar{\Lambda}}(s), \tag{13}$$

where Γ_0 is a constant and it includes the contributions from the other channels, while $\Gamma_{\Lambda\bar{\Lambda}}(s)$ is the contribution from the $\Lambda\bar{\Lambda}$ channel. For example, with s-wave coupling[55] for the new state with $J^{\text{PC}} = 1^{--}$ to the $\Lambda\bar{\Lambda}$ channel, one can obtain (in general, there should also be contributions from d-wave):

$$\Gamma_{\Lambda\bar{\Lambda}}(s) = \frac{g^2_{\Lambda\bar{\Lambda}}}{4\pi}\sqrt{\frac{s}{4} - M^2_{\Lambda}}, \tag{14}$$

where $g_{\Lambda\bar{\Lambda}}$ is an unknown s-wave coupling constant.

Then we have performed six-parameter (γ, $\beta_{\omega\phi}$, β_x, m_x, Γ_0 and $g_{\Lambda\bar{\Lambda}}$) χ^2 fits. Indeed, we can also obtain a good fit. The fitted parameters are $\gamma = 0.57 \pm 0.21\,\text{GeV}^{-2}$, $\beta_{\omega\phi} = -0.30 \pm 0.31$, $\beta_x = -0.03 \pm 0.09$, $m_x = 2237.7 \pm 50.2\,\text{MeV}$, $\Gamma_0 = 8.8^{+75.9}_{-8.8}\,\text{MeV}$, and $g_{\Lambda\bar{\Lambda}} = 3.0 \pm 1.9$. One can find that the fitted errors for the model parameters are very large. On the other hand, one can also look for poles for the Breit–Wigner function, $1/(s - m_x^2 + im_x\Gamma_x)$, which is parameterized by the Flatté form on the complex plane of $\sqrt{s}$. With the above fitted central values of m_x, Γ_0 and $g_{\Lambda\bar{\Lambda}}$, we get a pole at $\sqrt{s} = Z_R = M_R - i\Gamma_R/2 = (2096.2, -9.9)\,\text{MeV}$.

As discussed before, only a few experimental data which are very close to and above the reaction threshold need the contribution of the new state. In fact, the Flatté formulae would push down the Breit–Wigner mass, and there will be a clear drop at the $\Lambda\bar{\Lambda}$ threshold if the value of $g_{\Lambda\bar{\Lambda}}$ is large.[56] However, we do not have any information below the $\Lambda\bar{\Lambda}$ mass threshold, which means that the mass and width of the state still cannot be well determined if we choose the Flatté formulae. Indeed, we can get good fits by including $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, or the $\phi(2170)$ with a Flatté form and taking their mass m_x and width Γ_0 as quoted in PDG. On the other hand, it is worth mentioning that, for very wide regions for these values of m_x and Γ_0 one can always get a good fit by adjusting the value of $g_{\Lambda\bar{\Lambda}}$, this is because we do not have information below the $\Lambda\bar{\Lambda}$ mass threshold. In this work, since the $\Lambda\bar{\Lambda}$ channel is opened in the considering energy region, we just use a constant total decay width for this new state, in such a way we can also reduce the number of free parameters. This work constitutes a first step in this direction.

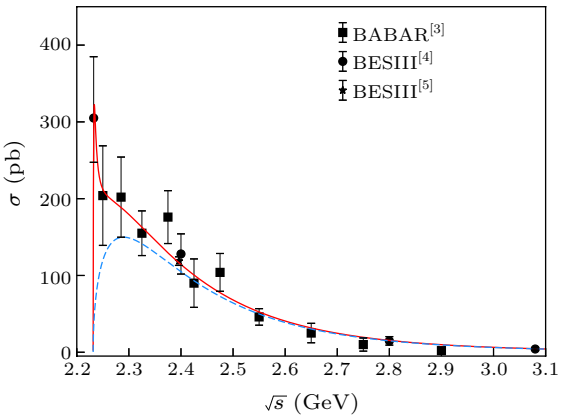

Fig. 2. The total cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ reaction compared with the experimental data measured by BABAR collaboration[3] and BESIII collaboration.[4,5]

Next, we pay attention to the total cross sections of $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. In Fig. 2, the theoretical fitted results of the total cross sections of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction in the energy range from the reaction threshold to $\sqrt{s} = 3.1\,\text{GeV}$ are shown and compared to the experimental data taken from Refs. [4,5]. In this figure, the red solid line displays the theoretical fitted result with total contributions from ω, ϕ, and the new state $X(2231)$, while the blue dotted cure represents the results without the contribution from $X(2231)$ state. Again, one can see that the near threshold enhancement structure is well reproduced thanks to a signif-

icant contribution from a very narrow state $X(2231)$ with mass about 2231 MeV. The narrow peak of this state is clearly seen.

Finally, in Fig. 3 we show the form factor ratio $|G_{\rm E}/G_{\rm M}|$ obtained with the fitted parameters given in Table 1, as a function of $\sqrt{s}$. The experimental data points are taken from Refs. [4,5]. This ratio is determined to be one at the threshold due to the kinematical restriction, which can be easily obtained from Eqs. (2) and (3).

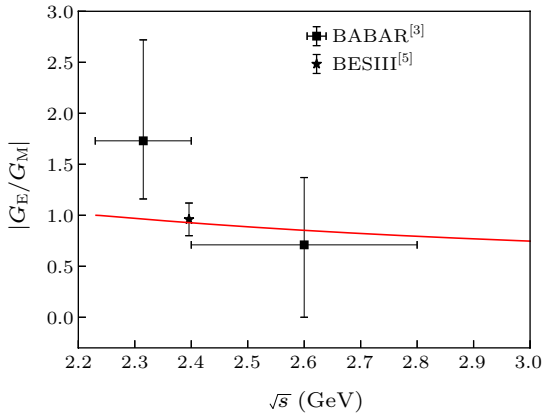

Fig. 3. Electromagnetic form factor ratio $|G_{\rm E}/G_{\rm M}|$ compared with the experimental data taken from Refs. [3,5].

We find that a narrow vector meson, $X(2231)$, whose mass is close to the mass threshold of $\Lambda\bar{\Lambda}$, is needed to describe the threshold enhancement of $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. However, its width cannot be well determined through the VMD model by fitting the current experimental data. This state could be a quasi-bound state of $\Lambda\bar{\Lambda}$, which has significant couplings to the $\Lambda\bar{\Lambda}$ and e^+e^- channels. In fact, from the analysis of the $p\bar{p} \to \Lambda\bar{\Lambda}$ reaction near threshold, the authors of Ref. [57] also predicted a narrow $\Lambda\bar{\Lambda}$ subthreshold state with quantum numbers $J^{\rm PC} = 1^{--}$ and has a width of a few MeV. However, a later high-statistics measurement of the $p\bar{p} \to \Lambda\bar{\Lambda}$ reaction[58] ruled out the existence of such a $\Lambda\bar{\Lambda}$ resonance as predicted in Ref. [57], since there is no structure in these new measurements of the $p\bar{p} \to \Lambda\bar{\Lambda}$ reaction near threshold, and the total cross section is observed to grow smoothly from threshold with a mix of s- and p-wave productions.

The near threshold region of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction was also investigated with specific emphasis on the important role played by the $\Lambda\bar{\Lambda}$ final state interaction in Ref. [9], where the $\Lambda\bar{\Lambda}$ potentials were constructed for the analysis of the $p\bar{p} \to \Lambda\bar{\Lambda}$ reaction. The total cross sections reported by the BABAR collaboration[3] can be well reproduced. However, the new results from the BESIII collaboration[4] are very difficult to be obtained by the theoretical calculations of Ref. [9]. As discussed above, there is no structure in the $p\bar{p} \to \Lambda\bar{\Lambda}$ reaction, and the BESIII results indicate that we do need to include such a narrow state which has significant coupling to the $\Lambda\bar{\Lambda}$ channel (To explain the new BESIII results, a very narrow resonance with mass around the $\Lambda\bar{\Lambda}$ was also discussed in Ref. [9]). Yet, such a narrow state may couple weakly to the $p\bar{p}$ channel, thus it does not appear in the $p\bar{p} \to \Lambda\bar{\Lambda}$ reaction.[58]

Moreover, it was found that in the processes of $e^+e^- \to K^+K^-K^+K^-$ and $e^+e^- \to \phi K^+K^-$, the cross sections are unusually large at $\sqrt{s} = 2.2324$ GeV, which indicates that there should be contributions from a narrow state with mass about 2232.4 MeV.[28] This state could be the vector meson $X(2231)$ that we proposed here. On the other hand, in the charmed sector, the $Y(4630)$ and $Y(4660)$ have been studied in the $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ reaction by taking into account also the $\Lambda_c\bar{\Lambda}_c$ final state interaction.[59–62]

Finally, one knows that $G_{\rm E}$ and $G_{\rm M}$ are complex in the time-like region, and there is a relative phase angle $\Delta\Phi$ between these two electromagnetic form factors. In addition to the ratio of $|G_{\rm E}/G_{\rm M}|$, a rather large phase $\Delta\Phi = 37^\circ \pm 12^\circ \pm 6^\circ$ was also obtained at $\sqrt{s} = 2.396$ GeV by the BESIII collaboration.[5] The fitted width of $X(2231)$ is so narrow that we cannot reproduce this large phase at $\sqrt{s} = 2.396$ GeV. The large phase will be described by considering these vector mesons with higher masses around 2.3–2.4 GeV and wide widths as predicted in Refs. [13,15]. Indeed, a broad vector meson with mass of around 2.34 GeV was introduced to explain the energy dependent behavior of the cross sections of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction above threshold. Clearly, a further improved investigation needs to consider the contributions from these higher mass resonances. It should noted that, including such contributions, the electromagnetic form factors of Λ hyperon would become more complex due to additional parameters from the vector meson dominance model, and we cannot determine or constrain these parameters. In the present work, we focus on the near threshold enhancement of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction. Thus, we leave these contributions to future studies when more precise experimental data become available.

In summary, we have studied the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction near threshold and the electromagnetic form factors of the Λ hyperon within the modified vector meson dominance model. In addition to these contributions from ground ω and ϕ meson, we also introduce a new narrow vector meson $X(2231)$ with mass around the mass threshold of $\Lambda\bar{\Lambda}$, and its width is about a few MeV. It is found that we can describe the effective form factor $G_{\rm eff}$ and the electromagnetic form factor ratio $|G_{\rm E}/G_{\rm M}|$ of Λ hyperon quite well. Especially, the threshold enhancement of the total cross sections of the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction at $\sqrt{s} = 2.2324$ GeV can be well reproduced. This narrow state could be a $\Lambda\bar{\Lambda}$

quasi-bound state with quantum numbers $J^{\rm PC}=1^{--}$. Further data in the very close to threshold region with better mass resolution would be very useful to confirm this narrow resonance.

On the other hand, if one takes a Flatté form for the total decay width of $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, and $\phi(2170)$, the experimental data can also be well reproduced with a strong coupling of these resonances to the $\Lambda\bar{\Lambda}$ channel.

The proposed formalism and conclusion here would give insight into the electromagnetic form factors of the Λ hyperon and the near threshold enhancement of the $e^+e^-\to\Lambda\bar{\Lambda}$ reaction. The proposed formalism attributes the $e^+e^-\to\Lambda\bar{\Lambda}$ non-vanishing cross sections near threshold to the contribution of a new narrow vector meson $X(2231)$, which could be the peak structure seen in the $e^+e^-\to K^+K^-K^+K^-$ and $e^+e^-\to\phi K^+K^-$ reactions at $\sqrt{s}=2.2324\,\text{GeV}$. It is expected that this conclusion can be distinguished and may be tested by the future experiments with improved precision at BESIII or the planned Super tau-charm Facility in China.[63–65]

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