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Generalized Rashba Coupling Approximation to a Resonant Spin Hall Effect of the Spin–Orbit Coupling System in a Magnetic Field

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We introduce a generalized Rashba coupling approximation to analytically solve confined two-dimensional electron systems with both the Rashba and Dresselhaus spin–orbit couplings in an external magnetic field. A solvable Hamiltonian is obtained by performing a simple change of basis, which has the same form as that with only Rashba coupling. Each Landau state becomes a new displaced-Fock state instead of the original Harmonic oscillator Fock state. Analytical energies are consistent with the numerical ones in a wide range of coupling strength even for a strong Zeeman splitting, exhibiting the validity of the analytical approximation. By using the eigenstates, spin polarization correctly displays a jump at the energy-level crossing point, where the corresponding spin conductance exhibits a pronounced resonant peak. As the component of the Dresselhaus coupling increases, the resonant point shifts to a smaller value of the magnetic field. In contrast to pure Rashba couplings, we find that the Dresselhaus coupling and Zeeman splittings tend to suppress the resonant spin Hall effect. Our method provides an easy-to-implement analytical treatment to two-dimensional electron gas systems with both types of spin–orbit couplings by applying a magnetic field.

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Spin–orbit coupling (SOC) enables a wide variety of fascinating phenomena, which brings out a new growing research field of spin-orbitronics.[1,2] A prominent example is the spin Hall effect, which is associated with the SOC especially in semiconductors.[3−5] The spin Hall effect is the transverse pure spin current response to an electric field, with the spin current polarization perpendicular to both the spin current and the electric field.[6−13] In the presence of a perpendicular magnetic field, a resonance spin Hall effect has attracted a lot of discussion due to the interplay of the Zeeman splitting and various spin–orbit interactions,[14−16] which may have potential applications in spintronics.

There are basically two types of SOCs in nature, i.e., the Rashba term with structural inversion asymmetry[17] and the Dresselhaus term due to the bulk inversion asymmetry[18] or interface inversion asymmetry.[19] Usually, both types of SOCs coexist in a material, such as GaAs-AlGaAs quantum wells and heterostructures,[20−25] but which one plays a major part depends on properties of the material. It has been recognized that Rashba and Dresselhaus SOC can interfere with each other, and leads to a number of interesting phenomenon by tuning the ratio between them, such as anisotropic transport,[26] spin splitting,[27,28] control of spin precession,[29] and light scattering.[30]

In the presence of a magnetic field, many efforts have been devoted to the resonance spin Hall effect in two-dimensional electron gas systems (2DEGs) with the Rashba SOC, which is caused by the Landau level crossing near the Fermi energy. A zero-field spin splitting induced by the Rashba SOC competes with the Zeeman splitting in a magnetic field, and then compromises a resonant spin Hall effect at certain magnetic field.[31,32] In contrast, Dresselhaus SOC enhances the Zeeman splitting and results in a suppression to the resonance.[33] There is an analytical solution for the system with only either Rashba or Dresselhaus coupling.[33,34] While considering both of them together, an analytical solution is currently not available, since the Hilbert space is not closed with both SOCs. Despite the fact that an exact solution has been given in terms of an infinite number of Landau levels,[35] numerical calculations are complicated. Analytical approximations and perturbation treatments[33−37] have been developed, which are valid for weak Zeeman splittings or for weak Dresselhaus SOC strengths. An easy-to-use analytical solution is currently missing for the system with both of the Rashba and the Dresselhaus SOC. An analytical solution in a closed form is actually highly non-trivial to explore the resonance spin Hall effect with two types of SOCs.

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In this work we develop a generalized Rashba coupling approximation (GRCA) to give an analytical solution to the 2DEGs with both of the Rashba and Dresselhaus SOCs in a magnetic field. The reformulated Hamiltonian including both types of SOCs retains the mathematical simplicity of the pure Rashba term. This easily implemented approach gives analytical expressions for eigenvalues and closed-form eigenstates in the transformed displaced-Fock subspace. Energy levels and the spin polarization obtained analytically are consistent with numerical ones in a wide range of the coupling parameters. By comparing to the system with pure Rashba SOC, we find that the spin Hall conductance exhibits a pronounced resonant peak at a larger value of the inverse of the magnetic field. The critical condition of the resonant spin Hall effect induced by the energy-level crossing is given analytically. Our analytical approach is applicable to capture the resonant spin Hall effect even for a strong Zeeman splitting.

In this Letter, first we derive expressions for the quantized Hamiltonian of the 2DEGs with the Rashba and Dresselhaus SOC in a magnetic field. Then, we obtain the analytical solution of the effective Hamiltonian for arbitrary ratio between the Rashba and Dresselhaus SOCs. Furthermore, we study the charge and spin Hall conductances analytically with the first-order corrections when an electric field is applied. Finally, a brief summary is given.

Hamiltonian. We consider a 2DEGs confined in the x–y plane subjected to a perpendicular magnetic field $\boldsymbol{B} = -B\hat{e}_z$. The Hamiltonian of a single electron with the Rashba and Dresselhaus SOC is given by ($\hbar = 1$)

$$H_0 = \frac{1}{2m}\Big(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\Big)^2 - \frac{1}{2}g\mu_{\rm B}B\sigma_z + H_{\rm R} + H_{\rm D}, \quad (1)$$

where g is the Landé factor of the electron with the effective mass m, $\mu_{\rm B}$ is the Bohr magneton, and σ_k are the Pauli matrices. The Landau gauge is chosen as $\boldsymbol{A} = B\boldsymbol{r}\times\hat{e}_z = (yB, 0, 0)$, giving the canonical momentum $\boldsymbol{\Pi} = \boldsymbol{p} + e\boldsymbol{A}/c$. The Rashba SOC $H_{\rm R}$ originates from the structure inversion asymmetry of the semiconductor material, $H_{\rm R} = \alpha(\Pi_x\sigma_y - \Pi_y\sigma_x)$, and the coupling strength α can be tuned by an electric field. $H_{\rm D} = \beta(\Pi_x\sigma_x - \Pi_y\sigma_y)$ are the linear Dresselhaus SOC, and the coupling strength β is determined by the geometry of the hetero-structure that stems from the bulk-inversion asymmetry of the semiconductor material.

Due to the gauge choice, the system is translational invariant along the x direction, and $p_x = k$ is a good quantum number. The orbit part of the wave function is obtained as $\psi(x, y) = \exp(ikx)\varphi(y - y_0)$, where $\varphi(y - y_0)$ is the harmonic oscillator wave function with the orbit center coordinate $y_0 = l_{\rm b}^2 k$ and the magnetic length $l_{\rm b} = \sqrt{c/eB}$. By introducing the ladder operator $a = (\Pi_x + i\Pi_y)l_{\rm b}/\sqrt{2}$ for the harmonic oscillator

$$a = \frac{1}{\sqrt{2}l_{\rm b}}\Big[y + \frac{c(p_x + ip_y)}{eB}\Big], \quad (2)$$

one obtains the Hamiltonian

$$H_0 = H_{\rm RSOC} + \frac{\sqrt{2}\beta}{l_{\rm b}}(a^\dagger\sigma_- + a\sigma_+), \quad (3)$$

$$H_{\rm RSOC} = \omega\Big(a^\dagger a + \frac{1}{2}\Big) - \frac{\Delta}{2}\sigma_z + i\frac{\sqrt{2}\alpha}{l_{\rm b}}(a\sigma_- - a^\dagger\sigma_+), \quad (4)$$

where $\omega = eB/mc$ is the cyclotron frequency and $\Delta = g\mu_{\rm B}B$ is the Zeeman splitting energy. It can be mapped to the quantum Rabi model in the quantum optics in which rotating-wave and counter-rotating-wave terms have different coupling strengths.[38–41] The quantum Rabi model also lacks a full closed-form solution, since the Hilbert space is not closed. Much effort has been devoted to developing analytical solutions, including an exact solution with complicated transcendental functions,[42,43] a generalized rotating-wave approximation,[44–46] and adiabatic approximation,[47,48] which are valid in a certain coupling regime.

When only the Rashba SOC term is present, i.e., $\beta = 0$, this Hamiltonian $H_{\rm RSOC}$ can be solved analytically in a closed subspace, which is the so-called Rashba SOC (RSOC) approximation. With the basis $\{|\downarrow, n\rangle, |\uparrow, n+1\rangle\}$, one obtains $H_{\rm RSOC}$ in a matrix form as

$$H_{\rm RSOC} = \begin{pmatrix} \omega(n+1/2) + \Delta/2 & i\sqrt{2(n+1)}\alpha/l_{\rm b} \\ -i\sqrt{2(n+1)}\alpha/l_{\rm b} & \omega(n+1+1/2) - \Delta/2 \end{pmatrix}, \quad (5)$$

which gives closed-form eigenvectors consisting of $|\downarrow, n\rangle$ and $|\uparrow, n+1\rangle$. However, once including the additional Dresselhaus SOC term (rotating-wave term in quantum Rabi model), the subspace related to n is not closed, rendering the complication of the solution. In that case, each Landau level is coupled to an infinite number of other Landau levels, and thus the exact analytic solution is not available. Therefore, it is non-trivial to develop an efficient analytical solution in a closed form for the system with both of the Rashba and Dresselhaus SOCs.

Analytical Solution. Inspired by the closed-form solution of the RSOC, it is interesting to explore a solvable Hamiltonian with the same form to $H_{\rm RSOC}$ by including both types of SOCs. The crucial thing is to establish a new set of basis states.

To facilitate the study, we write the Hamiltonian as

$$\begin{aligned} H_0 =& \omega\Big(a^\dagger a + \frac{1}{2}\Big) - \frac{\Delta}{2}\sigma_z + g_1\sigma_x(a^\dagger e^{-i\theta} + ae^{i\theta}) \\ & - g_1 i\sigma_y(a^\dagger e^{i\theta} - ae^{-i\theta}), \end{aligned} \quad (6)$$

with $g_1 = \sqrt{2}\sqrt{\beta^2+\alpha^2}/l_{\rm b}$, and $e^{i\theta} = (\beta + i\alpha)/\sqrt{\beta^2+\alpha^2}$. By performing a unitary transformation $U = \exp[\sigma_x(a^\dagger\gamma - a\gamma^*)]$ with a dimensionless variational displacement γ (γ^*), we obtain a transformed Hamiltonian $H_1 = UHU^\dagger = H_0' + H_1'$,

$$\begin{aligned} H_0' =& \omega a^\dagger a + \eta_0 + \sigma_z\{\eta_1 \cosh[2(a^\dagger\gamma - a\gamma^*)] \\ &+ g_1 \sinh[2(a^\dagger\gamma - a\gamma^*)](a^\dagger e^{i\theta} - ae^{-i\theta})\}, \quad (7) \\ H_1' =& \sigma_x[a^\dagger(g_1 e^{-i\theta} - \omega\gamma) + a(g_1 e^{i\theta} - \omega\gamma^*)] \\ &+ i\sigma_y\{-\eta_1 \sinh[2(a^\dagger\gamma - a\gamma^*)] - g_1 \cosh[2(a^\dagger\gamma \\ &- a\gamma^*)](a^\dagger e^{i\theta} - ae^{-i\theta})\}, \quad (8) \end{aligned}$$

where $\eta_0 = \omega/2 - g_1(\gamma^* e^{-i\theta} + \gamma e^{i\theta}) + \omega\gamma\gamma^*$ and $\eta_1 = -\Delta/2 - g_1(\gamma e^{-i\theta} - \gamma^* e^{i\theta})$ given after the conclusion paragraph. The displacement shift γ (γ^*) is associated with the Rashba SOC and Dresselhaus SOC strengths, which captures the displacement of the harmonic oscillator states for essential physics.

Since the even hyperbolic cosine function can be expanded as $\cosh[2(a^\dagger\gamma - a\gamma^*)] = 1 + \frac{1}{2!}[2(a^\dagger\gamma - a\gamma^*)]^2 + \frac{1}{4!}[2(a^\dagger\gamma - a\gamma^*)]^4 + \cdots$, it is approximated by keeping the terms that only contain the number operator $\hat{n} = a^\dagger a$ as[40]

$$\cosh[2(a^\dagger\gamma - a\gamma^*)] \approx G(a^\dagger a) + O(\gamma^2\gamma^{*2}). \quad (9)$$

The coefficient $G(a^\dagger a)$ can be expressed in the harmonic oscillator basis $|n\rangle$ as

$$G_{n,n} = \langle n|\cosh[2(a^\dagger\gamma - a\gamma^*)]|n\rangle = e^{-2\gamma\gamma^*} L_n(4\gamma\gamma^*), \quad (10)$$

with the Laguerre polynomials $L_n^{m-n}(x) = \sum_{i=0}^{\min\{m,n\}}(-1)^{n-i}\frac{m!x^{n-i}}{(m-i)!(n-i)!i!}$. Here, the higher order excitations such as $a^{\dagger 2}$, a^2, $\cdots$ are neglected in the approximation. Similarly, we expand the odd function $\sinh[2(a^\dagger\gamma - a\gamma^*)]$ by keeping the one-excitation terms as

$$\sinh[2(a^\dagger\gamma - a\gamma^*)] \approx R(a^\dagger a)a^\dagger - aR(a^\dagger a) + O(\gamma^3\gamma^{*3}). \quad (11)$$

Since the terms $R(a^\dagger a)a^\dagger$ and $aR(a^\dagger a)$ are conjugated to each other, which correspond to create and eliminate a single excitation of the oscillator. It yields

$$\begin{aligned} R_{n,n+1} =& -\frac{1}{\sqrt{n+1}}\langle n|\sinh[2(a^\dagger\gamma - a\gamma^*)]|n+1\rangle \\ =& \frac{2\gamma^*}{n+1} e^{-2\gamma\gamma^*} L_n^1(4\gamma\gamma^*) = R_{n+1,n}^*. \quad (12) \end{aligned}$$

Similarly, the other operators can be expanded by keeping leading terms as follows:

$$\sinh[2(a^\dagger\gamma - a\gamma^*)]B \approx F(a^\dagger a) + O(\gamma^2\gamma^{*2}), \quad (13)$$

$$\cosh[2(a^\dagger\gamma - a\gamma^*)]B \approx T(a^\dagger a)a^\dagger - aT(a^\dagger a), \quad (14)$$

with the operator $B = a^\dagger e^{i\theta} - ae^{-i\theta}$. The corresponding coefficients can be expressed in terms of the oscillator basis $|n\rangle$

$$\begin{aligned} F_{n,n} =& \langle n|\sinh[2(a^\dagger\gamma - a\gamma^*)]B|n\rangle \\ =& -e^{i\theta}(n+1)R_{n,n+1} - ne^{-i\theta}R_{n,n-1}, \quad (15) \\ T_{n,n+1} =& \frac{-\langle n|\cosh[2(a^\dagger\gamma - a\gamma^*)]B|n+1\rangle}{\sqrt{n+1}} \\ =& e^{-i\theta}G_{n,n} - \frac{\sqrt{n+2}}{\sqrt{n+1}}e^{i\theta}G_{n,n+2}, \quad (16) \end{aligned}$$

with $G_{n,n+2} = \langle n|\cosh[2(a^\dagger\gamma - a\gamma^*)]|n+2\rangle = (2\gamma^*)^2\exp[-2\gamma\gamma^*]L_n^2(4\gamma\gamma^*)/\sqrt{(n+1)(n+2)}$.

Finally, we obtain the reformulated Hamiltonian $\tilde{H}_1 = \tilde{H}_{\rm GRCA} + \tilde{H}_{\rm D}$, consisting of

$$\begin{aligned} \tilde{H}_{\rm GRCA} =& \omega a^\dagger a + \eta_0 + \sigma_z\widetilde{\Delta} + \widetilde{\alpha}a^\dagger\sigma_+ + \widetilde{\alpha}^* a\sigma_-, \quad (17) \\ \tilde{H}_{\rm D} =& \widetilde{\beta}a^\dagger\sigma_- + \widetilde{\beta}^* a\sigma_+, \quad (18) \end{aligned}$$

where the effective Zeeman energy is renormalized as $\widetilde{\Delta} = \eta_0 + \eta_1 G(a^\dagger a) + g_1 F(a^\dagger a)$. The effective Rashba and Dresselhaus SOCs strength are derived as $\widetilde{\alpha} = \{g_1 e^{-i\theta} - \omega\gamma - \eta_1 R(a^\dagger a) - g_1 T(a^\dagger a)\}$ and $\widetilde{\beta} = \{g_1 e^{-i\theta} - \omega\gamma + \eta_1 R(a^\dagger a) + g_1 T(a^\dagger a)\}$.

Interestingly, the transformed Dresselhaus terms $\tilde{H}_{\rm D}$ in Eq. (18) can be adjusted to vanish by varying the displacement γ. It is reasonable to make the matrix element $\langle n, +z|\tilde{H}_{\rm D}|n+1, -z\rangle$ be 0 dependent on γ. Consequently, the variational parameter γ is determined by solving

$$0 = g_1 e^{i\theta} - \omega\gamma^* + \eta_1 R_{n,n+1} + g_1 T_{n,n+1}. \quad (19)$$

Especially for the case of weak SOC interactions, γ (γ^*) is smaller compared with the unit and it approximately leads to $L_n(4\gamma\gamma^*) \simeq 1$, $L_n^1(4\gamma\gamma^*) \simeq n+1$, and $L_n^2(4\gamma\gamma^*) \simeq (n+1)(n+2)/2$. The above equation can be simplified to $g_1 e^{i\theta} - \omega\gamma^* + \gamma^*\Delta + g_1 e^{-i\theta} = 0$, which results in the displacement γ,

$$\gamma \approx \frac{2g_1\beta}{(\omega+\Delta)\sqrt{\alpha^2+\beta^2}}. \quad (20)$$

Therefore, one obtains the solvable Hamiltonian $\tilde{H}_{\rm GRCA}$ retaining the mathematical structure of the Hamiltonian $H_{\rm RSOC}$ in Eq. (5) only with Rashba SOC terms $a\sigma_-$ and $a^\dagger\sigma_+$. It is the so-called GRCA. The transformed Hamiltonian contains both of the Rashba and Dresselhaus SOCs, which plays an important role in the spin Hall effects. The simplicity of the method is based on its analytical eigenstates and eigenvalues.

One can easily diagonalize the effective Hamiltonian $\tilde{H}_{\rm GRCA}$ on the bases of $|n, -z\rangle$ and $|n+1, +z\rangle$,

$$\tilde{H}_{\rm GRCA} = \begin{pmatrix} \omega n + \widetilde{\Delta}_{-,n} & \sqrt{n+1}\widetilde{\alpha}_{n,n+1} \\ \sqrt{n+1}\widetilde{\alpha}_{n,n+1}^* & \omega(n+1) + \widetilde{\Delta}_{+,n+1} \end{pmatrix}. \quad (21)$$

Similar to the Hamiltonian $H_{\rm RSOC}$ in Eq. (5) with only the Rashba SOC, the eigenvalues are obtained as

$$E_{n,\pm} = \omega\Big(n+\frac{1}{2}\Big) + \frac{1}{2}[\widetilde{\Delta}_{+,n+1} + \widetilde{\Delta}_{-,n}] \pm \frac{1}{2}\sqrt{[\widetilde{\Delta}_{+,n+1} - \widetilde{\Delta}_{-,n} + \omega]^2 + 4(n+1)|\widetilde{\alpha}_{n,n+1}|^2}\,. \tag{22}$$

The corresponding eigenstates are expressed in the closed form as

$$|\varphi_{+,n}\rangle = \cos\frac{\theta_n}{2}|n+1\rangle|+z\rangle + \sin\frac{\theta_n}{2}|n\rangle|-z\rangle, \tag{23}$$

$$|\varphi_{-,n}\rangle = \sin\frac{\theta_n}{2}|n+1\rangle|+z\rangle - \cos\frac{\theta_n}{2}|n\rangle|-z\rangle, \tag{24}$$

where $\theta_n = \arccos[\delta_n/\sqrt{\delta_n^2 + 4(n+1)|\widetilde{\alpha}_{n,n+1}|^2}]$ with $\delta_n = \omega + \widetilde{\Delta}_{+,n+1} - \widetilde{\Delta}_{-,n}$.

The ground state is $|0,+z\rangle$ with the eigenvalue

$$E_0 = \eta_0 + (\eta_1 - 2\gamma^* g_1 e^{i\theta})e^{-2\gamma\gamma^*}. \tag{25}$$

As a consequence, the corresponding wave functions of the original full Hamiltonian H_0 in Eq. (6) can be obtained using the unitary transformation as $|\Psi_{\pm,n}\rangle = U^\dagger|\varphi_{\pm,n}\rangle$,

$$|\Psi_{+,n}\rangle = \frac{1}{\sqrt{2}}\Big[\Big(\cos\frac{\theta_n}{2}|-\gamma,n+1\rangle_d + \sin\frac{\theta_n}{2}|-\gamma,n\rangle_d\Big)|+\rangle_x + \Big(\cos\frac{\theta_n}{2}|\gamma,n+1\rangle_d - \sin\frac{\theta_n}{2}|\gamma,n\rangle_d\Big)|-\rangle_x\Big], \tag{26}$$

$$|\Psi_{-,n}\rangle = \frac{1}{\sqrt{2}}\Big[\Big(\sin\frac{\theta_n}{2}|-\gamma,n+1\rangle_d - \cos\frac{\theta_n}{2}|-\gamma,n\rangle_d\Big)|+\rangle_x + \Big(\sin\frac{\theta_n}{2}|\gamma,n+1\rangle_d + \cos\frac{\theta_n}{2}|\gamma,n\rangle_d\Big)|-\rangle_x\Big], \tag{27}$$

where $|\pm\rangle_x = (|+\rangle_z \pm |-\rangle_z)/\sqrt{2}$ is the eigenstate of σ_x. Each Landau state becomes the displaced-Fock state $|n\rangle$

$$|\mp\gamma,n\rangle_d = e^{\mp(\gamma a^\dagger - a\gamma^*)}|n\rangle, \tag{28}$$

which is the displacement transformation of the Fock state $|n\rangle$. Especially it reduces to the coherent state $|\mp\gamma,0\rangle_d = e^{\mp(\gamma a^\dagger - a\gamma^*)}|0\rangle$, which can be expanded as a superposition state of Fock states.

Figure 1 displays eight low-lying energy levels as a function of the effective coupling strength $\eta_{\rm R}/\omega = \sqrt{2}\alpha/(l_b\omega)$ for various values of the Dresselhaus coupling strength β. In the absence of the spin–orbit coupling $\eta_R = 0$, one observes two separated nth Landau levels induced by the Zeeman energy $\Delta = 0.5\omega$, in which the lower level is the spin-up state and the higher level corresponds to the spin-down electron state. As η_R increases, the higher level of the nth Landau level state becomes lower due to the hybridization of the nth and $(n+1)$th displaced-Fock states induced by both types of SOCs. Comparing with the Rashba SOC approximation, the energy crossing occurs at a larger value of the coupling strength as a consequence of the Dresselhaus SOC. It demonstrates that the Dresselhaus SOC enhances Zeeman splitting, while the Rashba SOC interplay with the Zeeman splitting in opposite ways.

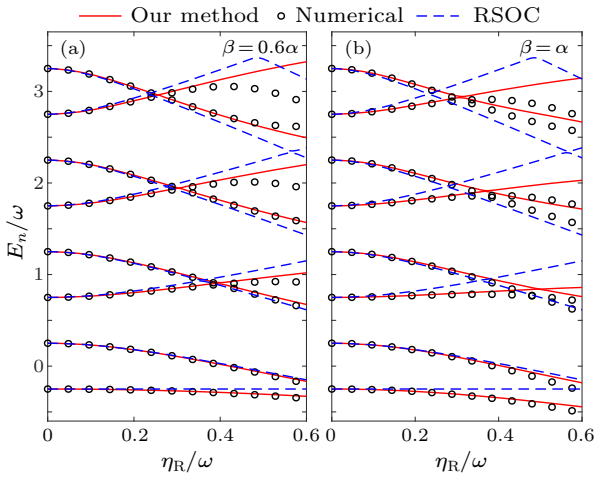

Fig. 1. Energy levels E_n/ω obtained in our method (red solid line) as a function of effective coupling strength $\eta_{\rm R}/\omega = \sqrt{2}\alpha/(l_{\rm b}\omega)$ for different ratios between the Dresselhaus and Rashba SOCs strength (a) $\beta/\alpha = 0.6$ and (b) $\beta/\alpha = 1$. The results obtained by the numerical exact diagonalization method (black circles) and under the RSOC approximation (blue dashed line) are listed for comparison. The parameters are $\Delta/\omega = 0.5$, $l_{\rm b} = 1$ and $\omega = 1$.

For the ratio between the Dresselhaus and Rashba SOC strengths $\beta/\alpha = 0.6$, our analytical approach is in good agreement with the numerical results for the coupling strength $\eta_R/\omega < 0.4$ for a strong Zeeman splitting $\Delta/\omega = 0.5$ in Fig. 1(a). Eigenvalues $E_{n,\pm}$ in Eq. (22) correctly describe the energy crossing, which is crucial to study the divergence or singularities of physical observables. There is noticeable deviation of high-level energies, since higher-order terms such as $a^2(a^{\dagger 2})$ in the transformed Hamiltonian are neglected. The RSOC approximation is valid in the weak coupling regime $\eta_R/\omega < 0.3$. When the ratio increases to $\beta/\alpha = 1$ in Fig. 1(b), the GRCA works well in a wide range of coupling strength, while the Rashba SOC approximation fails to capture the energy crossing point, which is crucial to the resonant spin Hall effect in the following.

Spin Current in an Electric Field. Since the competition of the SOCs and the Zeeman splitting induces an energy crossing, a resonant spin Hall effect is closely related to the level crossing. For the 2DEGs with N_e electrons, the spin Hall effect is related to a transverse spin current response to an external electric field. To

verify the validity of the eigen-states, we study the interesting phenomenon of the spin current and the corresponding spin Hall conductance.

As the electric field E is applied along the y axis, the Hamiltonian becomes $H = H_0 + eEy$ with the original full Hamiltonian H_0 defined in Eq. (1). Using the replacement of y by $y + eE/m\omega^2$ in the oscillator operator a, one obtains the quantized Hamiltonian

$$H = H_0 + H_E, H_E = -E\Big[\frac{ke}{m\omega} + \frac{e}{\omega}(\alpha\sigma_y + \beta\sigma_x)\Big], \quad (29)$$

where the constant $-e^2E^2/2m\omega^2$ is dropped. Using the same unitary transformation $U = \exp[\sigma_x(a^\dagger\gamma - a\gamma^*)]$, one obtains

$$\begin{aligned}\tilde{H}_E =& UH_EU^\dagger \\ =& -E\frac{ke}{m\omega} - E\frac{\beta e}{\omega}\sigma_x - E\frac{\alpha e}{\omega}\{\sigma_y G(a^\dagger a) \\ &+ i\sigma_z[R(a^\dagger a)a^\dagger - aR(a^\dagger a)]\}. \end{aligned} \quad (30)$$

The wave function for the Hamiltonian with the electric field can be given to the first-order correction in the perturbation in $\tilde{H}_E$ as

$$|\varphi^{(1)}_{\pm,n}\rangle = |\varphi_{\pm,n}\rangle + \sum_{n\neq k,l}\frac{\langle\varphi_{l,k}|\tilde{H}_E|\varphi_{\pm,n}\rangle}{E_{n,\pm} - E_{l,k}}|\varphi_{l,k}\rangle, (l = \pm), \quad (31)$$

where the eigenvalues $E_{n,\pm}$ and eigenstates $|\varphi_{\pm,n}\rangle$ are given in Eqs. (22)–(24).

The charge current operator of a single electron is given by

$$j_{\rm c} = -ev_x, \quad (32)$$

$$v_x = \frac{1}{i}[x, H] = \frac{p_x}{m} + \omega y + \alpha\sigma_y + \beta\sigma_x, \quad (33)$$

and the spin-z component current operator is

$$\begin{aligned} j^z_{\rm s} =& \frac{\hbar}{2}(S^z v_x + v_x S^z) \\ =& \frac{1}{2}\Big[\sqrt{\frac{\omega}{2\,m}}(a^\dagger + a) - \frac{eE}{m\omega}\Big]\sigma_z. \end{aligned} \quad (34)$$

For the system with N_e electrons, the average current density is given by

$$I_{\rm c(s)} = \frac{1}{L_xL_y}\sum_{nl}\langle j_{\rm c(s)}\rangle_{nl} f(E_{nl}), (l = \pm 1), \quad (35)$$

where $f(E_{nl})$ is the Fermi distribution function, and $N_e = \sum_{nl} f(E_{nl})$, and L_xL_y is the area of x–y plane. The charge or spin Hall conductance is

$$G_{\rm c(s)} = I_{\rm c(s)}/E. \quad (36)$$

Under the first-order perturbation, the corresponding spin/charge current can be expressed as $\langle j_{\rm c(s)}\rangle_{\pm,n} = \langle j^{(0)}_{\rm c(s)}\rangle_{\pm n} + \langle j^{(1)}_{\rm c(s)}\rangle_{\pm n}$, where

$$\langle j^{(0)}_{\rm c(s)}\rangle_{\pm n} = \langle\varphi_{\pm,n}|Uj_{\rm c(s)}U^\dagger|\varphi_{\pm,n}\rangle, \quad (37)$$

$$\langle j^{(1)}_{\rm c(s)}\rangle_{\pm n} = \sum_{n\neq k,l}\frac{\langle\varphi_{\pm,n}|\tilde{H}_E|\varphi_{l,k}\rangle\langle\varphi_{l,k}|Uj_{\rm c(s)}U^\dagger|\varphi_{\pm,n}\rangle}{E_{k,l} - E_{n,\pm}} + {\rm H.c.} \quad (38)$$

Under the zeroth approximation, one obtains analytical solutions

$$\langle j^{z(0)}_{\rm s}\rangle_{\pm,n} = -\frac{eE}{2m\omega}\langle\sigma_z\rangle_{\pm n}, \quad (39)$$

where $\langle\sigma_z\rangle_{\pm n}$ is given after the conclusion paragraph.

With the average current density $I^z_{\rm s}$, the spin Hall conductance can be derived from Eq. (34) under the zeroth-order correction

$$G^{z(0)}_{\rm s} = -\frac{\langle S_z\rangle}{L_xL_y}\frac{e}{m\omega} = -\frac{\langle S_z\rangle G_{\rm c}}{e}, \quad (40)$$

where the expected value of spin polarization is

$$\langle S_z\rangle = \sum_{nl}\frac{1}{2}\langle\sigma_z\rangle_{nl} f(E_{n,l}), (l = \pm). \quad (41)$$

The Hall conductance can be derived from the charge current given after the conclusion paragraph,

$$G_{\rm c} = e^2N_e/(2\pi N_\phi), \quad (42)$$

which is a constant dependent on the filling factor N_e/N_ϕ with the degeneracy $N_\phi = L_xL_y/(2\pi l_b)$.[14]

Figure 2 shows the energy levels $E_{n,\pm}$ obtained from Eq. (22) and the spin polarization $\langle S_z\rangle$ under the zeroth approximation. It is observed that the energy $E_{n,+}$ firstly enters into the Fermi energy region, then it gives rise to the energy $E_{n+1,-}$ in Figs. 2(a) and 2(c). As the energy gap between $E_{n,+}$ and $E_{n+1,-}$ becomes smaller, it yields energy crossing at certain magnetic field B_0, which is determined by $E_{n+1,-} = E_{n,+}$ given after the conclusion paragraph. When the magnetic field exceeds the energy-crossing point, $E_{n,-}$ emerges firstly, and then $E_{n+1,+}$ enters into the Fermi energy region. Especially, for the pure Rashba SOC ($\beta = 0$), the energy crossing point for the critical magnetic field B_0 is given explicitly,

$$\begin{aligned} 2\omega =& \sqrt{(\omega - \Delta)^2 + 4(n+1)\eta_R^2} \\ &+ \sqrt{(\omega - \Delta)^2 + 4(n+2)\eta_R^2}, \end{aligned} \quad (43)$$

which recovers the previous results.[14]

To explore the spin-z component current, Figs. 2(b) and 2(d) show the corresponding spin polarization $\langle S_z\rangle$. It reaches maxima at odd integers $n = \ldots, 9, 11, 13$, and minima at even integers n. A discontinuous jump occurs at the critical value B_0 induced by the energy crossing. Below the field B_0, the maximal value of $\langle S_z\rangle$ occurs at even integers $n = 18$, 20. The jump of the spin polarization ascribes to the

energy crossing of two eigenstates with almost opposite spins. By comparing the results with only Rashba SOC, the jump of the spin polarization for $\beta/\alpha = 0.5$ occurs at a larger value of $1/B_0$ in Fig. 2(d).

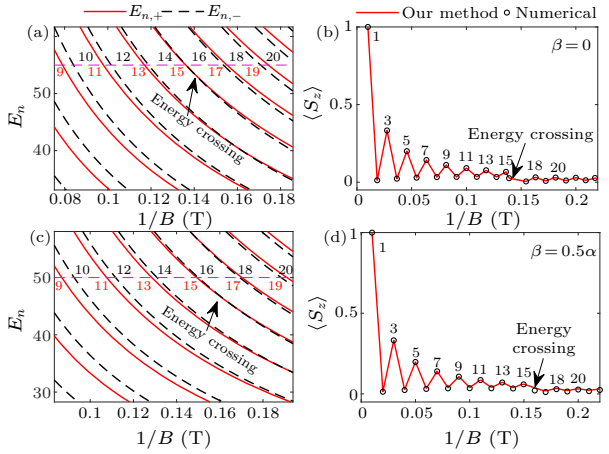

Fig. 2. Energy levels E_n and spin polarization for the spin current $\langle S_z \rangle$ obtained analytically for an electron as a function of $1/B$ for $\beta = 0$ [(a), (b)] and $\beta/\alpha = 0.5$ [(c), (d)] with $\Delta/\omega = 0.5$ and $\alpha = 0.25\omega$. The results of $\langle \sigma_z \rangle$ obtained by the numerical exact diagonalization (black circles) are listed for comparison.

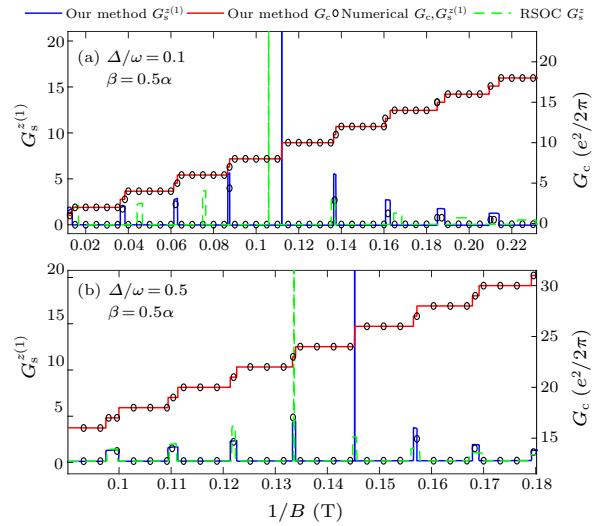

Fig. 3. Spin Hall conductance $G_s^{z(1)}$ (blue solid lines) and charge conductance G_c (red solid line) obtained with first-order corrections as a function of $1/B$ for different Zeeman splitting energy (a) $\Delta/\omega = 0.1$ and (b) $\Delta/\omega = 0.5$. The ratio between the Dresselhaus and Rashba SOCs is $\beta/\alpha = 0.5$ with the Rashba SOC strength $\alpha = 0.25\omega$. $G_s^{z(1)}$ and G_c obtained by the numerical exact diagonalization (black circles) and G_s^z under the RSOC approximation (green dashed lines) are listed for comparison. The external electronic field is $E/\omega = 0.1\, N/C$.

In the presence of the electric field, we calculate the spin Hall conductance $G_s^{z(1)}$ using the first-order corrections as well as the charge Hall conductance G_c. When an even number of Landau levels $E_{n,\pm}$ are occupied in Fig. 2(c), $G_s^{z(1)}$ exhibits some peaks, and the corresponding charge conductance G_c increases with a narrow plateau in Fig. 3. When an odd number of Landau levels are occupied, $G_s^{z(1)}$ tends to be zero, and the plateaus of G_c becomes wider. Interestingly, $G_s^{z(1)}$ becomes divergent with a resonant peak at the energy-crossing point, where the narrow plateau of G_c vanishes. The resonant spin effect ascribes the energy degenerate of the spin current $\langle j_s^{(1)} \rangle$ in Eq. (38) at the critical point. The resonance point coincides with the jump point of $\langle S_z \rangle$ in Fig. 2(d). Compared to the results with only the Rashba SOC, the spin Hall conductance $G_s^{z(1)}$ exhibits a shift value of $1/B_0$, which is induced by the Dresselhaus SOC effects. For a large Zeeman splitting energy $\Delta/\omega = 0.5$, the resonant point of $1/B_0$ shifts to a larger value in Fig. 3(b). It demonstrates that the Dresselhaus coupling and Zeeman splittings play a role in suppressing the resonant spin Hall effect. Fortunately, the charge and spin conductance obtained by our approach agree well with the numerical results, exhibiting the validity of our approach.

Conclusion. When both the Rashba and Dresselhaus spin-orbit couplings are considered, we develop the generalized Rashba coupling approximation to give an analytical solution in closed form. In the transformed displaced-Fock space, a solvable Hamiltonian containing two types of couplings is obtained in a form similar to that with only the Rashba term. Eigen-energies obtained analytically are valid in a wide range of coupling strengths, which correctly captures the energy-levels crossing even for a strong Zeeman splitting energy. With the analytical closed-form eigenstates, spin polarization displays a jump at the crossing point and the corresponding spin conductance becomes divergent with a resonant peak, which are consistent with numerical ones. By increasing the ratio of the Dresselhaus coupling, the resonant point shifts to a smaller value of the magnetic field, which reveals that the Dresselhaus SOC tends to suppress the resonant spin Hall effect. Therefore, our method captures the nature physics of the displacement influences on the wave function induced by the two types of SOC interactions. The easy-to-implement analytical solution facilitates us to analyze the observables such as spin current and spin Hall conductance in 2DEGs, which will have a potential application in future studies of spin-orbitronics and interacting fractional quantum Hall systems.

Deviation of the Hamiltonian by the Displacement Transformation. We perform a unitary transformation $U = \exp[\sigma_x(a^\dagger\gamma - a\gamma^*)]$ to the Hamiltonian H_0 in Eq. (6). One easily obtains $UaU^\dagger = a - \gamma\sigma_x$ and $Ua^\dagger U^\dagger = a^\dagger - \gamma^*\sigma_x$. The first and second terms of H_0 in Eq. (6) can be transformed into

$$Ua^\dagger aU^\dagger = a^\dagger a - \sigma_x(a^\dagger\gamma + a\gamma^*) + \gamma\gamma^*, \tag{44}$$

$$U\sigma_z U^\dagger = \sigma_z\left\{1 + \frac{1}{2}\sigma_z[2(a^\dagger\gamma - a\gamma^*)]^2 + \cdots\right\} - i\sigma_y$$

$$\times\left\{2(a^\dagger\gamma-a\gamma^*)+\frac{1}{3!}[2(a^\dagger\gamma-a\gamma^*)]^3+\cdots\right\}$$
$$=\sigma_z\cosh[2(a^\dagger\gamma-a\gamma^*)]-i\sigma_y\sinh[2(a^\dagger\gamma-a\gamma^*)]. \quad (45)$$

Meanwhile, two SOCs terms of H_0 are derived explicitly as

$$U\sigma_x(a^\dagger e^{-i\theta}+ae^{i\theta})U^\dagger$$
$$=\sigma_x(a^\dagger e^{-i\theta}+ae^{i\theta})-(\gamma^* e^{-i\theta}+\gamma e^{i\theta}), \quad (46)$$

and

$$Ui\sigma_y(a^\dagger e^{i\theta}-ae^{-i\theta})U^\dagger$$
$$=i\sigma_y B-\sigma_z[2AB-(\gamma e^{-i\theta}-\gamma^* e^{i\theta})]$$
$$+\frac{1}{2!}i\sigma_y[4A^2B-4A(\gamma e^{-i\theta}-\gamma^* e^{i\theta})]$$
$$-\frac{1}{3!}\sigma_z[8A^3B-12A^2(\gamma e^{-i\theta}-\gamma^* e^{i\theta})^2]$$
$$+\frac{1}{4!}i\sigma_y[16A^4B-32A^3(\gamma e^{-i\theta}-\gamma^* e^{i\theta})]+\cdots$$
$$=i\sigma_y[\cosh(2A)B-(\gamma e^{-i\theta}-\gamma^* e^{i\theta})\sinh(2A)]$$
$$-\sigma_z[\sinh(2A)B-(\gamma e^{-i\theta}-\gamma^* e^{i\theta})\cosh(2A)],$$

where the operators are given by $A=a^\dagger\gamma-a\gamma^*$ and $B=a^\dagger e^{i\theta}-ae^{-i\theta}$. Thus, the transformed Hamiltonian is given in terms of H_0' and H_1' in Eqs. (7) and (8).

By expanding the even and odd functions $\cosh[2(a^\dagger\gamma-a\gamma^*)]$ and $\sinh[2(a^\dagger\gamma-a\gamma^*)]$, the corresponding coefficients are derived as

$$G_{n,n}=\langle n|\cosh[2(a^\dagger\gamma-a\gamma^*)]|n\rangle$$
$$=\frac{1}{2}\langle n|\{\exp[2(a^\dagger\gamma-a\gamma^*)]$$
$$+\exp[-2(a^\dagger\gamma-a\gamma^*)]\}|n\rangle$$
$$=e^{-2\gamma\gamma^*}\sum_{i=0}^{n}(-1)^{n-i}\frac{n!}{(n-i)!(n-i!)i!}(4\gamma\gamma^*)^{n-i}$$
$$=e^{-2\gamma\gamma^*}L_n(4\gamma\gamma^*), \quad (47)$$

and

$$\sqrt{n+1}R_{n+1,n}$$
$$=\langle n+1|\sinh[2(a^\dagger\gamma-a\gamma^*)]|n\rangle$$
$$=\frac{1}{2}\langle n|\{\exp[2(a^\dagger\gamma-a\gamma^*)]-\exp[-2(a^\dagger\gamma-a\gamma^*)]\}|n\rangle$$
$$=\frac{2\gamma e^{-2\gamma\gamma^*}}{\sqrt{n!(n+1)!}}\sum_{i=0}^{n}(-1)^{n-i}$$
$$\cdot\frac{n!(n+1)!}{(n-i)!(n+1-i!)i!}(4\gamma\gamma^*)^{n-i}$$
$$=\frac{2\gamma}{\sqrt{n+1}}e^{-2\gamma\gamma^*}L_n^1(4\gamma\gamma^*). \quad (48)$$

Spin Current under the Zeroth Corrections. In the presence of the electric field, the spin current is derived as

$$\langle j_{\rm s}^z\rangle_{\pm n}=\frac{1}{2}\langle\varphi_{\pm,n}|U\Big[\sqrt{\frac{\hbar\omega}{2m}}(a^\dagger+a)-\frac{eE}{m\omega}\Big]\sigma_z U^\dagger|\varphi_{\pm,n}\rangle. \quad (49)$$

Using the eigenstates $|\varphi_{\pm,n}\rangle$ in Eqs. (23) and (24), one obtains

$$\langle(a^\dagger+a)\sigma_z\rangle_{+n}$$
$$=\langle\varphi_{+,n}|U(a^\dagger+a)\sigma_z U^\dagger|\varphi_{+,n}\rangle$$
$$=\langle\varphi_{+,n}|[(a^\dagger+a)-(\gamma+\gamma^*)\sigma_x]U\sigma_z U^\dagger|\varphi_{+,n}\rangle$$
$$=\langle n|(a^\dagger+a)\sinh[2(a^\dagger\gamma-a\gamma^*)]\sin^*\frac{\theta_n}{2}\cos\frac{\theta_n}{2}|n+1\rangle$$
$$-\langle n+1|(a^\dagger+a)\sinh[2(a^\dagger\gamma-a\gamma^*)]\cos^*\frac{\theta_n}{2}\sin\frac{\theta_n}{2}|n\rangle$$
$$=0, \quad (50)$$

and

$$\langle(a^\dagger+a)\sigma_z\rangle_{-n}=\langle\varphi_{-,n}|U(a^\dagger+a)\sigma_z U^\dagger|\varphi_{-,n}\rangle=0. \quad (51)$$

Thus, the spin current is simplified to

$$\langle j_{\rm s}^z\rangle_{\pm n}=-\frac{eE}{2m\omega}\langle\sigma_z\rangle_{\pm n}, \quad (52)$$

where the average values of $\langle\sigma_z\rangle_{\pm n}$ are derived in the following:

$$\langle\sigma_z\rangle_{+n}$$
$$=\langle\varphi_{+,n}|U\sigma_z U^\dagger|\varphi_{+,n}\rangle$$
$$=\cos\frac{\theta_n}{2}\cos^*\frac{\theta_n}{2}G_{n+1,n+1}-\sin\frac{\theta_n}{2}\sin^*\frac{\theta_n}{2}G_{n,n}$$
$$-\sqrt{n+1}\Big(\sin\frac{\theta_n}{2}\cos^*\frac{\theta_n}{2}R_{n+1,n}$$
$$+\sin^*\frac{\theta_n}{2}\cos\frac{\theta_n}{2}R_{n,n+1}\Big), \quad (53)$$

and

$$\langle\sigma_z\rangle_{-n}$$
$$=\langle\varphi_{-,n}|U\sigma_z U^\dagger|\varphi_{-,n}\rangle$$
$$=\sin\frac{\theta_n}{2}\sin^*\frac{\theta_n}{2}G_{n+1,n+1}-\cos\frac{\theta_n}{2}\cos^*\frac{\theta_n}{2}G_{n,n}$$
$$+\sqrt{n+1}\Big(\cos\frac{\theta_n}{2}\sin^*\frac{\theta_n}{2}R_{n+1,n}$$
$$+\cos^*\frac{\theta_n}{2}\sin\frac{\theta_n}{2}R_{n,n+1}\Big). \quad (54)$$

Meanwhile, the charge current $j_{\rm c}=-ev_x$ can be expressed in terms of the harmonic oscillator a ($a^\dagger$) as

$$j_{\rm c}=-e\Big[\sqrt{\frac{\hbar\omega}{2m}}(a^\dagger+a)+\alpha\sigma_y+\beta\sigma_x-\frac{eE}{m\omega}\Big]. \quad (55)$$

The average value of the charge current is given by

$$\langle j_{\rm c}\rangle_{\pm n}=\langle\varphi_{\pm,n}|Uj_{\rm c}U^\dagger|\varphi_{\pm,n}\rangle$$
$$=\langle\varphi_{\pm,n}|\frac{e^2E}{m\omega}|\varphi_{\pm,n}\rangle=\frac{e^2E}{2\pi N_\phi}L_xL_y. \quad (56)$$

Here $Uj_cU^\dagger = -e[\sqrt{\frac{\omega}{2m}}(a^\dagger + a) - \sqrt{\frac{\omega}{2m}}(\gamma + \gamma^*)\sigma_x + \alpha\sigma_y \cosh(2A) + \alpha i\sigma_z \sinh(2A) + \beta\sigma_x - eE/(m\omega)]$. Then we obtain the charge Hall conductance $G_c = I_c/E$ as

$$G_c = \frac{e^2}{2\pi N_\phi}\sum_{nl} f_{(nl)} = \frac{e^2 N_e}{2\pi N_\phi}. \tag{57}$$

Energy-Crossing Conditions. Since the $(n+1)$th Landau level of spin down and the nth Landau level of spin-up cross near the Fermi energy, this results in the resonance peak of the spin Hall conductance at the energy crossing point. It leads to the energy crossing at certain magnetic field B_0, which satisfies $E_{n+1,-} = E_{n,+}$. It yields

$$\begin{aligned} &2\omega + f(n+2) - 2f(n+1) + f(n) \\ &= \sqrt{[f(n+1) + f(n) + \omega]^2 + 4(n+1)|\widetilde{\alpha}_{n,n+1}|^2} \\ &\quad + \sqrt{[f(n+2) + f(n+1) + \omega]^2 + 4(n+2)|\widetilde{\alpha}_{n+1,n+2}|^2}. \end{aligned} \tag{58}$$

Especially, when the Dresselhaus SOC is neglected by setting $\beta = 0$, the displacement variable reduces to $\gamma = 0$. One can simplify the parameters as $R_{n,n+1} = R_{n+1,n} = 0$, $T_{n,n+1} = -i\alpha/\sqrt{\alpha^2+\beta^2}$, $\eta_0 = \omega/2$, $\eta_1 = -\Delta/2$, $f(n) = -\Delta/2$, and $\widetilde{\alpha}_{n,n+1} = i\eta_R$. It leads to the eigenvalues

$$E_{n,+} = \omega n + \omega + \frac{1}{2}\sqrt{(\omega-\Delta)^2 + 4(n+1)\eta_R^2}\,, \tag{59}$$

$$E_{n+1,-} = \omega(n+1) + \omega - \frac{1}{2}\sqrt{(\omega-\Delta)^2 + 4(n+2)\eta_R^2}\,. \tag{60}$$

Thus, the energy-levels crossing is given by $E_{n+1,-} - E_{n,+} = 0$, resulting in

$$\begin{aligned} 2\omega &= \sqrt{(\omega-\Delta)^2 + 4(n+1)\eta_R^2} \\ &\quad + \sqrt{(\omega-\Delta)^2 + 4(n+2)\eta_R^2}\,. \end{aligned} \tag{61}$$

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