CHIN. PHYS. LETT. Vol. 38, No. 6 (2021) 062101

Relativistic Chiral Description of the $^1S_0$ Nucleon–Nucleon Scattering

Xiu-Lei Ren(任修磊)[1,2], Chun-Xuan Wang(王春宣)[3], Kai-Wen Li(李凯文)[3,4], Li-Sheng Geng(耿立升)[3,5,6*], and Jie Meng(孟杰)[1,3,4*]

[1]*State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*
[2]*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[3]*School of Physics, Beihang University, Beijing 102206, China*
[4]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[5]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*
[6]*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou 450001, China*

(Received 9 February 2021; accepted 8 April 2021; published online 25 May 2021)

Recently, a relativistic chiral nucleon–nucleon interaction was formulated up to leading order, which provides a good description of the phase shifts of $J \leq 1$ partial waves [Chin. Phys. C 42 (2018) 014103]. Nevertheless, a separable regulator function that is not manifestly covariant was used in solving the relativistic scattering equation. In the present work, we first explore a covariant and separable form factor to regularize the kernel potential and then apply it to study the simplest but most challenging $^1S_0$ channel which features several low-energy scales. In addition to being self-consistent, we show that the resulting relativistic potential can describe quite well the unique features of the $^1S_0$ channel at leading order, in particular the pole position of the virtual bound state and the zero amplitude at the scattering momentum ∼340 MeV, indicating that the relativistic formulation may be more natural from the viewpoint of effective field theories.

DOI: 10.1088/0256-307X/38/6/062101

Chiral nuclear forces have been studied extensively[1−4] since Weinberg extended chiral perturbation theory (ChPT),[5] an effective field theory (EFT) of low-energy QCD, to describe nucleon–nucleon (NN) scattering in the 1990s.[6,7] Using the Weinberg power counting, two famous chiral forces were constructed up to next-to-next-to-next-to-leading order a decade ago,[8,9] which have been applied in the *ab initio* descriptions of nuclear structure and reactions (see, e.g., Refs. [10–22]). Recently, the Bochum-Jülich and Indaho groups have constructed chiral forces at the fifth order and shown that the resulting description of NN phase shifts and scattering data[23−26] is comparable to the high precision phenomenological nuclear potentials (such as Reid93,[27] Argonne V_{18},[28] and CD-Bonn[29]) with a χ^2/datum ∼ 1.

On the other hand, with the developments of covariant density functional theories and covariant chiral perturbation theory, relativistic effects are found to play an important role in nuclear structure[30] and one-baryon and heavy-light systems[31−34] (for a short review see Ref. [35]). Therefore, we proposed to construct a relativistic nuclear force and baryon-baryon interactions in covariant ChPT,[36−44] which would provide essential inputs for relativistic many-body calculations, such as the Dirac–Brueckner–Hartree–Fock theory.[45−48] In Ref. [36], we explored a covariant power counting of NN scattering at LO and found that a good description of phase shifts of angular momentum $J = 0,\ 1$ can be achieved by solving the relativistic three-dimensional reduction of the Bethe–Salpeter equation,[49] i.e. the Kadyshevsky equation.[50] Since the intermediate momentum in the scattering equation runs from zero to infinity, the kernel potential must be damped by a regulator function at high momentum to avoid divergence. Such regulator functions are usually referred to as form factors (FFs). In principle, the choice of FFs is rather *ad hoc*[51] and the impact on physical observables should be removed or minimized.

In literature, there are several kinds of form factors, such as the following one,

$$f(\boldsymbol{q}^2) = \frac{\Lambda^2}{\Lambda^2 + \boldsymbol{q}^2}, \tag{1}$$

given by Ueda,[52] where $\boldsymbol{q}$ is the three-momentum transfer, and Λ denotes the cutoff. In Ref. [53], the Bonn potential was constructed with monopole ($n =$

Supported by the China Scholarship Council, the National Natural Science Foundation of China (Grant Nos. 11375024, 11522539, 11735003, 11335002, and 11775099), NSFC and DFG through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD" (NSFC Grant No. 11621131001, DFG Grant No. TRR110), the China Postdoctoral Science Foundation (Grant Nos. 2016M600845 and 2017T100008), the Major State 973 Program of China (Grant No. 2013CB834400), and the Fundamental Research Funds for the Central Universities.
*Corresponding authors. Email: lisheng.geng@buaa.edu.cn; mengj@pku.edu.cn
© 2021 Chinese Physical Society and IOP Publishing Ltd

1) or dipole ($n = 2$) FFs:

$$f(\boldsymbol{q}^2) = \Big[\frac{\Lambda^2 - m_\phi^2}{\Lambda^2 - \boldsymbol{q}^2}\Big]^n, \tag{2}$$

for different meson (ϕ)-nucleon–nucleon vertices. In Refs. [54,55], an eikonal form factor[56] was used to construct the one-boson-exchange potentials. A Gaussian FF,

$$f(\boldsymbol{q}^2) = \exp\Big[-\frac{\boldsymbol{q}^2}{\Lambda^2}\Big], \tag{3}$$

was firstly employed by the Nijmegen group[57] and then applied in the study of chiral forces.[58] In Ref. [59], Epelbaum *et al.* proposed a separable form factor (SFF),

$$f(\boldsymbol{p}) = \exp\Big[-\Big(\frac{\boldsymbol{p}^2}{\Lambda^2}\Big)^n\Big], \tag{4}$$

which only depends on the initial (final) three-momenta $\boldsymbol{p}$ ($\boldsymbol{p}'$). In comparison with the $\boldsymbol{q}^2$-dependent FFs, which introduce additional angular dependence to partial wave potentials and thus affect the interpretation of contact interactions of chiral nuclear forces,[60,61] this separable form factor is better suited in constructing chiral forces.[8,9,25] Recently, a regulator function more proper for the long-range interaction in coordinate space was proposed.[62]

In our previous study,[36] we took the SFF to regularize the kernel of the relativistic LO chiral potential. However, this form factor is not covariant. In order to maintain the self-consistency, a covariant FF is favored in the relativistic framework. Therefore, in this work, we explore a separable form factor which is manifestly covariant.

As an application, we study the simplest but most challenging $^1S_0$ channel with this new form factor. There are several particular features in the $^1S_0$ wave, such as the large variance of phase shifts from 60° to −10° with the laboratory energy $T_{\rm lab} \leq 300\,{\rm MeV}$, a significantly larger scattering length ($a = 23.7\,{\rm fm}$) than the pion Compton wave length, the zero amplitude, namely the zero phase shift of $^1S_0$ with the center of mass (c.m.) momentum $k_0 \sim 340\,{\rm MeV}$, a virtual bound state around $i\gamma = -i10\,{\rm MeV}$. In Ref. [63], van Kolck pointed out that a different kind of fine tuning is needed to produce the zero amplitude of $^1S_0$ in contrast with the virtual bound state. Since these typical energy scales, such as $-i10\,{\rm MeV}$ and $340\,{\rm MeV}$ (in Ref. [64], the chiral expansion of NN potential is performed around the scattering momentum of the zero amplitude, $k_0 = 340\,{\rm MeV}$) are smaller than the chiral symmetry breaking scale ($\Lambda_\chi \sim 1\,{\rm GeV}$), they should be roughly reproduced at the lowest order according to the principle of effective field theories, as explored in Ref. [65]. Thus, the description of these quantities may be considered as a criterion to test a natural power counting for the NN interaction. Inspired by Ref. [65], where the unique features of $^1S_0$ are well described simultaneously by rearranging the short-range interactions in non-relativistic ChPT, we extend our previous work[36] to perform a systematic study of the $^1S_0$ channel up to leading order in the relativistic framework to describe/predict the related quantities with the covariant and separable form factor.

The manuscript is organized as follows. We first present the $^1S_0$ potential in the relativistic formulation, and compare the covariant FF with the non-covariant one. We show the description of the $^1S_0$ phase shifts and the predicted low energy quantities, followed by a short summary.

Theoretical Framework. The relativistic chiral force is formulated up to LO in Ref. [36], where the $^1S_0$ potential reads

$$V_{1S0}(p', p) = 4\pi C_{1S0} + 2\pi(C_{1S0} + \hat{C}_{1S0})\frac{E_p E_p' - m_N^2}{m_N^2} + \frac{\pi g_{\rm A}^2}{f_\pi^2}\int_{-1}^{1} dz \frac{1}{q^2 - m_\pi^2 + i\epsilon}(E_p E_{p'} - p\,p'\,z - m_N^2), \tag{5}$$

with the axial vector coupling $g_{\rm A} = 1.267$ and the pion decay constant $f_\pi = 92.4\,{\rm MeV}$.[66] The four vector q represents the momentum shift with $q^0 = (E_{p'} - E_p)$ and $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$, $\boldsymbol{p}$ ($\boldsymbol{p}'$) is the spatial component of initial (final) momentum of the nucleon in the center of mass (c.m.) frame with $E_p = \sqrt{\boldsymbol{p}^2 + m_N^2}$ ($E_{p'} = \sqrt{\boldsymbol{p}'^2 + m_N^2}$), p and p' are defined as the magnitudes of the momenta, $p = |\boldsymbol{p}|$, $p' = |\boldsymbol{p}'|$, and z denotes the cosine of the angle between $\boldsymbol{p}$ and $\boldsymbol{p}'$. It should be noted that there are two unknown parameters, C_{1S0} and $\hat{C}_{1S0}$, which are the combinations of the five LECs, C_S, $C_{\rm A}$, C_V, $C_{\rm AV}$, C_T, appearing in the lowest order chiral NN Lagrangian[67,68] with $C_{1S0} = C_S + C_V + 3C_{\rm AV} - 6C_T$ and $\hat{C}_{1S0} = 3C_V + C_{\rm A} + C_{\rm AV} - 6C_T$.

Because the nuclear force is non-perturbative, one has to re-sum the above potential via a scattering equation, such as the Bethe–Salpeter equation[49] or its three-dimensional reductions.[69] In Ref. [36], we employed the Kadyshevsky equation[50] (our numerical results would remain almost the same, if the Thompson equation[70] and the Blankenbecler–Sugar equation[71] were employed instead) in the c.m. frame, which reads

$$T_{1S0}(p', p|W) = V_{1S0}(p', p|W) + \int_0^{+\infty} \frac{k^2 dk}{(2\pi)^3} V_{1S0}(p', k|W) \cdot \frac{m_N^2}{2E_k^2(E_p - E_k + i\epsilon)} T_{1S0}(k, p|W), \tag{6}$$

for the $^1S_0$ partial wave, where $W = (\sqrt{s}/2, \boldsymbol{0})$ is half of the total four-momentum with the total energy

$\sqrt{s} = 2E_p = 2E_{p'}$. In solving the scattering equation, one has to use a form factor to regularize the kernel potential as mentioned in the introduction,

$$V_{1S0}(p,p'|W) \to f(\boldsymbol{p}|W)\; V_{1S0}(p,p'|W)\; f(\boldsymbol{p}'|W). \quad (7)$$

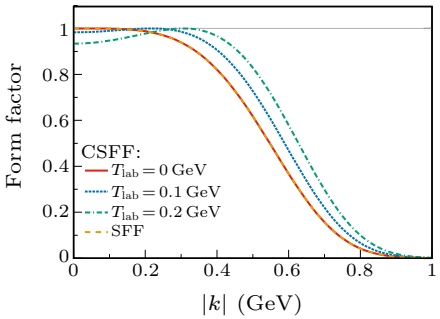

Fig. 1. Form factors as a function of momenta with the cutoff $\Lambda = 600$ MeV. The solid, dotted, and dot-dashed lines represent the CSFF for $T_{\rm lab} = 0,\ 100,\ 200$ MeV, respectively. The dashed line denotes the SFF.

Here, we introduce a covariant and separable form factor (CSFF), which has the exponential form

$$f^{\rm CSFF}(\boldsymbol{p}|W) = \exp\Big[-\Big(\frac{\frac{s}{4} - \boldsymbol{p}^2 - m_N^2}{\Lambda^2}\Big)^2\Big]. \quad (8)$$

One can see that CSFF is trivial (equal to unity) for on-shell potentials with $\boldsymbol{p}^2 = E_p^2 - m_N^2$. However, for the half-/full-off shell potentials appearing in the scattering equation, this form factor becomes

$$f^{\rm CSFF}(\boldsymbol{k}|W) = \exp\Big[-\Big(\frac{\frac{1}{2}m_N T_{\rm lab} - \boldsymbol{k}^2}{\Lambda^2}\Big)^2\Big], \quad (9)$$

where $T_{\rm lab}$ is the laboratory kinetic energy with $s = 4m_N^2 + 2m_N T_{\rm lab}$. The behavior of the CSFF as a function of $|\boldsymbol{k}|$ is shown in Fig. 1 with $T_{\rm lab} = 0,\ 100,\ 200$ MeV, and the cutoff Λ is fixed at 600 MeV. For comparison, the SFF [Eq. (4)] with $n = 2$ is also given. One can see that for $T_{\rm lab} = 0$, the SFF and CSFF are the same. While for $T_{\rm lab} > 0$, the CSFF is not monotonically decreasing in contrast to the SFF. The CSFF is smaller than the SFF and slightly increases to 1.0 for $|\boldsymbol{k}|$ between 0 and $\sqrt{1/2m_N T_{\rm lab}}$. For $|\boldsymbol{k}| > \sqrt{1/2m_N T_{\rm lab}}$, the CSFF is larger than the SFF and decreases to zero for $|\boldsymbol{k}|$ approaching 1 GeV.

Results and Discussion. In order to fine-tune the $^1S_0$ potential, the two parameters, C_{1S0} and $\hat{C}_{1S0}$, are determined by fitting to the six data points of the Nijmegen phase shifts for laboratory energy $T_{\rm lab} \leq 100$ MeV, as carried out in Ref. [36]. To calculate the phase shifts, the covariant and separable form factor is employed to regularize the LO chiral potential and to solve the Kadyshevsky equation. We find that the minimum χ^2, $\sum_{i=1}^{6}(\delta_i^{\rm LO} - \delta_i^{\rm Nij})^2$, is 1.64 for 4 degrees of freedom when the cutoff is 460 MeV. For comparison, we also use the SFF, as in Ref. [36], to describe the $^1S_0$ phase shifts. The best fit result, $\chi^2 = 7.86$, locates at $\Lambda = 695$ MeV. This shows that different types of form factors could affect the phase shifts considerably. Particularly, in our relativistic framework, a self-consistent CSFF achieves a rather good description of the $^1S_0$ phase shifts. In Fig. 2, the evolution of fit-χ^2 is shown with the cutoff changing from 300 MeV to 850 MeV. One can see that the description of phase shifts is almost the same for the two FFs when the cutoff is around 300 MeV. As Λ increases, the fit-χ^2 with the CSFF decreases more quickly than its counterpart with the SFF. The CSFF result shows a plateau, $\chi^2 \sim 2.0$ with Λ ranging from 350 MeV to 500 MeV (in principle, a small cutoff is favored to avoid the appearance of deeply bound states when one applied chiral nuclear forces to perform many-body studies). On the other hand, as the cutoff increases beyond 550 MeV, the CSFF-χ^2 becomes larger and increases faster than the SFF-χ^2, which smoothly approaches to its minimum at $\Lambda = 695$ MeV and then starts to increase. (The sharp increase of the χ^2 with increasing cutoff can be traced back to the Wigner bound.[72] In plain words, it means that with only contact interactions, one cannot simultaneously reproduce the scattering length and effective range (or the phase shifts) of the $^1S_0$ channel with a large cutoff.)

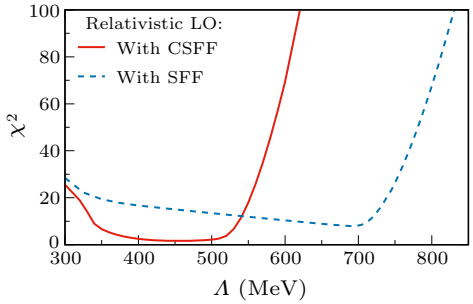

Fig. 2. Description of the $^1S_0$ phase shifts, χ^2, as a function of the cutoff Λ. The red solid and blue dashed lines denote the relativistic LO results obtained with the CSFF and the SFF, respectively.

In Fig. 3, the corresponding LECs C_{1S0} and $\hat{C}_{1S0}$, obtained with the CSFF and the SFF, are plotted as a function of the cutoff Λ. For the CSFF case, the magnitudes of both couplings decrease smoothly with the cutoff Λ and do not show any special structure. On the other hand, the results obtained with the SFF show opposite trends. It is interesting to note that the coupling constants obtained with both kinds of FFs agree more or less with each other in the cutoff range of 500–600 MeV. Furthermore, the large differences between the couplings with the CSFF and those with the SFF for small cutoffs are related to the fact that the largest difference between both form factors appears for small cutoffs. On the other hand, the sharp decrease of the couplings with the SFF around $\Lambda = 700$ MeV indicates the emerging impact of the Wigner bound, as can be seen in Fig. 2, i.e., the sharp increase of the

fit χ^2. On the other hand, for the CSFF case, the stabilization of the couplings around $\Lambda = 550$ MeV indicates the emerging impact of the Wigner bound, as also corroborated by Fig. 2.

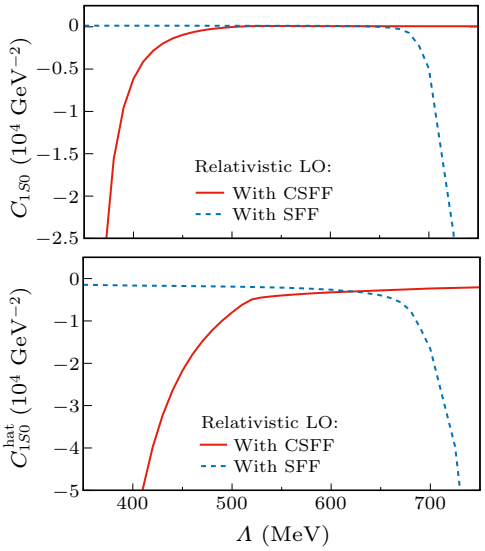

Fig. 3. Evolution of the LECs, C_{1S0} and $\hat{C}_{1S0}$, as a function of the cutoff Λ with the CSFF and the traditional SFF.

Next, we take the LECs from the best fit to plot the phase shifts up to $T_{\rm lab} = 300$ MeV in Fig. 4. We see that both strategies yield a similar description of phase shifts with $T_{\rm lab} \leq 100$ MeV. Especially, in the very low energy region ($T_{\rm lab} < 1$ MeV), the phase shifts show a drastic increase from 0° to 62°, which is the same as the data of the NN-online database.[73] For the $T_{\rm lab} > 100$ MeV region, the calculated phase shifts with the CSFF agree better with the Nijmegen results in comparison with the SFF phase shifts. It is interesting to note that the zero amplitude is well reproduced around the c.m. momentum $k \sim 340$ MeV.

In addition, we predict the coefficients of the effective range expansion of the $^1S_0$ phase shifts,

$$p\cot[\delta_{1S0}(p)] = -\frac{1}{a} + \frac{1}{2}rp^2 + v_2p^4 + v_3p^6 + v_4p^8 + \cdots, \tag{10}$$

where a, r, and $v_{2,3,4,\ldots}$ denote the scattering length, effective range, and curvature parameters, respectively. Their values are tabulated in Table 1 in comparison with the results of the Nijmegen partial wave analysis (PWA) extracted from the NijmII potential.[27,74] One can see that both the scattering length and effective range agree well with the Nijmegen PWA with a deviation of about 5%. On the other hand, the discrepancy of the shape parameters $v_{2,3,4}$ is slightly larger, which may be due to the lack of two-pion exchange contributions in the present study.[75]

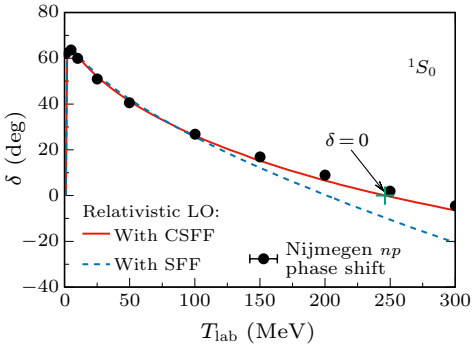

Fig. 4. $^1S_0$ phase shifts δ as a function of laboratory energy $T_{\rm lab}$. The red solid line denotes the best fitting results from the relativistic chiral NN potential with the covariant and separable form factor, while the blue dashed line is the result with the non-covariant form factor. The cross sign denotes the zero phase shift. Solid dots represent the Nijmegen np phase shifts.[76]

We also predict the binding momentum, γ, of the virtual bound state in the $^1S_0$ channel, which can be determined by searching for poles in the scattering amplitude,

$$p_{\rm B}\cot[\delta_{1S0}(p_{\rm B})] \equiv ip_{\rm B}, \tag{11}$$

with $p_{\rm B} = |\boldsymbol{p}_{\rm B}| = i\gamma$ below threshold. In Table 1, the pole position of the virtual bound state is given. One can see that our result agrees with the empirical value.

We note in passing that a simultaneous fit of the partial wave phase shifts with angular momentum $J \leq 1$ using the covariant form factor yields a χ^2/d.o.f. ≈ 10, slightly larger than that of Ref. [36] obtained with the traditional separable form factor (however, the small difference between the fits with the CSFF and SFF should be better viewed as systematic uncertainties for the present study). Nevertheless, the description of the $^1S_0$ partial wave phenomena remains similar.

Table 1. Predicted effective range expansion coefficients of the $^1S_0$ phase shifts (from the best fit results mentioned in the text) in comparison with the values from the Nijmegen PWA.[27,74]

	Λ (MeV)	a (fm)	r (fm)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)	$i\gamma$ (MeV)
Nijmegen PWA		−23.7	2.70	−0.50	4.0	−20	$-i$10.0
Rel.LO-CSFF	460	−23.0	2.61	−0.66	5.5	−32	$-i$8.2
Rel.LO-SFF	695	−22.0	2.53	−0.75	5.9	−34	$-i$8.5

Finally, it should be noted that our current results are still cutoff dependent, as shown in Fig. 2. This unwelcome phenomenon was first noticed around 2000 s,[60,77] and even today remains an open question. There are many works trying to construct a cutoff independent chiral NN potential by modifying the Weinberg power counting, see, e.g., Refs. [65,75,78–90]. It will be interesting to investigate whether one

can achieve cutoff independence in our relativistic framework in the future.

In summary, we have explored a covariant and separable form factor to construct a covariant chiral nucleon–nucleon interaction and found that a slightly better description of the $^1S_0$ phase shifts can be achieved using the covariant form factor. The resulting scattering length and effective range are in good agreement with the empirical values of the Nijmegen PWA. In addition, a simultaneous description of the zero amplitude and the very shallow virtual bound state is obtained at leading order, which indicates that the relativistic $^1S_0$ potential is more consistent with the basic principle of EFTs, namely, being able to describe the several typical small scale quantities simultaneously at leading order. It should be noted that the leading order relativistic chiral description of the $^1S_0$ phenomena, as well as the $J \leq 1$ partial wave phase shifts, is qualitatively similar to whether the covariant or the traditional separable form factor is adopted. It will be interesting to check what happens at higher chiral orders.

X.-L. Ren thanks Professor Bingwei Long for the fruitful discussion.

References

[1] Bedaque P F and van Kolck U 2002 *Ann. Rev. Nucl. Part. Sci.* **52** 339
[2] Epelbaum E, Hammer H W and Meißner U G 2009 *Rev. Mod. Phys.* **81** 1773
[3] Machleidt R and Entem D R 2011 *Phys. Rep.* **503** 1
[4] Epelbaum E and Meißner U G 2012 *Ann. Rev. Nucl. Part. Sci.* **62** 159
[5] Weinberg S 1979 *Physica A* **96** 327
[6] Weinberg S 1990 *Phys. Lett. B* **251** 288
[7] Weinberg S 1991 *Nucl. Phys. B* **363** 3
[8] Entem D R and Machleidt R 2003 *Phys. Rev. C* **68** 041001
[9] Epelbaum E, Glockle W and Meißner U G 2005 *Nucl. Phys. A* **747** 362
[10] Epelbaum E, Krebs H, Lee D and Meißner U G 2011 *Phys. Rev. Lett.* **106** 192501
[11] Tews I, Krüger T, Hebeler K and Schwenk A 2013 *Phys. Rev. Lett.* **110** 032504
[12] Epelbaum E, Krebs H, Lähde T A, Lee D and Meißner U G 2013 *Phys. Rev. Lett.* **110** 112502
[13] Hergert H, Bogner S K, Binder S, Calci A and Langhammer J 2013 *Phys. Rev. C* **87** 034307
[14] Hergert H, Binder S, Calci A, Langhammer J and Roth R 2013 *Phys. Rev. Lett.* **110** 242501
[15] Epelbaum E, Krebs H, Lähde T A, Lee D and Meißner U G 2014 *Phys. Rev. Lett.* **112** 102501
[16] Jansen G R, Engel J, Hagen G, Navratil P and Signoracci A 2014 *Phys. Rev. Lett.* **113** 142502
[17] Bogner S K, Hergert H, Holt J D, Schwenk A and Binder S 2014 *Phys. Rev. Lett.* **113** 142501
[18] Lynn J E, Carlson J, Epelbaum E, Gandolfi S and Gezerlis A 2014 *Phys. Rev. Lett.* **113** 192501
[19] Hagen G *et al.* 2016 *Nat. Phys.* **12** 186
[20] Elhatisari S, Lee D, Rupak G, Epelbaum E and Krebs H 2015 *Nature* **528** 111
[21] Lapoux V, Somà V, Barbieri C, Hergert H and Holt J D 2016 *Phys. Rev. Lett.* **117** 052501
[22] Elhatisari S, Epelbaum E, Krebs H, Lähde T A and Lee D 2017 *Phys. Rev. Lett.* **119** 222505
[23] Epelbaum E, Krebs H and Meißner U G 2015 *Phys. Rev. Lett.* **115** 122301
[24] Reinert P, Krebs H and Epelbaum E 2018 *Eur. Phys. J. A* **54** 86
[25] Entem D R, Kaiser N, Machleidt R and Nosyk Y 2015 *Phys. Rev. C* **91** 014002
[26] Entem D R, Machleidt R and Nosyk Y 2017 *Phys. Rev. C* **96** 024004
[27] Stoks V G J, Klomp R A M, Terheggen C P F and de Swart J J 1994 *Phys. Rev. C* **49** 2950
[28] Wiringa R B, Stoks V G J and Schiavilla R 1995 *Phys. Rev. C* **51** 38
[29] Machleidt R 2001 *Phys. Rev. C* **63** 024001
[30] 2016 *International Review of Nuclear Physics*, in *Relativistic Density Functional for Nuclear Structure* (Singapore: World Scientific) vol 10
[31] Geng L S, Martin C J, Alvarez-Ruso L and Vicente V M J 2008 *Phys. Rev. Lett.* **101** 222002
[32] Ren X L, Geng L S, Martin C J, Meng J and Toki H 2012 *J. High Energy Phys.* **2012** 73
[33] Ren X L, Geng L S and Meng J 2015 *Phys. Rev. D* **91** 051502
[34] Altenbuchinger M, Geng L S and Weise W 2014 *Phys. Rev. D* **89** 014026
[35] Geng L S 2013 *Front. Phys.* **8** 328
[36] Ren X L, Li K W, Geng L S, Long B W and Ring P 2018 *Chin. Phys. C* **42** 014103
[37] Li K W, Ren X L, Geng L S and Long B W 2018 *Chin. Phys. C* **42** 014105
[38] Song J, Li K W and Geng L S 2018 *Phys. Rev. C* **97** 065201
[39] Li K W, Hyodo T and Geng L S 2018 *Phys. Rev. C* **98** 065203
[40] Xiao Y, Geng L S and Ren X L 2019 *Phys. Rev. C* **99** 024004
[41] Bai Q Q, Wang C X, Xiao Y and Geng L S 2020 *Phys. Lett. B* **809** 135745
[42] Xiao Y, Wang C X, Lu J X and Geng L S 2020 *Phys. Rev. C* **102** 054001
[43] Song J, Xiao Y, Liu Z W, Wang C X and Li K W 2020 *Phys. Rev. C* **102** 065208
[44] Liu Z W, Song J, Li K W and Geng L S 2021 *Phys. Rev. C* **103** 025201
[45] Brockmann R and Machleidt R 1990 *Phys. Rev. C* **42** 1965
[46] Shen S, Hu J, Liang H, Meng J and Ring P 2016 *Chin. Phys. Lett.* **33** 102103
[47] Shen S, Liang H, Meng J, Ring P and Zhang S 2017 *Phys. Rev. C* **96** 014316
[48] Shen S, Liang H, Long W H, Meng J and Ring P 2019 *Prog. Part. Nucl. Phys.* **109** 103713
[49] Salpeter E E and Bethe H A 1951 *Phys. Rev.* **84** 1232
[50] Kadyshevsky V G 1968 *Nucl. Phys. B* **6** 125
[51] Erkelenz K 1974 *Phys. Rep.* **13** 191
[52] Ueda T and Green A E S 1968 *Phys. Rev.* **174** 1304
[53] Machleidt R, Holinde K and Elster C 1987 *Phys. Rept.* **149** 1
[54] Jackson A D, Riska D O and Verwest B 1975 *Nucl. Phys. A* **249** 397
[55] Holinde K and Machleidt R 1976 *Nucl. Phys. A* **256** 479
[56] Woloshyn R M and Jackson A D 1972 *Nucl. Phys. A* **185** 131
[57] Nagels M M, Rijken T A and de Swart J J 1978 *Phys. Rev. D* **17** 768
[58] Ordonez C, Ray L and van Kolck U 1994 *Phys. Rev. Lett.* **72** 1982
[59] Epelbaum E, Gloeckle W and Meißner U G 2000 *Nucl. Phys. A* **671** 295
[60] Nogga A, Timmermans R G E and van Kolck U 2005 *Phys. Rev. C* **72** 054006

[61] Epelbaum E 2000 *PhD Thesis* (Julich, Forschungszentrum)
[62] Epelbaum E, Krebs H and Meißner U G 2015 *Eur. Phys. J. A* **51** 53
[63] van Kolck U 1999 *Nucl. Phys. A* **645** 273
[64] Lutz M 2000 *Nucl. Phys. A* **677** 241
[65] Sánchez S M, Yang C J, Long B and van Kolck U 2018 *Phys. Rev. C* **97** 024001
[66] Olive K A 2016 *Chin. Phys. C* **40** 100001
[67] Polinder H, Haidenbauer J and Meißner U G 2006 *Nucl. Phys. A* **779** 244
[68] Djukanovic D, Gegelia J, Scherer S and Schindler M R 2007 *Few Body Syst.* **41** 141
[69] Woloshyn R M and Jackson A D 1973 *Nucl. Phys. B* **64** 269
[70] Thompson R H 1970 *Phys. Rev. D* **1** 110
[71] Blankenbecler R and Sugar R 1966 *Phys. Rev.* **142** 1051
[72] Wang C X, Geng L S and Long B 2021 *Chin. Phys. C* **45** 054101
[73] NN-OnLine, http://nn-online.org
[74] Pavon V M and Ruiz A E 2005 *Phys. Rev. C* **72** 044007
[75] Epelbaum E and Gegelia J 2012 *Phys. Lett. B* **716** 338
[76] Stoks V G J, Klomp R A M, Rentmeester M C M and de Swart J J 1993 *Phys. Rev. C* **48** 792
[77] Kaplan D B, Savage M J and Wise M B 1998 *Phys. Lett. B* **424** 390
[78] Birse M C 2006 *Phys. Rev. C* **74** 014003
[79] Timoteo V S, Frederico T, Delfino A and Tomio L 2005 *Phys. Lett. B* **621** 109
[80] Birse M C 2007 *Phys. Rev. C* **76** 034002
[81] Yang C J, Elster C and Phillips D R 2008 *Phys. Rev. C* **77** 014002
[82] Yang C J, Elster C and Phillips D R 2009 *Phys. Rev. C* **80** 034002
[83] Yang C J, Elster C and Phillips D R 2009 *Phys. Rev. C* **80** 044002
[84] Valderrama M P 2011 *Phys. Rev. C* **83** 024003
[85] Pavon V M 2011 *Phys. Rev. C* **84** 064002
[86] Long B and Yang C J 2011 *Phys. Rev. C* **84** 057001
[87] Long B and Yang C J 2012 *Phys. Rev. C* **86** 024001
[88] Epelbaum E, Gasparyan A M, Gegelia J and Krebs H 2015 *Eur. Phys. J. A* **51** 71
[89] Baru V, Epelbaum E, Gegelia J and Ren X L 2019 *Phys. Lett. B* **798** 134987
[90] Ren X L, Epelbaum E and Gegelia J 2020 *Phys. Rev. C* **101** 034001