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A Noise-Robust Pulse for Excitation Transfer in a Multi-Mode Quantum Memory

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Multi-mode quantum memory is a basic element required for long-distance quantum communication, as well as scalable quantum computation. For on-demand readout and long storage times, control pulses are crucial in order to transfer atomic excitations back and forth into spin excitations. Here, we introduce noise-robust composite pulse sequences for high-fidelity excitation transfer in multi-mode quantum memory. These pulses are robust to the deviations in amplitude and the detuning parameters of realistic conditions. We show the efficiency of these composite pulses with a typical rare-earth ion-doped system. This approach could be applied to a variety of quantum memory schemes.

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Quantum memory comprises specially designed quantum processors which exploit the storage and manipulation of flying qubits and stationary qubits. They constitute the essential building blocks in various forms of quantum information processing: facilitating the synchronization of probabilistic photonic processes for quantum computing, functioning as quantum repeaters for long-distance quantum communication.[1−6] The implementation of quantum memory requires efficient interaction between flying qubits (photons) and stationary qubits (matter qubits). A variety of physical systems have been demonstrated as platforms for quantum memories, such as atomic ensembles,[7] single atoms in cavities,[8] spins in quantum dots[9,10] or NV centers,[11] and rare-earth ions doped in solids.[12] Rare-earth ion-doped systems are promising candidates for use in quantum memory, since on the one hand, they can achieve great efficiency by mapping photons onto collective atomic excitations, and on the other hand, solid state systems offer the potential for integration and scalability in conjunction with existing technology.

Quantum memory offers a variety of advantages, in areas including storage time, fidelity, multi-mode capacity, and on-demand readout.[1,3] Rare-earth ions in solids exhibit excellent coherent properties in terms of spin states. Furthermore, they demonstrate a significant inhomogeneous broadening of optical transitions, which could be a useful resource for multiplexed quantum protocols, e.g., temporal or spectral multi-mode storage. Recent experimental advances in this system include a long coherence time, with time scales measured in hours,[13] and the mapping and retrieval of 64 multiplexed temporal modes,[14] or 26 multiplexed spectral modes,[15] as well as the storage of entanglement,[16,17] polarization[18−20] and the orbital angular momentum of photons.[21]

The atomic frequency combs spin-wave (AFC-SW) protocol has been proposed as a useful resource, combining massive multiplexing, long storage time, and the capacity for on-demand readout of stored states.[22−24] On-demand readout can be achieved by transferring optical atomic excitations to collective spin excitations using control pulses. This also results in a much longer storage time, since the coherence time of spin excitations is far superior to that of atomic excitations. As such, the performance of quantum memory based on the AFC-SW protocol is significantly determined by the control pulses to transfer the excitations. In particular, the fidelity of the control pulses is crucial in applications where quantum memory is used as a quantum processing node in a network.[6] However, the achievement of fully functional quantum memory with high fidelity has so far remained elusive, due to limitations such as noise, and imperfections in the control pulses.

In this work, we propose a scheme to design noise-robust pulses for excitation transfer with high fidelity. Firstly, we provide an overview of the AFC-SW protocol, focusing on the transfer process. We then develop a control model, taking into account the main sources of noise. We utilize a composite pulse approach to achieve the desired transfer operations, via analytic formulas and numerical simulations. This

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type of pulses can realize the desired population transfer between atomic excitations and spin excitations with high fidelity, even under realistic noise conditions. Many works have addressed the issue of composite pulses. The most common composite pulses are designed by employing various techniques, such as the Magnus expansion,[25,26] quaternion algebra, and geometric phases.[27] In addition, most composite pulses are designed to achieve particular classes of gate operations,[25–27] and to tackle a specific source of errors.[25,26] In contrast, our pulse sequences are constructed in a simple iterative manner, can achieve general quantum gate operations, and can systematically eliminate both amplitude and detuning errors. Therefore, our composite pulses are flexibly applicable to various quantum memory protocols.

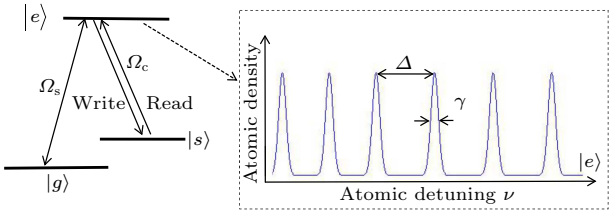

Fig. 1. Schematic of AFC-SW-based quantum memory. A signal photon field induces the optical transition between atomic levels $|g\rangle$ and $|e\rangle$. Next, a pair of control pulses transfer the excitations between levels $|e\rangle$ and $|s\rangle$, thereby facilitating on-demand readout and long-term storage. The capacity for multi-mode storage is based on the spectrally shaped combs on $|e\rangle$, which are composed of narrow peaks with width γ and separation Δ.

We consider the typical system of rare-earth ions, doped in crystal, such as Pr^{3+}:Y_2SO_5,[23] or Eu^{3+}:Y_2SO_5.[24] Such a system can be described as an ensemble of atoms with three levels, as shown in Fig. 1. Since the levels are inhomogeneously broadened, the transition between the ground state $|g\rangle$ and the excited state $|e\rangle$ can be spectrally shaped. The atomic distribution could therefore consist of a frequency comb, where narrow peaks with a characteristic width γ are separated by Δ, and span a large frequency range Γ. When a signal photon enters the crystal, it should be absorbed by the crystal. It would then be stored in an excitation mode, delocalized over all the atoms, which could be expressed by the Dicke-type state, as given in Ref. [22]: $\sum_{j=1}^{N} e^{-i\upsilon_j t} e^{-ik_s z_j} |g_1 \ldots e_j \ldots g_N\rangle$. Here, N is the total number of atoms in the crystal, υ_j is the detuning parameter, z_j is the position, j corresponds to the jth atom, and k_s is the wave number of the absorbed signal photon field. If the comb peaks are sharp enough, the detuning parameter υ_j can be approximated as $\upsilon_j = m_j\Delta$. After an evolution time $2\pi/\Delta$, all the atomic components are in phase, leading to the collective emission of stored photons.

The multi-mode capacity of this type of quantum memory can be implemented as a train of input signal modes. The duration of one mode is limited by the frequency range of the comb as $12\pi/\Gamma$, and the duration of the mode train is limited by the rephasing time as $2\pi/\Delta$. Thus the total number of the input modes that can be stored could be given by the ratio between the train duration and the duration of one mode, as $n = \Gamma/6\Delta$. In rare-earth ionic materials, it is easy to prepare hundreds of comb peaks (i.e., small Δ with large Γ) making it possible to store many modes.

As shown in Fig. 1, once the signal photon is absorbed, a "write" pulse is applied to transfer the atomic excitations in the $|g\rangle$ and $|e\rangle$ levels to the spin excitations in the $|g\rangle$ and $|s\rangle$ levels.[22,28] Since the spin states have a much longer coherence time, T_s, than the optical coherence time, the protocol allows for long-term storage. Next, a "read" pulse is applied to transfer the spin excitations back to atomic excitations, and a re-emission of the stored photon occurs at time $2\pi/\Delta+T_s$. In this way, the whole process leads to on-demand storage and retrieval capability in the signal photon. The control pulses for excitation transfer play an important role in the AFC-SW protocol, which determines the remarkable features of quantum memory, i.e., the capacity for long-term storage and on-demand readout.

In the AFC-SW protocol, the control pulses are applied to transfer collective excitations between the $|e\rangle$ and $|s\rangle$ levels. Under the rotating-wave approximation, the Hamiltonian of the system could be expressed as

$$H(t) = \frac{\omega(t)}{2}\sigma_x + \frac{\upsilon(t)}{2}\sigma_z, \tag{1}$$

where σ_x and σ_z are the Pauli operators, defined in the subspace of $|e\rangle$ and $|s\rangle$; $\upsilon(t)$ and $\omega(t)$ are the detuning and the amplitude of the control pulse, respectively. Thus, the dynamics of the system is determined by the propagator

$$\begin{aligned} R(\omega\hat{x} + \upsilon\hat{z}, \theta) &= \exp\Big[-i\int_0^t dt H(t)\Big] \\ &= \exp\Big[-i\Big(\frac{\omega}{2}\sigma_x + \frac{\upsilon}{2}\sigma_z\Big)\frac{\theta}{\sqrt{\omega^2+\upsilon^2}}\Big]. \end{aligned} \tag{2}$$

Here θ is the rotating angle around the axis $\omega\hat{x} + \upsilon\hat{z}$ on the Bloch sphere.

We begin with the case of the widely-used π pulse. If the detuning maintains a value of $\upsilon(t) = 0$, the propagator has an explicit form, as

$$R(\hat{x}, \theta) = \begin{pmatrix} \cos(A/2) & -i\sin(A/2) \\ -i\sin(A/2) & \cos(A/2) \end{pmatrix}, \tag{3}$$

where $A = \int_0^t \omega(t)dt$ is the pulse area. When we set $A = \pi$, we achieve a π pulse capable of realizing excitation transfer between the $|e\rangle$ and $|s\rangle$ levels. This pulse is a rotation around x axis on the Bloch sphere. Since the π pulse explicitly depends on the control parameters, it is not robust with respect to variations in

these parameters. As such, the π pulse struggles to achieve high fidelity under realistic conditions.

In the following, we outline the following scheme to design noise-robust pulses for excitation transfer:

Step 1: Choose the target operation. As described above, quantum memory based on the AFC-SW protocol utilizes a range of frequency detuning to produce the atomic frequency comb structure. As such, we require a control pulse at a fixed detuning, v_c. The ordinary π pulse around the x axis is denoted by $R(\hat{x},\pi)$. We could achieve $R(\hat{x},\pi)$ via

$$R(\hat{x},\pi)=R(\hat{x}+\hat{z},\pi)R(\hat{z},\pi)R(\hat{x}+\hat{z},\pi). \tag{4}$$

To implement a rotation $R(\hat{x}+\hat{z},\pi)$ around the axis $\hat{x}+\hat{z}$ with a π angle, we could set $\omega(t)=v_c$, and the time duration as $\pi/\sqrt{\omega(t)^2+v_c^2}$. For a rotation $R(\hat{z},\pi)$ around the axis $\hat{z}$ with a π angle, we could set $\omega(t)=0$ and time duration as $\pi/\sqrt{0^2+v_c^2}$. Putting these three rotations together, we can obtain the target π pulse around the x axis.

Step 2: Model the noise effects. There are two types of noise: fluctuation $\delta\omega$ in the pulse amplitude ω, and fluctuation δv in the detuning v. Inserting two kinds of noises into the Hamiltonian, we get

$$H^1(t)=\frac{\omega(t)+\delta\omega}{2}\sigma_x+\frac{v+\delta v}{2}\sigma_z. \tag{5}$$

Since both amplitude and frequency noise are typically orders of magnitude slower than operation times, the resulting perturbations about ω and v are treated as quasi-static noise in the following analysis. The evolution operator is now rewritten as

$$\begin{aligned}&U(\omega\hat{x}+v\hat{z},\theta)\\&=\exp\Big[-i\int_0^t dtH^1(t)\Big]\\&=\exp\Big[-i\Big(\frac{\omega+\delta\omega}{2}\sigma_x+\frac{v+\delta v}{2}\sigma_z\Big)\frac{\theta}{\sqrt{\omega^2+v^2}}\Big].\end{aligned} \tag{6}$$

Table 1. Parameters for the designed composite pulse.

Rotation parameters	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5
Values (MHz)	8.1028	43.2337	0	30.9351	3.9232	0

In order to address the effects of noise, we need to separate the noise terms from the operation term. We therefore perform an expansion of the evolution operator to the first order of noise, given as

$$\begin{aligned}U(\omega\hat{x}+v\hat{z},\theta)=&\exp\Big[-i\Big(\frac{\omega}{2}\sigma_x+\frac{v}{2}\sigma_z\Big)\frac{\theta}{\sqrt{\omega^2+v^2}}\Big]\\&\times(I-i\sum_k\Delta_k\sigma_k)\\=&R(\omega\hat{x}+v\hat{z},\theta)(I-i\sum_k\Delta_k\sigma_k).\end{aligned} \tag{7}$$

Here, $R(\omega\hat{x}+v\hat{z},\theta)$ is the desired rotation, I is the identity operator, σ_k are the Pauli operators, and the error terms are given as

$$\begin{aligned}\Delta_x&=\delta\omega\frac{\omega^2\theta+v^2\sin\theta}{2(\omega^2+v^2)^{3/2}}+\delta v\frac{\omega v(\theta-\sin\theta)}{2(\omega^2+v^2)^{3/2}},\\\Delta_y&=\delta\omega\frac{v(\cos\theta-1)}{2(\omega^2+v^2)}+\delta v\frac{\omega(1-\cos\theta)}{2(\omega^2+v^2)},\\\Delta_z&=\delta\omega\frac{\omega v(\theta-\sin\theta)}{2(\omega^2+v^2)^{3/2}}+\delta v\frac{v^2\theta+\omega^2\sin\theta}{2(\omega^2+v^2)^{3/2}}.\end{aligned} \tag{8}$$

These formulas demonstrate how noise affects the evolution of the system, resulting in small errors in the target rotation, which are characterized as error terms Δ_k in three degrees of freedom, x,y,z.

Step 3: Construct a composite pulse to cancel the noise effects. We have a rotation around the $\hat{z}$ axis, together with two rotations around the $\hat{x}+\hat{z}$ axis to achieve the desired rotation, but these three rotations would induce various noise effects during the rotation process. How to eliminate noise is the key question in the field of quantum information processing. To resolve this problem, we exploit the idea of the composite pulse, as given in Refs. [29–32], to construct a composite 2π rotation. The composite 2π rotation can be viewed as a series of rotations with different axes and 2π angles. Specifically, we construct a five-level pulse sequence for the composite 2π rotation as:

$$\begin{aligned}\tilde{I}^{(5)}=&U\Big(\omega_5\hat{x}+v_c\hat{z},\pi+\frac{3\pi}{2}\Big)\\&\times U(\omega_4\hat{x}+v_c\hat{z},\pi)U(\omega_3\hat{x}+v_c\hat{z},\pi)\\&\times U(\omega_2\hat{x}+v_c\hat{z},\pi)U(\omega_1\hat{x}+v_c\hat{z},\pi)\\&\times U(\omega_0\hat{x}+v_c\hat{z},2\pi)\\&\times U(\omega_1\hat{x}+v_c\hat{z},\pi)U(\omega_2\hat{x}+v_c\hat{z},\pi)\\&\times U(\omega_3\hat{x}+v_c\hat{z},\pi)U(\omega_4\hat{x}+v_c\hat{z},\pi)\\&\times U\Big(\omega_5\hat{x}+v_c\hat{z},\frac{\pi}{2}\Big).\end{aligned} \tag{9}$$

Combined with this composite 2π rotation, we can then find the entire desired rotation, expressed as

$$U_{\rm NRP}=U(\hat{z}+\hat{x},\pi)\tilde{I}^{(5)}U(\hat{z},\pi)U(\hat{z}+\hat{x},\pi). \tag{10}$$

This composite pulse sequence would also give rise to noise, as

$$U_{\rm NRP}=R(\hat{x},\pi)\Big(I-i\sum_k\eta_k\sigma_k\Big), \tag{11}$$

where η_k are error terms for the composite pulse sequence due to fluctuations in $\delta\omega$ and δv. We then select the five control parameters, ω_i, to ensure that $\eta_k=0$. As shown in Eqs. (10) and (11), the composite 2π rotation plays an important role in this scheme: Firstly, it offers free parameters ω_i to allow for noise cancellation of the whole pulse sequence. Secondly, it is similar to an identity operator, in that it does not affect the overall target rotation.

For concreteness, we consider a detailed example, based on the AFC-SW protocol in Pr^{3+}:Y_2SO_5,[23] or Eu^{3+}:Y_2SO_5.[24] On the one hand, the commonly used π pulse for excitation transfer is shown in Fig. 2(a). The pulse amplitude is $\omega(t) = \pi$ MHz, and the pulse detuning is $v(t) = 0$. On the other hand, we choose the frequency detuning parameter $v_c = 4\pi$ MHz, and numerically solve the coupled equations for the amplitude parameters, ω_i. The results are shown in Table 1. With these parameter values, we are able to construct a noise-robust composite pulse for excitation transfer, as illustrated in Fig. 2(b). Figure 2(c) shows the population probability on the $|e\rangle$ and $|s\rangle$ levels as a function of composite pulse duration.

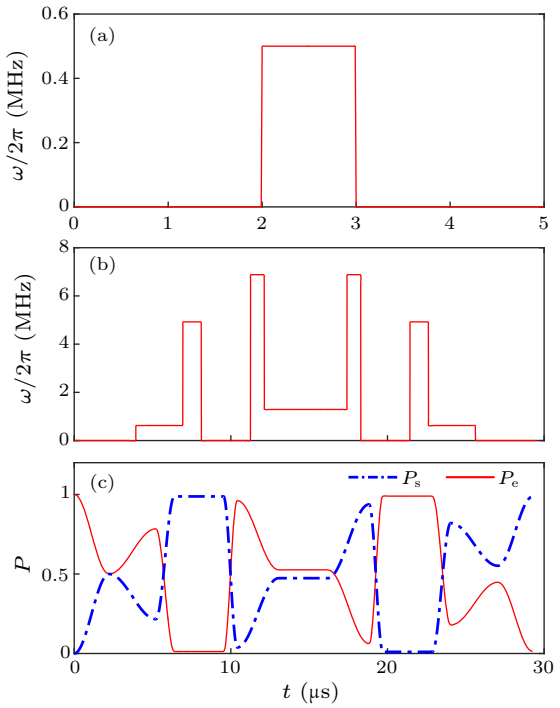

Fig. 2. (a) Commonly used π pulse with $v(t) = 0$ and $\omega(t) = \pi$ MHz. (b) Noise robust pulse with detuning $v_c = 4\pi$ MHz. (c) Population transfer between levels $|e\rangle$ and $|s\rangle$, using the noise robust pulse.

Transfer fidelity is defined as the overlap between the ideal transferred state, and the realistic transferred state under conditions of noise. We use the formula

$$F = \mathrm{Tr}\sqrt{\sqrt{\rho'}\rho\sqrt{\rho'}}, \tag{12}$$

where ρ is the density matrix, as $\rho = |\psi\rangle\langle\psi|$ without noise, and $\rho' = |\psi'\rangle\langle\psi'|$ is the density matrix including noise. We calculate the infidelity $1 - F$ as a function of amplitude noise $\delta\omega$ and detuning noise δv. We note that in realistic situations, the ratio of noise to the control field ($\frac{\delta\omega}{\omega}$ or $\frac{\delta v}{v}$) is between 0 and 0.2.

Figure 3 shows the infidelity for the commonly used π pulse (denoted as OP), and the noise-robust π pulse (denoted as NRP), under different noise conditions. In Figs. 3(a) and 3(a) small variation $\delta\omega$ in the detuning induces an error in transfer fidelity on the order of 10^{-2} using the ordinary π pulse, whereas the transfer infidelity with the noise-robust π pulse remains one to two orders smaller than the ordinary π pulse over a wide range of noise values. The noise-robust π pulse permits fidelity beyond 99% even with respect to a large noise value of $\delta\omega/\omega = 0.2$. This high fidelity value is essential in many quantum network applications involving local operations requiring to be performed fault-tolerantly.[6] Figure 3(b) shows the infidelity as a function of driving detuning noise, where the noise-robust π pulse remains more robust than the ordinary π pulse.

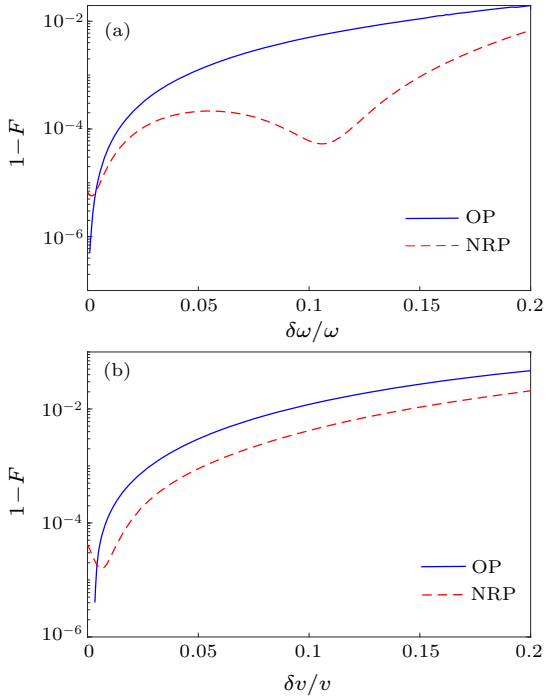

Fig. 3. Transfer infidelity versus various forms of noise for an ordinary π pulse (denoted as OP), and a noise-robust pulse (denoted as NRP). (a) Infidelity as a function of fluctuations in amplitudes $\delta\omega/\omega$. The calculations are preformed based on a detuning noise of $\delta v/v = 0.01$. (b) Infidelity as a function of fluctuations in detuning $\delta v/v$. The calculations are performed based on an amplitude noise of $\delta\omega/\omega = 0.01$.

In conclusion, we have developed composite pulse sequences for high-fidelity excitation transfer in an AFC-SW based quantum memory. These composite pulses are designed to compensate for variations in various experimental parameters (e.g., intensity or amplitude, detuning or frequency shifts). We theoretically simulate the fidelity of the transfer process in realistic rare-earth ion-doped systems. In particular, our results show high fidelity beyond 99% across a broad range of experimental parameters, and for different cases. The operational fidelity of these composite pulses are significantly larger compared to conventional π pulses. The designed control pulses would be a highly accurate and robust tool for a wide va-

riety of quantum memory applications (e.g., EIT,[33] DLCZ,[34] GEM[35] or microwave[36,37]), and would be particularly valuable under conditions involving significant experimental uncertainties.

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