CHIN. PHYS. LETT. Vol. 38, No. 4 (2021) 042501

Higher-Twist Effect in Pion Parton Distribution

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(Received 7 December 2020; accepted 1 February 2021; published online 6 April 2021)

A higher-twist modified parton evolution equation is used to evolve the initial valence quark distributions in pions, which are derived based on light-front quantization via BLFQ collaboration. The results are consistent with the valence quark distributions of the E615 experiment, and the pion structure function of the H1 experiment. The structure function data highlight the necessity for a higher-twist modification in the small x region. Comparisons with some other models are also given.

DOI: 10.1088/0256-307X/38/4/042501

As the lightest QCD bound state, the pion plays an important role in the study of strong interactions. Although the pion is the simplest $q\bar{q}$ state, we are not clear about its parton distribution functions (PDFs), the reason for this is that there is no fixed pion target. Current understanding of pion PDFs originates primarily from two types of experiment: one involves pion-nucleon scattering, as performed at CERN[1] and Fermilab[2] via a process producing prompt photons or dileptons. This type of process effectively determines the valence quark distributions of the pion. The other is a process of leading neutron production in electron-proton deep inelastic scattering, as performed at HERA,[3,4] in which pion exchange is believed to dominate the processes of neutrons at large $x_{\rm L}$ (longitudinal momentum fraction carried by neutron) and low transverse momentum $p_{\rm T}$. In this mechanism, protons first evolve to neutrons and pions, where the pions can be approximately regarded as on-shell. By means of scattering electrons on the pion and exchanging virtual photons, the pion structure can be studied indirectly, and the distributions of sea quarks and gluons in pions can be explored.

There has been a great deal of research into the analysis of parton distributions in pions. The first global QCD analysis of the pion PDF was performed in Ref. [5]. Some detailed analyses of lattice QCD have also been performed,[6–12] together with some phenomenological models to study pion PDFs, such as the Nambu–Jona–Lasinio (NJL) model,[13] the constituent quark model,[14] the chiral quark model,[15] the anti-de Sitter (AdS)/QCD models,[16–19] and the Dyson–Schwinger equation/Bethe-Salpeter equation(DSE/BSE) model.[20] Extensions of the continuum analysis in Ref. [21] have yielded new achievements,[22] leading to the first parameter-free predictions of the valence quark, gluon, and sea quark distributions within the pion,[23,24] unifying these with electromagnetic pion elastic and transition form factors,[25–30] etc. Among all these works, few compare with the structure function (F_2) data of HERA,[3,4] the BLFQ collaboration[31] calculated the F_2 of pions, but apparently produced disappointing results. We believe that one of the main reasons for this is that, with the evolution of the standard Dokshitzer–Gribov–Levin–Altarelli–Parisi (DGLAP)[32] equation, most recent models cannot adequately fit the F_2 data.

In the models researching for pion PDFs, some works give their initial parton distributions at a certain low-momentum scale; using the DGLAP equation to evolve, one can obtain parton distributions at higher scales. There are two types of initial parton distributions: one involves only the valence quark input, such as that in Refs. [31,33] (BLFQ collaboration) and in Refs. [23,24] (Ding *et al.*); the other type includes all the partons, such as the GRS model,[34,35] the SMRS model[36] and the model of Novikov *et al.*[37] These types of models generally contain many parameters, which can be determined by fitting the experiments. The BLFQ–NJL model obtained valence quark distribution at low Q^2, starting from the effective light cone Hamiltonian of the constituent quarks. This model has only one adjustable parameter, i.e., the initial scale, Q_0^2. With the evolution of the DGLAP equation, the team obtained valence quark distributions at higher scales, together with differential cross sections, and the ratio of valence quarks of pions and kaons xu_v^π/xu_v^k, these agree well with the experimental data. However, when compared with the pion structure function, it was found that the predicted result is significantly higher than that found in the experimental data for the small $x(x < 10^{-2})$ region, with x being the longitudinal momentum fraction of pion carried by the parton. This indicates that for the small x region,

Supported by the National Natural Science Foundation of China (Grant No. 11851303).
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the sea quark distribution is too large. In other words, the splitting of gluons into quarks proceeds too fast, based on the evolution of the DGLAP equation for the small x region, indicating that the evolution equation also needs to take parton recombination effects into account.

The Gribov–Levin–Ryskin (GLR) equation[38] was the first to modify the DGLAP equation with a gluon recombination effect at twist-4 level. Later, Muller and Qiu (MQ) performed a perturbative calculation of the recombination probabilities via the double leading logarithmic approximation (DLLA),[39,40] facilitating the application of the GLR-MQ equation. However, the GLR-MQ equation was further improved by Zhu–Ruan–Shen to restore momentum conservation in Refs. [41–44], and we therefore refer to the improved equation as the ZRS equation.

The ZRS equation[41–44] used the time ordered perturbation theory (TOPT) to establish the relationship between the amplitudes of twist-4 processes under the leading logarithmic (Q^2) approximation [LL(Q^2)A], and obtain the complete recombination function.[44] If considering only recombination between gluons, and taking the 'most contribution' part (i.e., the recombination of two gluons with the same longitudinal momentum faction), the equation can be expressed as

$$\begin{aligned}\frac{dxG(x,Q^2)}{d\ln Q^2} =& P_{gg}^{\rm AP}\otimes G(x,Q^2)+P_{gq}^{\rm AP}\otimes S(x,Q^2)\\ &+\frac{\alpha_s^2}{4\pi R^2Q^2}\int_{x/2}^{x}dx_1xx_1G^2(x_1,Q^2)\\ &\cdot\sum_i P_i^{gg\to g}(x_1,x)\\ &-\frac{\alpha_s^2}{4\pi R^2Q^2}\int_{x}^{1/2}dx_1xx_1G^2(x_1,Q^2)\\ &\cdot\sum_i P_i^{gg\to g}(x_1,x),\end{aligned}\tag{1a}$$

for gluon distribution, and

$$\begin{aligned}\frac{dxS(x,Q^2)}{d\ln Q^2} =& P_{qg}^{\rm AP}\otimes G(x,Q^2)+P_{qq}^{\rm AP}\otimes S(x,Q^2)\\ &+\frac{\alpha_s^2}{4\pi R^2Q^2}\int_{x/2}^{x}dx_1xx_1G^2(x_1,Q^2)\\ &\cdot\sum_i P_i^{gg\to q}(x_1,x)\\ &-\frac{\alpha_s^2}{4\pi R^2Q^2}\int_{x}^{1/2}dx_1xx_1G^2(x_1,Q^2)\\ &\cdot\sum_i P_i^{gg\to q}(x_1,x),\end{aligned}\tag{1b}$$

for sea quark distributions. Here, the first two terms on the right-hand side of the above two equations refer to the DGLAP equation, and $P^{\rm AP}$ denotes the evolution kernels of the linear DGLAP equation. The third and fourth terms are the contributions of the twist-4 correction. The third term is known as the antishadowing term, and the fourth term is the shadowing term; $P^{gg\to g}$ and $P^{gg\to q}$ represent the recombination function. Considering that the contribution of the leading term, $P^{gg\to g}$, is much greater than $P^{gg\to q}$ in the recombination functions ($P^{gg\to q}/P^{gg\to g}=1/27$ for the leading term), we ignore the contribution of the recombination term in Eq. (1b), meaning that the higher-twist effects in sea quarks are mainly ascribed to gluons in Eq. (1a). In addition, in Eq. (1a), if only the contribution of the leading term is to be kept, the evolution equation for gluons can then be simplified to

$$\begin{aligned}\frac{dxG(x,Q^2)}{d\ln Q^2} =& \,{\rm DGLAP}\\ &+\frac{81\alpha_s^2}{16\pi R^2Q^2}\int_{x/2}^{x}\frac{dx_1}{x_1}[x_1G(x_1,Q^2)]^2\\ &-\frac{81\alpha_s^2}{16\pi R^2Q^2}\int_{x}^{1/2}\frac{dx_1}{x_1}[x_1G(x_1,Q^2)]^2,\\ &\left({\rm if}\ x\le\frac{1}{2}\right),\end{aligned}\tag{2a}$$

$$\begin{aligned}\frac{dxG(x,Q^2)}{d\ln Q^2} =& \,{\rm DGLAP}\\ &+\frac{81\alpha_s^2}{16\pi R^2Q^2}\int_{x/2}^{1/2}\frac{dx_1}{x_1}[x_1G(x_1,Q^2)]^2,\\ &\left({\rm if}\ \frac{1}{2}\le x\le 1\right).\end{aligned}\tag{2b}$$

The occurrence of Eq. (2b) arises because there is only an antishadowing effect in the region of $\frac{1}{2}\le x\le 1$. Here, $\alpha_{\rm s}$ is the strong interaction coupling constant, and R is the effective distance between the recombination gluons. In this work, we adopt this simplified form.

The DGLAP evolution equation is momentum-conserved. The authors of Ref. [41] proved that the complete ZRS equation satisfies momentum conservation. For formula (2), it is easy to prove

$$\begin{aligned}&-\int_0^{1/2}dx\int_x^{1/2}\frac{dx_1}{x_1}[x_1G(x_1,Q^2)]^2\\ &+\int_0^{1/2}dx\int_{x/2}^{x}\frac{dx_1}{x_1}[x_1G(x_1,Q^2)]^2\\ &+\int_{1/2}^{1}dx\int_{x/2}^{1/2}\frac{dx_1}{x_1}[x_1G(x_1,Q^2)]^2\\ &=0.\end{aligned}\tag{3}$$

This momentum conservation is independent of the specific form of the distribution function $x_1G(x_1,Q^2)$.

Since the first experiment showing π^+ incident on an isospin scalar target,[34] a great deal of research has focused on the PDFs of pions. In this work, we first consider the initial parton distributions taken from the

BLFQ collaboration.[31] For π^+, the valence quark distribution is

$$f_{u_V}(x) = f_{\bar{d}_V}(x) = B^{-1}(1+\alpha, 1+\beta)x^{\alpha}(1-x)^{\beta}, \quad (4)$$

where $\alpha = \beta = 0.5961$ and $B(1+\alpha, 1+\beta)$ is the Euler Beta function, which ensures that the above formula satisfies normalization criteria. In addition,

$$\int_0^1 x f_{u_V}(x)dx + \int_0^1 x f_{\bar{d}_V}(x)dx = 1. \quad (5)$$

The valence quarks carry all the momentum of the meson, and there are no sea quarks or gluons at this scale. While the BLFQ-NJL model cannot give an exact value of this momentum scale, in Ref. [31] the team took $Q_0^2 = 0.12\,\mathrm{GeV}^2$ for the leading logarithm approximation.

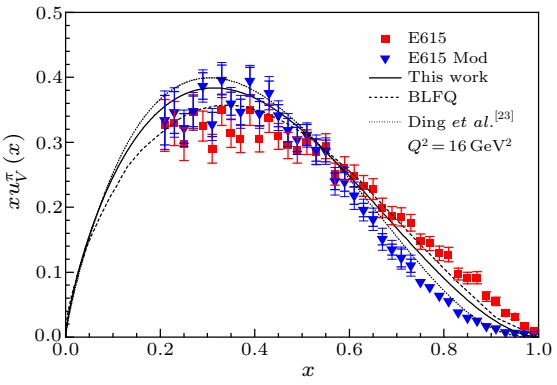

Fig. 1. Characteristics of $xu_V^{\pi}(x)$ as a function of x for the pion at $Q^2 = 16\,\mathrm{GeV}^2$. The solid line represents our result. The dashed line is the prediction of the BLFQ-NJL model,[31] and the dotted line shows the result of Ding *et al.*[23,24] The red data point is the original analysis of the E615 experimental result[2] and the blue data point is the reanalysis of the E615 experimental result.[45]

We use Eq. (4) as the initial input of valence quark PDFs, and perform the evolution according to Eq. (2). In contrast to the BLFQ-NJL model,[31] we take $Q_0^2 = 0.09\,\mathrm{GeV}^2$, and choose $R = 1.45\,\mathrm{GeV}^{-1}$, these two parameters are determined primarily by the structure function data of H1.[4] In Fig. 1, we show the valence quark distribution at $Q^2 = 16.0\,\mathrm{GeV}^2$. The solid line is our result, and the dashed and dotted lines denote the results from BLFQ[31] and Ding *et al.*[23,24] respectively. The red experimental points represent the original data from experiment E615,[2] and the blue points represent the reanalysis of the experimental E615 data via the BSE method, including the resummation of soft gluons.[45,46] It can be seen that our results are in the reasonable region between the original data and the results of its reanalysis. Our valence quark distribution is slightly different from that of the BLFQ-NJL model, even though we take the same input distributions; this is because we take the evolution starting scale at $Q_0^2 = 0.09\,\mathrm{GeV}^2$, which is smaller than their $Q_0^2 = 0.12\,\mathrm{GeV}^2$, so that we have a longer evolution range, and our result is smaller in the large x region, and larger in small x region. The result of Ding *et al.* is slightly larger than our result for the small x region.

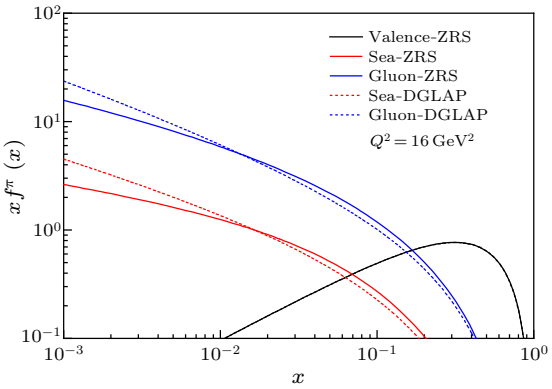

Fig. 2. Comparison of PDFs obtained using the ZRS equation and those from the DGLAP equation at $Q^2 = 16\,\mathrm{GeV}^2$. The valence quark distributions are the same for the two conditions.

In order to show the higher-twist effects in different parton distributions, we perform the calculation with the linear DGLAP equation using the same input as Eq. (4), at the same initial scale $Q^2 = 0.09\,\mathrm{GeV}^2$. The results are shown in Fig. 2. The solid lines and dashed lines represent the results of the ZRS and DGLAP equations respectively. For the valence quark, the two results are the same, since the valence quark has no relation to the higher-twist effect. For sea quarks and gluons, the solid lines are lower in the smaller x region, and bigger in the larger x region; this constitutes direct evidence of shadowing and antishadowing effects. If the equation contains only the shadowing part, without the antishadowing part, as in the GLR-MQ equation, the solid lines will always be lower than the dashed lines, and momentum conservation will not be maintained.

In deep inelastic e-p scattering, for the leading neutron production at large x_{L} and small p_{T}, pion exchange is considered to be the main mechanism and can be regarded as on-shell. The scattering cross section can be expressed as:[4]

$$d\sigma(ep \to e'nX) = f_{\pi^+/p}(x_{\mathrm{L}}, t) \cdot d\sigma(e\pi^+ \to e'X), \quad (6)$$

where $f_{\pi^+/p}(x_{\mathrm{L}}, t)$ represents the pion flux associated with the splitting of a proton into a $\pi^+ n$ system, and $d\sigma(e\pi^+ \to e'X)$ is the cross section of the e-pion interaction. Here, t is the squared four-momentum transfer between the incident proton and the final state neutron.

Both ZEUS[3] and H1[4] at HERA put forward the structure function data of the pion, while selecting different flux factors in Eq. (6). H1 adopted a one-pion exchange model,[47] choosing data from $x_{\mathrm{L}} = 0.73$

(center of region $0.68 < x_{\rm L} < 0.77$) and $p_{\rm T} < 0.2\,{\rm GeV}$. ZEUS took two kinds of flux factors:[48,49] one is an effective one-pion-exchange model (EF), and the other is an additive quark model (AQM), so that they obtained two sets of data. They choose a broader region of $x_{\rm L}$ ($0.64 < x_{\rm L} < 0.82$), and the center is also $x_{\rm L} = 0.73$. The neutron transverse momentum $p_{\rm T}^{\max}$ of ZEUS is much larger than that of H1. The main reason for our selection of the H1 data from which to set our parameters is that the smaller values of $p_{\rm T}^{\max}$ are expected to enhance the relative contribution of pion exchange,[4] whereas for ZEUS, the pion flux is more complicated to determine.

In the leading-order approximation, the relationship between the structure function and the parton distribution function is

$$F_2^{\pi}(x,Q^2) = \sum_q e_q^2 x[f_q^{\pi}(x,Q^2) + f_{\bar{q}}^{\pi}(x,Q^2)], \quad (7)$$

where q is the flavor index, e_q is the charge of the flavor quark, and x is the fraction of the momentum of the pion carried by a parton interacting with a virtual photon.

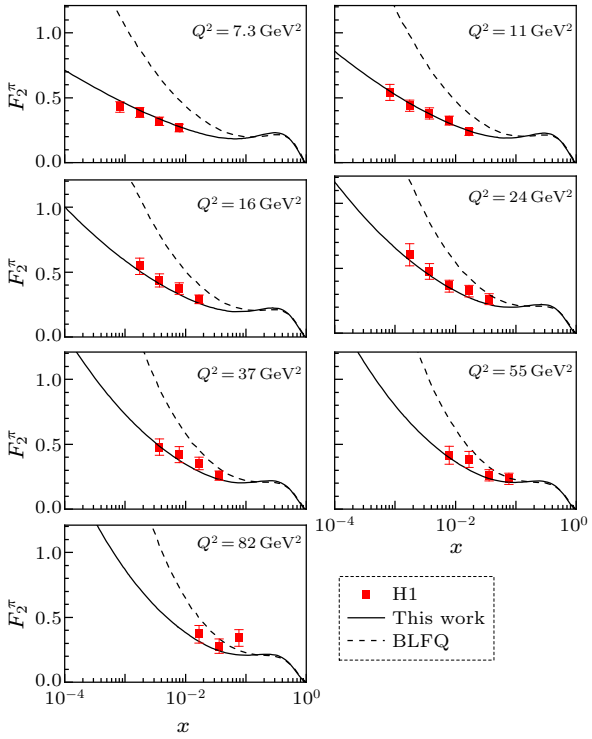

Fig. 3. Structure function $F_2^{\pi}(x,Q^2)$ for the pion as a function of x, at fixed experimental values of Q^2. The data are taken from Ref. [4] by the H1 collaboration at DESY–HERA. The solid line is our result, and the dashed line is the prediction of BLFQ.[31]

Our structure function F_2^{π} results, as compared with the experimental data of H1[4] and ZEUS[3] are given in Figs. 3 and 4. In the two sets of graphs, the solid lines are our results, obtained via the ZRS equation, and the dashed lines are the results of BLFQ collaboration.[31] In Fig. 3, our results are in good agreement with the experiment; the dashed lines (obtained via the evolution of the DGLAP equation) are clearly higher than those for the experimental data. We note that the higher-twist correction played an important role in the small x region. In Fig. 4, our lines are in-between those of the two sets of data. The results of BLFQ are much closer to the data of the additive quark model, while their line slope is obviously larger than that of the data, particularly in the small x region. It seems that higher-twist corrections cannot be neglected, even in comparison with this group of data.

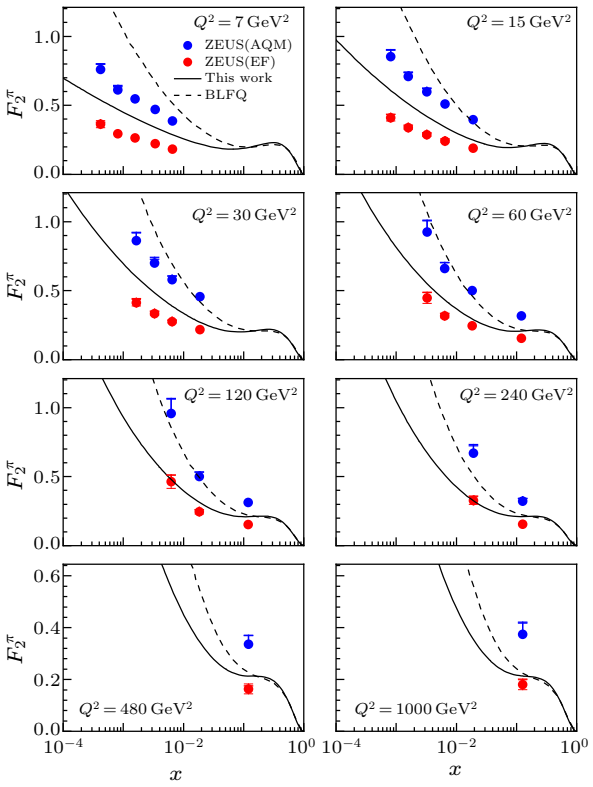

Fig. 4. Comparison of ZEUS and BLFQ results. Structure function $F_2^{\pi}(x,Q^2)$ for the pion as a function of x, at fixed experimental values of Q^2. The data are taken from Ref. [3] by the ZEUS collaboration. The solid line is our result, and the dashed line is the prediction of BLFQ.[31]

The difference between the solid lines and the dashed lines in Figs. 3 and 4 can be ascribed primarily to sea quark distribution. We compare the contribution of each part of F_2^{π} at $Q^2 = 55\,{\rm GeV}^2$ in Fig. 5. Here, the solid lines are our results, and the dashed lines are those of BLFQ. We can see that, in the region where $x < 0.01$, the structure function is determined by sea quark distributions, so for structure function data in the small x region, we are therefore able to determine whether or not the higher-twist correction is needed.

In other research works relating to pion parton distributions, the global analysis in Ref. [5] has demonstrated that both Drell–Yan data from the E615 col-

laboration and the HERA structure functions can be consistently described via the standard framework. However, we find that when they fit the leading neutron production structure function data, the teams only selected data for the two largest values of $x_{\rm L}$ (0.82 and 0.91 for H1, 0.85 and 0.94 for ZEUS, respectively), for which the data can be described within their framework. Furthermore, their result of $x_{\rm L} = 0.82$ for H1 compares unfavorably with $x_{\rm L} = 0.91$ for the small x region. Generally, it is considered that in the region where $x_{\rm L} > 0.7$, the pion-exchange process is dominant.[4] Since a smaller $x_{\rm L}$ is associated with smaller x, it is our view that the work of Ref. [5] failed to fully reflect the parton distributions in the small x region.

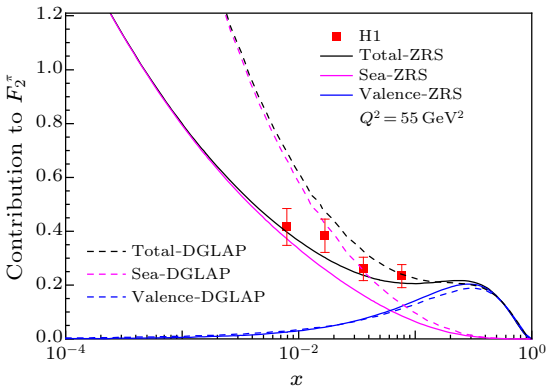

Fig. 5. Comparison of ZRS and BLFQ contributions to the F_2^π at $Q^2 = 55\,\mathrm{GeV}^2$. The data are taken from Ref. [4].

Similarly to the BLFQ-NJL model, Ding *et al.*[23,24] provided an initial valence quark distribution at $Q^2 = 0.09\,\mathrm{GeV}^2$. Compared with the BLFQ-NJL model, we find that, except for the small difference in valence quark distributions, their gluon and sea quark distributions will be virtually the same when taking the same $Q_0^2 = 0.09\,\mathrm{GeV}^2$ to evolve. Recently, Novikov *et al.* in Ref. [37] put forward a set of parton distributions at $Q^2 = 1.9\,\mathrm{GeV}^2$, containing valence quarks, sea quarks, and gluons. For comparison, we evolve the initial distributions of BLFQ [Eq. (4)] and Ding *et al.*[23,24] from $Q_0^2 = 0.09\,\mathrm{GeV}^2$ to $Q^2 = 1.9\,\mathrm{GeV}^2$ using the DGLAP and ZRS equations, respectively; the results are shown in Fig. 6. The green shadowed areas in Fig. 6 represent the region of PDFs in Ref. [37]. Figure 6(a) shows the valence quark distribution, where the black solid line is the evolution result of input distribution via Eq. (4), and the red dashed line shows the results of Ding *et al.*;[23,24] the green part is clearly different from the other two models. In Figs. 6(b) and 6(c), the corresponding sea quark and gluon distributions are given, where the solid lines and dashed lines are the results of the input of Eq. (4) and Ding *et al.*,[23,24] respectively. The solid lines and dashed lines are almost the same for the evolution of both DGLAP and ZRS equations. The reason for this is that they have the same initial Q_0^2, and similar input valence quark distributions, all the sea quarks and gluons come from the input valence quark, and this is an integral effect for them. Therefore, using the same parameter $R = 1.45\,\mathrm{GeV}^{-1}$, as given in Eq. (2), and the input distribution of Ding *et al.*,[23,24] we can fit the H1 data almost as well as in Fig. 3. However, the work in Ref. [37] features a very broad range for sea quark distributions, and we can not obtain clear information for sea quarks. If we take the center line of their distribution, the structure function is much smaller than that of the H1 data in the small x region, even with the evolution of the DGLAP equation, since most of the momentum of sea quarks is located in the large x region.

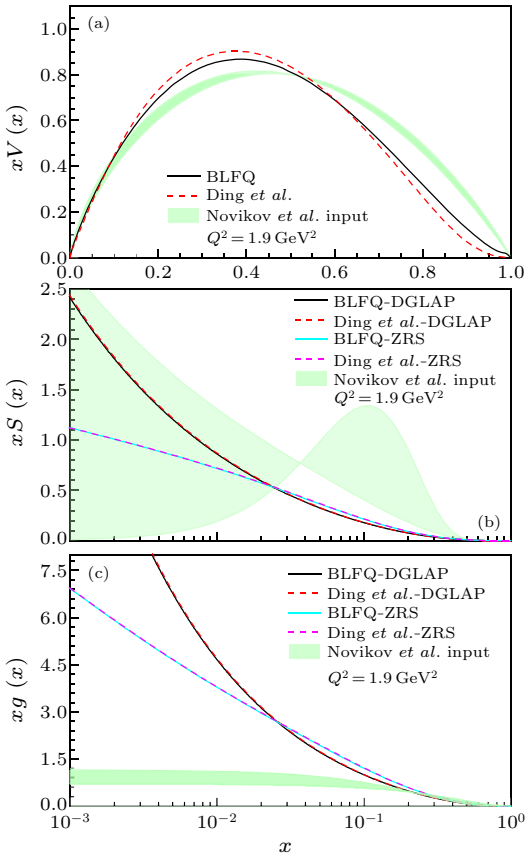

Fig. 6. Comparison between the pion PDFs obtained with the input PDFs of BLFQ[31,33] and Ding *et al.*[23,24] by using the DGLAP and ZRS equations, respectively, the green region is the input PDFs of Novikov *et al.*[37]

In Table 1, we show the momentum fraction of the pion carried by valence quark, sea quark and gluon at scale $Q^2 = 1.9\,\mathrm{GeV}^2$. We observe that the results of the input distribution of BLFQ[31] and Ding *et al.*[23,24] are almost the same, while the results of Novikov *et al.*[37] are very different from the other two models; in particular, the gluon momentum is much smaller. It should be noted that the momentum in this table may be slightly different from the original work, since different works adopt different strategies

for the running coupling constant α_s, we adopt the α_s as given in Ref. [34]. In addition, Eq. (2) is momentum conserved for different partons, so the higher-twist effect will not affect the total momentum of different partons, it will only change the shape of sea quark and gluon distributions. Thus, in Table 1, the momentum result of the DGLAP equation is the same as that of the ZRS equation for separate input distributions.

Table 1. Momentum fractions of the pion, carried by the valence quark, sea quark and gluon at $Q^2 = 1.9\,\mathrm{GeV}^2$.

	$\langle xV\rangle$	$\langle xS\rangle$	$\langle xg\rangle$
BLFQ (DGLAP)	0.532	0.068	0.400
BLFQ (ZRS)	0.532	0.068	0.400
Ding *et al.* (DGLAP)	0.530	0.069	0.401
Ding *et al.* (ZRS)	0.530	0.069	0.401
Novikov *et al.* input	0.56	0.21	0.23

Although there are many works relating to pion PDFs, we are still not very clear about the sea quark and gluon distributions of the pion. As well as the leading neutron production experiment in HERA,[3,4] we need further data in order to analyze the higher-twist effect in the small x region. It is hoped that the planned COMPASS experiment[50] and the Tagged DIS experiment at the Jefferson Laboratory[51] may detect the distribution function of pions in the smaller x region, and may also reveal the effects of twist-4 and even higher-twist parton correlations.

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