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Effects of Quantum Noise on Quantum Approximate Optimization Algorithm

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The quantum-classical hybrid algorithm is a promising algorithm with respect to demonstrating the quantum advantage in noisy-intermediate-scale quantum (NISQ) devices. When running such algorithms, effects due to quantum noise are inevitable. In our work, we consider a well-known hybrid algorithm, the quantum approximate optimization algorithm (QAOA). We study the effects on QAOA from typical quantum noise channels, and produce several numerical results. Our research indicates that the output state fidelity, i.e., the cost function obtained from QAOA, decreases exponentially with respect to the number of gates and noise strength. Moreover, we find that when noise is not serious, the optimized parameters will not deviate from their ideal values. Our result provides evidence for the effectiveness of hybrid algorithms running on NISQ devices.

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In recent years, there have been rapid developments towards building quantum computers. The expectation is that these will feature a quantum chip containing hundreds of qubits—in essence, a noisy-intermediate-scale quantum (NISQ)[1] device. However, some quantum algorithms will require enormous numbers of qubits, as well as a quantum circuit having a long circuit depth, to realize quantum supremacy.[2] When running complex quantum circuits, quantum error correction codes need to be executed, but this will further increase the required number of qubits and quantum circuit complexity. For example, to factor 2048-bit RSA integers requires 20 billion noisy qubits.[3] This requirement far exceeds the capacity of current quantum chips, and this is likely to remain the case for the near future.

One meaningful problem is to find a quantum algorithm capable of realizing quantum supremacy on NISQ devices. A quantum-classical hybrid algorithm shows promise in this regard. In recent years, several quantum-classical hybrid algorithms have been proposed,[4–12] a few of which have been demonstrated experimentally on real quantum chips.[5,13–20] In NISQ devices, quantum noise is inevitable, and quantum algorithms running on NISQ devices need to be noise tolerant. However, little research has been conducted to date to study how quantum noise affects the performance of these quantum-classical hybrid algorithms. In Ref. [21], Sharma *at al.* studied the effects of quantum noise on variational quantum compiling[22–24] and proposed optimal parameter resilience (OPR).

In this Letter, we focus on a specific quantum-classical hybrid algorithm—the quantum approximate optimization algorithm (QAOA).[25–29] There is currently limited research concerning the effects of quantum noise on the QAOA. In our work, we study specifically the effects of quantum noise on its performance. We analyze the effects of quantum noise on various aspects of QAOA, including the fidelity of its output state, its cost function, and its parameter optimization. We find that QAOA is noise tolerant, although the QAOA parameter space is somewhat flattened by noise. To demonstrate our conclusion, we choose three different quantum noise channels, testing the effects of each on the QAOA by means of numerical simulations. In the following, we show describe our work in further detail.

In our work, we consider quantum noise, for which the Kraus operators are written as

$$K = \{a_0 I, a_1 K_1, a_2 K_2, \ldots, a_s K_s\}, \tag{1}$$

where each a_i represents a real number, and I is the identity operator. In our numerical simulations, we choose three quantum noise channels: dephasing, bit flip, and depolarizing channels. The Kraus operators of each are given as follows.

Dephasing noise:

$$K_{\text{dephasing}} = \{\sqrt{1-p}I, \sqrt{p}Z\}. \tag{2}$$

Bit flip noise:

$$K_{\text{bitflip}} = \{\sqrt{1-p}I, \sqrt{p}X\}. \tag{3}$$

Depolarizing noise:

$$K_{\text{depolarizing}} = \left\{\sqrt{1-\frac{3}{4}p}I, \frac{\sqrt{p}}{2}X, \frac{\sqrt{p}}{2}Y, \frac{\sqrt{p}}{2}Z\right\}. \tag{4}$$

Here X, Y, Z represent the Pauli X, Y, Z matrices, respectively. These three channels have one parameter,

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p. We choose p to be in the range $[0.0001, 0.02]$; more specifically, we focus on a smaller interval, and therefore utilize the exponential interval,

$$p_i = 0.0001 \times 200^{0.1\times i},\ i = 0, 1, \ldots, 10. \tag{5}$$

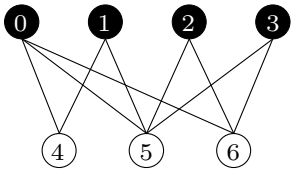

Fig. 1. Max-Cut graph.

Table 1. Edge weight table.

Edge	Weight	Edge	Weight
0,4	0.73	2,5	0.88
0,5	0.33	2,6	0.58
0,6	0.50	3,5	0.67
1,4	0.69	3,6	0.43
1,5	0.36		

We test the effects of this kind of quantum noise on QAOA performance using a treatment of the maximum cut (Max-Cut) problem in graph theory. The specific Max-Cut problem we treat corresponds to finding the maximum cut in an undirected graph. The undirected graph information of the selected Max-Cut problem is shown in Fig. 1 and Table 1.

Firstly, we analyze the effects of quantum noise on the state fidelity of the output state of a QAOA quantum circuit. As shown in the Supplemental Material, for a specific QAOA quantum circuit, $U(\boldsymbol{\gamma}, \boldsymbol{\beta})$; if there is no quantum noise, the output quantum state is

$$|\phi_{\text{out}}^{\text{ideal}}\rangle = U(\boldsymbol{\gamma}, \boldsymbol{\beta})|\phi_{\text{in}}\rangle. \tag{6}$$

We use N to denote the number of quantum gates in the QAOA quantum circuit. The noisy output quantum state is expressed as

$$\rho_{\text{out}}^{\text{noise}} = \sum_{i_1, i_2, \ldots, i_N} p_{1,i_1} p_{2,i_2} \cdots p_{N,i_N} \cdot |\phi_{i_1,i_2,\ldots,i_N}\rangle\langle\phi_{i_1,i_2,\ldots,i_N}|, \tag{7}$$

where $i_j = 0, 1, 2, \ldots, s$, p_{k,i_j} signifies the probability that the i_jth noise Kraus operator acts when the quantum state evolves to the kth quantum gate. The fidelity between $|\phi_{\text{out}}^{\text{ideal}}\rangle$ and $\rho_{\text{out}}^{\text{noise}}$ is

$$F = \langle\phi_{\text{out}}^{\text{ideal}}|\rho_{\text{out}}^{\text{noise}}|\phi_{\text{out}}^{\text{ideal}}\rangle. \tag{8}$$

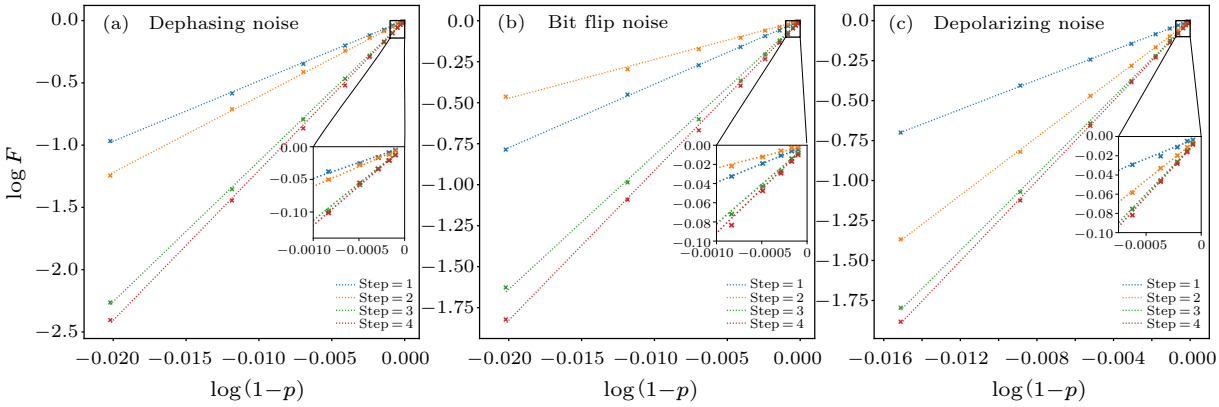

Fig. 2. Effect of quantum noise on output state fidelity. The horizontal axis gives the noise parameter $\log(1-p)$, the vertical axis is the output state fidelity. QAOA step $n = 1$–4, and the corresponding quantum gate numbers of the QAOA quantum circuit are $N = [59, 111, 163, 215]$, in which one double quantum gate is regarded as two quantum gates. Panels (a), (b), and (c) represent dephasing noise, bit flip noise, and depolarizing noise, respectively. The simulated δ and the corresponding variance are shown in Table 2.

Table 2. Parameter δ's fitting result.

Noise	Step	δ	Variance
Dephasing noise	1	0.820929	2.08951×10^{-5}
	2	0.549195	7.22086×10^{-6}
	3	0.692255	5.32020×10^{-6}
	4	0.559736	9.267858×10^{-6}
Bit flip noise	1	0.656347	5.06738×10^{-6}
	2	0.212916	8.12004×10^{-6}
	3	0.502418	1.86095×10^{-5}
	4	0.424866	8.76859×10^{-6}
Depolarizing noise	1	0.783912	4.44939×10^{-6}
	2	0.819920	6.71699×10^{-6}
	3	0.734511	8.06321×10^{-6}
	4	0.582604	2.22129×10^{-6}

The proportion of the ideal output in relation to noisy output is $p_{1,0}p_{2,0}\cdots p_{N,0} = (1-p)^N$, where $1 - p = a_0^2$. Except for $|\phi_{0,0,\ldots,0}\rangle$, only a small part of $|\phi_{i_1,i_2,\ldots,i_N}\rangle$ has a value for $|\langle\phi_{\text{out}}^{\text{ideal}}|\phi_{i_1,i_2,\ldots,i_N}\rangle|^2$, which is significantly larger than 0. We then perform a numerical simulation, finding that the output state fidelity can be approximated with the equation

$$F \approx (1-p)^{\delta N}, \tag{9}$$

for which δ is a constant, dependent on the quantum circuit architecture and quantum noise model. The numerical simulation results are shown in Fig. 2.

We also consider a further fidelity, i.e., the overlap of QAOA output state, and the exact solution of the Max-Cut problem; here, we use G to represent this

fidelity, with G defined as

$$G = \sum_{|\psi\rangle \in \boldsymbol{S}} \langle \psi | \rho_{\text{out}}^{\text{noise}} | \psi \rangle, \tag{10}$$

where $\boldsymbol{S} = \{|0001111\rangle, |1110000\rangle\}$, which is the solution set of the Max-Cut problem we chose. The effects of quantum noise on G are similar to the case of F, and G can be expressed approximately via the equation

$$G \approx G_0 (1-p)^{\eta N}, \tag{11}$$

where η is a constant, and G_0 represents the overlap of noise-free output state and the exact solution of the Max-Cut problem. The numerical simulation results are shown in Fig. 3.

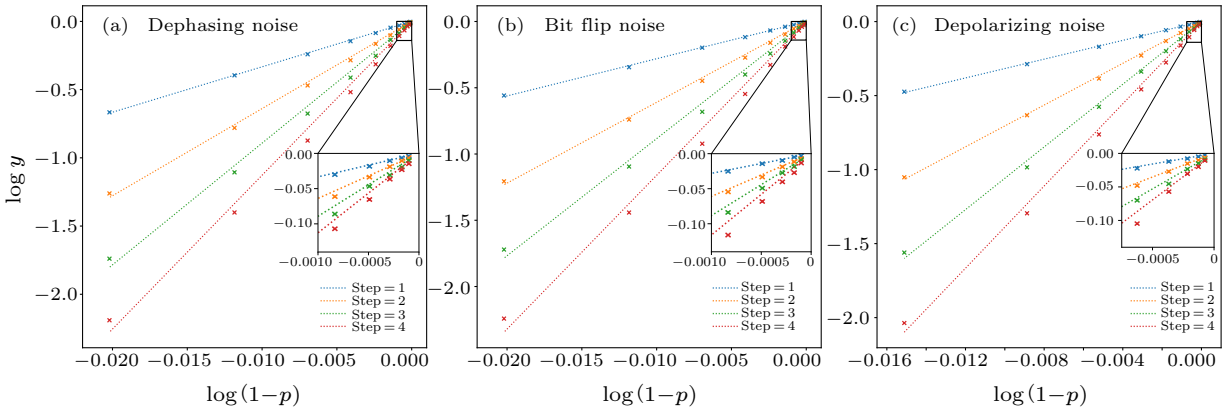

Fig. 3. Effect of quantum noise on G. The horizontal axis gives the noise parameter $\log(1-p)$, and the vertical axis is $y = \log(G/G_0)$. The QAOA step n = 1–4, and the corresponding G_0 is $G_0 = [0.206220, 0.503128, 0.710670, 0.858298]$. Panels (a), (b), and (c) represent dephasing noise, bit flip noise, and depolarizing noise, respectively. The simulated η and the corresponding variance are shown in Table 3.

Table 3. Parameter η's fitting result.

Noise	Step	δ	Variance
Dephasing noise	1	0.564126	1.07561×10^{-5}
	2	0.575603	3.85836×10^{-5}
	3	0.547389	8.08784×10^{-5}
	4	0.524444	8.93222×10^{-5}
Bit flip noise	1	0.475950	1.16587×10^{-5}
	2	0.549503	3.24104×10^{-5}
	3	0.542246	8.42643×10^{-5}
	4	0.539797	1.29240×10^{-4}
Depolarizing noise	1	0.537495	8.16338×10^{-6}
	2	0.635128	1.77014×10^{-5}
	3	0.649509	4.93053×10^{-5}
	4	0.645789	7.22830×10^{-5}

Secondly, we analyze the effects of quantum noise on the QAOA cost function defined in the following. In the Max-Cut problem, the problem Hamiltonian H_p is written as

$$H_p = \sum_{i,j} -C_{ij} \frac{1 - Z_i Z_j}{2}. \tag{12}$$

In our work, we redefined H_p; specifically, we discarded its constant term and multiplied the result by 2, yielding

$$H_p = \sum_{i,j} C_{ij} Z_i Z_j. \tag{13}$$

The performance and results of the QAOA are not influenced. One feature of the redefined H_p is that its statistical average $\langle H_p \rangle$ in the entire space satisfies

$$\langle H_p \rangle = \text{Tr}\left(\frac{H_p}{2^m}\right) = 0, \tag{14}$$

where m denotes the number of qubits.

For a specific QAOA quantum circuit $U(\boldsymbol{\gamma}, \boldsymbol{\beta})$ with N quantum gates, the cost function, if there is no quantum noise, is as follows:

$$f(\boldsymbol{\gamma}, \boldsymbol{\beta})^{\text{ideal}} = \langle \phi_{\text{out}}^{\text{ideal}} | H_p | \phi_{\text{out}}^{\text{ideal}} \rangle. \tag{15}$$

However, when quantum noise K acts on the QAOA quantum circuit, the output state density matrix changes. The noisy cost function is then written as

$$f(\boldsymbol{\gamma}, \boldsymbol{\beta})^{\text{noise}} = \sum_{i_1, i_2, \ldots, i_N} p_{1,i_1} p_{2,i_2} \cdots p_{N,i_N} \langle \phi_{i_1,i_2,\ldots,i_N} | H_p | \phi_{i_1,i_2,\ldots,i_N} \rangle. \tag{16}$$

We then perform further numerical simulations, finding that the noisy cost function can be approximated via the equation

$$f(\boldsymbol{\gamma}, \boldsymbol{\beta})^{\text{noise}} \approx (1-p)^{\alpha N} f(\boldsymbol{\gamma}, \boldsymbol{\beta})^{\text{ideal}}, \tag{17}$$

for which α is a constant, dependent on the quantum circuit architecture and quantum noise model. The numerical simulation results are shown in Fig. 4.

Finally, we test the effects of quantum noise on the QAOA optimization process. We consider a reasonable assumption: When $(1-p)^{\alpha N} > C$, where C is a constant, the optimal QAOA quantum circuit parameters are not changed.

We use the vanilla gradient descent algorithm to optimize the QAOA quantum circuit parameters. In the selected Max-Cut problem, we find that when the initial values of parameters $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ are close to 0, we obtain the best results after parameter optimization. As such, in our work, the initial values of $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ are randomly chosen in the range $[-0.01, 0.01]$.

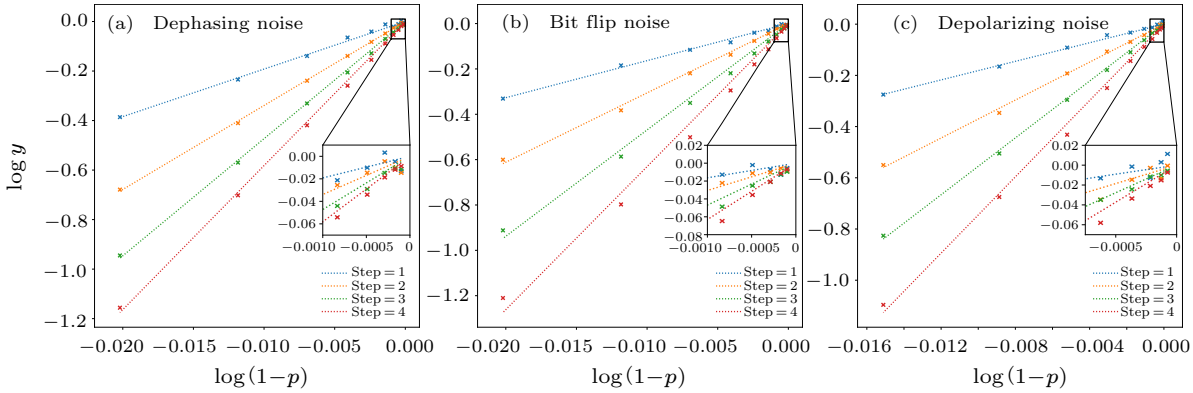

Fig. 4. Data fitting showing the effects of quantum noise on the QAOA cost function in the test. Here, p is the noise parameter, and $y = \frac{f(\boldsymbol{\gamma},\boldsymbol{\beta})^{\text{noise}}}{f(\boldsymbol{\gamma},\boldsymbol{\beta})^{\text{ideal}}}$. Panels (a), (b), and (c) correspond to dephasing, bit flip, and depolarizing quantum noise, respectively. The simulated α and the corresponding variance are shown in Table 4.

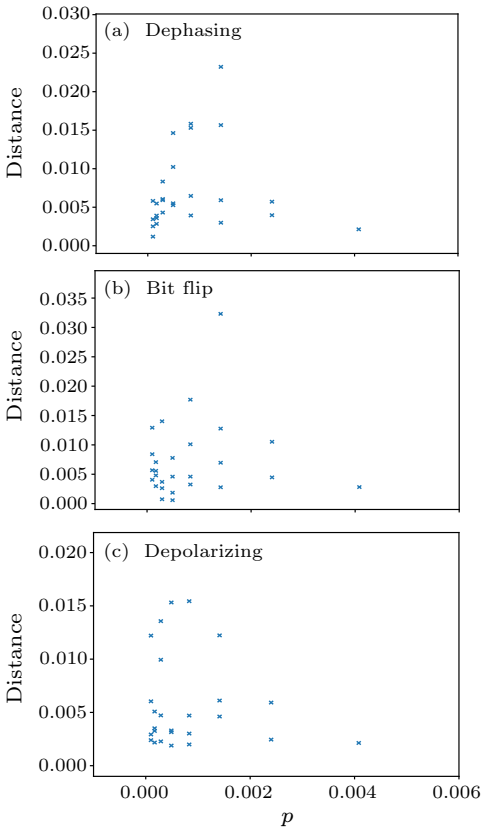

Fig. 5. Distance between noisy optimized parameters and ideal optimized parameters. Here, we select points which satisfy $(1-p)^{\alpha N} > 0.9$, where α is listed in Table 4. Panels (a), (b), and (c) correspond to dephasing, bit flip, and depolarizing quantum noise, respectively.

We compute the distance between the ideal optimized parameters and the noisy optimized parameters. We use the root mean square of the Euclidean distance to describe the distance between two parameter sequences. Explicitly, we have the distance $\mathcal{D}$:

$$\mathcal{D} = \sqrt{\frac{|\boldsymbol{\gamma}^{\text{noise}} - \boldsymbol{\gamma}^{\text{ideal}}|^2 + |\boldsymbol{\beta}^{\text{noise}} - \boldsymbol{\beta}^{\text{ideal}}|^2}{2n}}, \quad (18)$$

where $2n$ represents the parameter number of the QAOA quantum circuit. The results are shown in Fig. 5. The optimized parameters under conditions of quantum noise are close to those of ideal optimized parameters. From this, we infer that if we set sufficiently large execution times for the QAOA quantum circuit, the optimized parameters under conditions of quantum noise will be the same as those with ideal optimized parameters.

Table 4. Parameter α's fitting result.

Noise	Step	α	Variance
Dephasing noise	1	0.325484	3.61151×10^{-5}
	2	0.305575	4.13948×10^{-6}
	3	0.290244	4.21576×10^{-6}
	4	0.270603	5.36495×10^{-6}
Bitflip noise	1	0.277000	2.70421×10^{-5}
	2	0.275399	1.27296×10^{-5}
	3	0.287741	2.57187×10^{-5}
	4	0.293386	4.66970×10^{-5}
Depolarizing noise	1	0.307477	4.10423×10^{-5}
	2	0.333806	1.69716×10^{-5}
	3	0.340608	6.17305×10^{-6}
	4	0.346604	2.39438×10^{-5}

In summary, we have investigated the effects of a class of quantum noise channels on QAOA performance, and arrived at the following conclusions: (1) For a specific QAOA quantum circuit, the relations between quantum noise parameter p and the output state fidelity F and G are $F = (1-p)^{\delta N}$ and $G = G_0(1-p)^{\eta N}$. (2) For a specific QAOA quantum circuit, the relationship between quantum noise parameter p and the cost function is $\frac{f^{\text{noise}}}{f^{\text{ideal}}} = (1-p)^{\alpha N}$. (3) QAOA is a noise tolerant algorithm, when $(1-p)^{\alpha N}$ is large enough, for both noisy and ideal QAOA cost functions, the optimized QAOA parameters being virtually the same. To support these conclusions, we evaluated the effects of dephasing, bit flip, and depolarizing quantum noise channels on the QAOA output, the test results being consistent with our conclusions.

Our conclusions extend to other problems. The QAOA quantum circuit is a kind of multi-layer parameterized quantum circuit, and hence our conclusions apply equally to such circuits. Some unsolved problems remain. However. For example, our method is only suitable for certain low quantum noise chan-

nels, and is not applicable to arbitrary types. The effects of quantum noise on other NISQ quantum algorithms are also an open problem. In the future, we will extend our method to account for these considerations, and investigate the effects of quantum noise on other NISQ quantum algorithms.

Notes: During the review of our manuscript, Marshall *et al.*[30] followed our work and offered new conclusions, proposing a more reasonable relationship between quantum noise parameter p and state fidelity and cost function: Eqs. (9) and (17) are special cases of their conclusions.

References

[1] Preskill J 2018 *Quantum* **2** 79
[2] Preskill J 2012 arXiv:1203.5813[quant-ph]
[3] Gidney C and Ekerå M 2019 arXiv:1905.09749[quant-ph]
[4] Mitarai K, Negoro M, Kitagawa M and Fujii K 2018 *Phys. Rev. A* **98** 032309
[5] Havlíček V, Córcoles A D, Temme K, Harrow A W, Kandala A, Chow J M and Gambetta J M 2019 *Nature* **567** 209
[6] Lloyd S and Weedbrook C 2018 *Phys. Rev. Lett.* **121** 040502
[7] Dallaire-Demers P L and Killoran N 2018 *Phys. Rev. A* **98** 012324
[8] Romero J, Olson J P and Aspuru-Guzik A 2017 *Quantum Sci. Technol.* **2** 045001
[9] Amin M H, Andriyash E, Rolfe J, Kulchytskyy B and Melko R 2018 *Phys. Rev. X* **8** 021050
[10] Farhi E and Neven H 2018 arXiv:1802.06002[quant-ph]
[11] Schuld M and Killoran N 2019 *Phys. Rev. Lett.* **122** 040504
[12] Schuld M, Bocharov A, Svore K M and Wiebe N 2020 *Phys. Rev. A* **101** 032308
[13] Schuld M, Fingerhuth M and Petruccione F 2017 *Europhys. Lett.* **119** 60002
[14] Hamilton K E, Dumitrescu E F and Pooser R C 2019 *Phys. Rev. A* **99** 062323
[15] Zhu D, Linke N M, Benedetti M, Landsman K A, Nguyen N H, Alderete C H, Perdomo-Ortiz A, Korda N, Garfoot A and Brecque C *et al.* 2019 *Sci. Adv.* **5** eaaw9918
[16] Grant E, Benedetti M, Cao S, Hallam A, Lockhart J, Stojevic V, Green A G and Severini S 2018 *npj Quantum Inf.* **4** 65
[17] Tacchino F, Macchiavello C, Gerace D and Bajoni D 2019 *npj Quantum Inf.* **5** 26
[18] Rocchetto A, Aaronson S, Severini S, Carvacho G, Poderini D, Agresti I, Bentivegna M and Sciarrino F 2019 *Sci. Adv.* **5** eaau1946
[19] Ding Y, Lamata L, Sanz M, Chen X and Solano E 2019 *Adv. Quantum Technol.* **2** 1800065
[20] Benedetti M, Garcia-Pintos D, Perdomo O, Leyton-Ortega V, Nam Y and Perdomo-Ortiz A 2019 *npj Quantum Inf.* **5** 45
[21] Sharma K, Khatri S, Cerezo M and Coles P J 2020 *New J. Phys.* **22** 043006
[22] Khatri S, LaRose R, Poremba A, Cincio L, Sornborger A T and Coles P J 2019 *Quantum* **3** 140
[23] Jones T and Benjamin S C 2018 arXiv:1811.03147 [quant-ph]
[24] Heya K, Suzuki Y, Nakamura Y and Fujii K 2018 arXiv:1810.12745 [quant-ph]
[25] Farhi E, Goldstone J and Gutmann S 2014 arXiv:1411.4028 [quant-ph]
[26] Otterbach J S, Manenti R, Alidoust N, Bestwick A, Block M, Bloom B, Caldwell S, Didier N, Fried E S, Hong S, Karalekas P, Osborn C B, Papageorge A, Peterson E C, Prawiroatmodjo G, Rubin N, Ryan C A, Scarabelli D, Scheer M, Sete E A, Sivarajah P, Smith R S, Staley A, Tezak N, Zeng W J, Hudson A, Johnson B R, Reagor M, da Silva M P and Rigetti C 2017 arXiv:1712.05771[quant-ph]
[27] Guerreschi G G and Matsuura A Y 2019 *Sci. Rep.* **9** 9
[28] Crooks G E 2018 arXiv:1811.08419 [quant-ph]
[29] Zhou L, Wang S T, Choi S, Pichler H and Lukin M D 2020 *Phys. Rev. X* **10** 021067
[30] Marshall J, Wudarski F, Hadfield S and Hogg T 2020 *IOP SciNotes* **1** 025208

Supplementary Material: Effects of Quantum Noise on Quantum Approximate Optimization Algorithm

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I. QAOA

The QAOA was developed by Farhi, Goldstone, and Gutmann[1]. It gives a way to solve approximately in polynomial time combinatorial optimization problems such as the Max-Cut problem, the traveling salesman problem, and the 3-SAT problem .

The QAOA can be regarded as a simplified version of an adiabatic evolution, which involves the continuous evolution of a quantum state from a ground state of a Hamiltonian to a ground state of another Hamiltonian. As the QAOA is discrete, it drives the ground state of one Hamiltonian to the ground state of another Hamiltonian in discrete steps. In the respect, the QAOA is an approximation algorithm. In general, the output state of the QAOA is just close to the ground state of the target Hamiltonian. The QAOA quantum circuit has a parameterization. Changing the parameters of the circuit adjusts the direction of each step of the QAOA. In particular, the QAOA quantum circuit can be written

$$U(\vec{\gamma},\vec{\beta}) = \prod_{j=1}^{n} e^{-iH_d\beta_j} e^{-iH_p\gamma_j}, \tag{1}$$

where n denotes the QAOA step number, and H_p the problem Hamiltonian. For a specific problem, the ground state of H_p is the solution of the problem. H_d is the drive Hamiltonian of the QAOA, the initial state of the QAOA being the ground state of H_d. Generally, we define H_d in the form

$$H_d = \sum_i -X_i, \tag{2}$$

where X represents the Pauli X matrix. The initial state of the QAOA is then $|\phi_{in}\rangle = |+\rangle^{\otimes m}$, m being the qubit number of the QAOA quantum circuit, and $\vec{\gamma}$ and $\vec{\beta}$ the two $[n,1]$ parameter vectors. The purpose of the QAOA is to find the optimal values of $\vec{\gamma}$ and $\vec{\beta}$ that ensures the output quantum state is closest to the ground state of the problem Hamiltonian H_p. Generally, we use a classical optimization algorithm to find the optimal $\vec{\gamma}$ and $\vec{\beta}$. The cost function is the expectation of H_p and may be written

$$f(\vec{\gamma},\vec{\beta}) = \langle\phi_{in}|U^{\dagger}(\vec{\gamma},\vec{\beta})H_pU(\vec{\gamma},\vec{\beta})|\phi_{in}\rangle. \tag{3}$$

When solving specific problems, the whole execution of the QAOA comprises five steps:

(1) Construct problem Hamiltonian H_p. The ground state of H_p is the solution of the problem.

(2) Use H_p, H_d to construct the QAOA quantum circuit—the initial values of $\vec{\gamma}$ and $\vec{\beta}$ may be chosen randomly.

(3) Execute QAOA quantum circuit multiple times and compute the cost function $f(\vec{\gamma},\vec{\beta}) = \langle H_p\rangle$.

(4) Optimize $\vec{\gamma}$ and $\vec{\beta}$ to minimize the cost function $f(\vec{\gamma},\vec{\beta})$ with the classical optimization algorithm.

(5) Execute the optimized QAOA quantum circuits multiple times—the optimal result is the solution of the problem.

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II. SIMULATION METHOD

In our numerical simulations, we use a noisy quantum virtual machine (QVM) in the quantum programming architecture for the NISQ-device application (QPanda) as our noisy quantum simulator[2]. QPanda's noisy QVM supports all kinds of quantum noise models. Users can also define their own quantum noise model. QPanda also contains the variational quantum network (VQNet)—a framework to construct quantum-classical hybrid neural networks[3]. We use VQNet to realize the QAOA.

A. Noisy Simulator

First, we introduce the method of QPanda's noisy QVM. In noisy quantum circuits, the quantum state is a mixed state and represented by a density matrix. In QPanda's noisy QVM, we still use the state vector to represent the quantum state and combine the Monte Carlo method to simulate mixed-state evolution under noisy quantum operations[4]. The specific method is as follows:

(1) Assume the input state is a pure state denoted by $|\phi\rangle$ and U a unitary quantum gate; the quantum noise is represented by Kraus operators $K = \{K_i\}, i = 1, 2, ..., s$.

(2) Compute $p_i = \langle\phi|K_i^\dagger K_i|\phi\rangle, i = 1, 2, ..., s$. We note that $\sum_i p_i = 1$.

(3) Generate a uniformly distributed random number r in the range $[0, 1)$, then find l that satisfies

$$\sum_{i=1}^{l-1} p_i \leq r \leq \sum_{i=1}^{l} p_i. \tag{4}$$

Here we assume $\sum_{i=1}^{0} p_i = 0$.

(4) The expression for the evolution of state $|\phi\rangle$ is

$$|\phi_l\rangle \to U \frac{1}{\sqrt{p_l}} K_l|\phi\rangle. \tag{5}$$

(5) Repeat procedure (2) $\sim$ (4) M times. M_l is the number of times that K_l was selected. When M is large enough, we know that $M_l \approx M p_l$, and hence the output quantum state ρ becomes

$$\rho \approx \sum_i \frac{M_l}{M} |\phi_l\rangle\langle\phi_l|. \tag{6}$$

The noisy QVM realized by this method needs to run the quantum circuit multiple times. The density matrix of the output state may then be computed approximately from the statistical distribution of the results of multiple runs.

B. VQNet

We use VQNet to realize QAOA. VQNet supports ideal and noisy QVM. The flow chart of QAOA is shown in Fig. S1. $\vec{\gamma}$ and $\vec{\beta}$ are QAOA quantum circuit parameters, "Variational Quantum Circuit" represents the QAOA quantum circuit, "Quantum Operator" is a basic operator in VQNet, and the inputs of "Quantum Operator" are the "Variational Quantum Circuit" and the "Graph Hamiltonian". The output is the expectation of the input Hamiltonian after running the "Variational Quantum Circuit". The VQNet supports back propagation to get the derivatives of the cost function with respect to $\vec{\gamma}$ and $\vec{\beta}$. We then use the gradient descent algorithm to optimize parameters $\vec{\gamma}$ and $\vec{\beta}$.

[1] E. Farhi, J. Goldstone, and S. Gutmann (2014), arXiv: 1411.4028, URL http://arxiv.org/abs/1411.4028.
[2] Z.-Y. Chen and G.-P. Guo (2019), arXiv: 1901.08340, URL http://arxiv.org/abs/1901.08340.
[3] Z.-Y. Chen, C. Xue, S.-M. Chen, and G.-P. Guo (2019), arXiv: 1901.09133, URL http://arxiv.org/abs/1901.09133.
[4] *Noise and Quantum Computation — pyQuil 2.8.0 documentation*, URL https://pyquil.readthedocs.io/en/latest/noise.html.

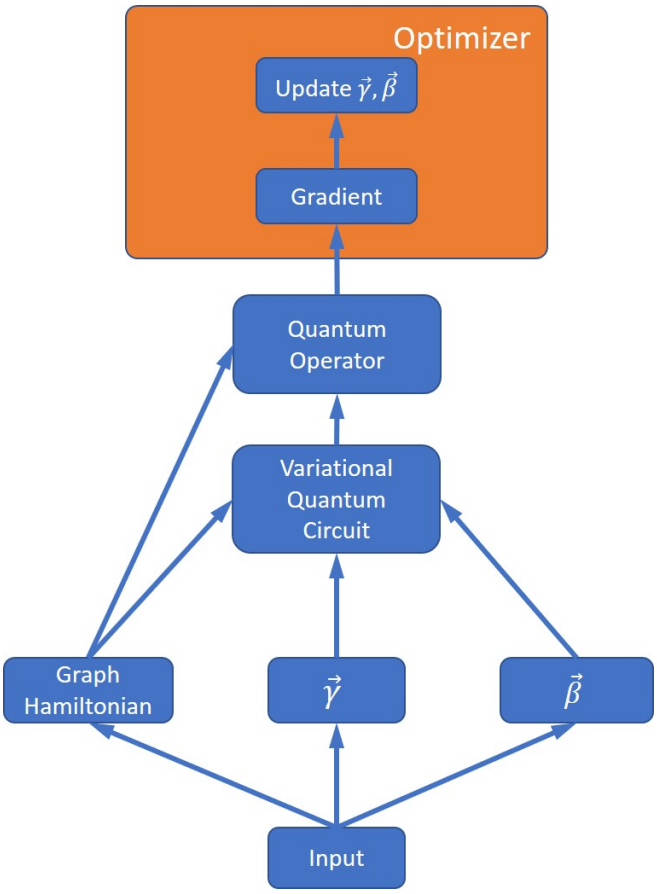

FIG. S1: QAOA Flow Chart