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Effect of Electron Correlation and Breit Interaction on Energies, Oscillator Strengths, and Transition Rates for Low-Lying States of Helium

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The transition energies, E1 transitional oscillator strengths of the spin-allowed as well as the spin-forbidden and the corresponding transition rates, and complete M1, E2, M2 forbidden transition rates for $1s^2$, $1s2s$, and $1s2p$ states of He I, are investigated using the multi-configuration Dirac–Hartree–Fock method. In the subsequent relativistic configuration interaction computations, the Breit interaction and the QED effect are considered as perturbation, separately. Our transition energies, oscillator strengths, and transition rates are in good agreement with the experimental and other theoretical results. As a result, the QED effect is not important for helium atoms, however, the effect of the Breit interaction plays a significant role in the transition energies, the oscillator strengths and transition rates.

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In the stellar corona and nebula environment, the line intensity ratios are sensitive to the radiation and collision processes, which are conformed as tools to diagnose the density.[1,2] The abundance of helium, oxygen and neon can determine the formation of bright planetary nebulae. Helium, as the simplest multi-electron atom and the second most abundant element in the universe, has always been one of the most concerning elements in plasma physics, atom physics, and astrophysics.[3–15] For instance, its energy level interval, transition probability, and oscillator strength are of great application to the manipulation of atoms or ions in quantum information, the extraction of proton radius, and the determination of fine structure constant.[7,9,16] Despite the great progress has been made in experimental and theoretical research, there are still unknown parameters such as the "proton radius puzzle",[17,18] which makes the precise measurement of atoms particularly significant. In the field of precision spectroscopy, the study of fine structure and transition process can be used to determine the radius of the nuclear charge and to test quantum electrodynamics (QED) theory.[11,19–23]

So far, there are many works on the fine structure, transition probability, and oscillator strength among $1s^2$, $1s2s$, and $1s2p$ states of the helium atom, such as the research of transition frequencies,[10,20,24,25] Shannon entropy[26] and transition rates,[9,27–30] which have improved the measurement of the fine structure constant[31,32] and the nuclear charge radius.[33] In 2008, Bo *et al.* merely obtained the splitting of fine energy levels $2^3P_1^o$–$2^3P_0^o$ and $2^3P_2^o$–$2^3P_0^o$, using the multi-configuration Dirac–Fock (MCDF) method.[34] A few years later, the fine structure and transition including the relativistic effect and the influence of finite nuclear mass were further studied by Morton *et al.*[27,29] using the relativistic many-body perturbation theory (RMBPT). In 2020, the electric dipole (E1) oscillator strengths of $1s2p$ states were investigated by Głowacki[30] using the configuration interaction Dirac–Fock (CIDF) approach. Although there are a lot of works[4,6,29] about magnetic quadrupole (M2) transition, it is still meaningful to study systematically the electric quadrupole (E2) and M2 transitions.

Furthermore, the forbidden magnetic dipole (M1) transition 2^3S_1–1^1S_0 was considered and its rate was measured to be $1.10 \times 10^{-4}\,\mathrm{s}^{-1}$ in the experiment of Woodworth *et al.*[3] It differs from the result $1.73 \times 10^{-4}\,\mathrm{s}^{-1}$ acquired by Lin *et al.*[35] using the relativistic random-phase approximation (RRPA) method. Indelicato,[36] Johnson *et al.*[4] and Łach *et al.*[6] improved the accuracy of transition rate by considering sufficient electronic correlation effect using the MCDF, RMBPT, and QED theory, their results were 1.279×10^{-4}, 1.266×10^{-4} and $1.272426 \times 10^{-4}\,\mathrm{s}^{-1}$, respectively, and had small differences compared with the experimental result.[3] Another extremely weak M1 2^1S_0–2^3S_1 transition has also attracted much attention. In 1977, Lin *et al.* studied the radiative transitions of the He I isoelectronic sequence with RRPA. In their work the transition 2^1S_0–2^3S_1 rate was predicted to be $1.51 \times 10^{-7}\,\mathrm{s}^{-1}$.[35] However, Baklanov and Denisov calculated this forbidden transition rate as $6.11 \times 10^{-8}\,\mathrm{s}^{-1}$ (Ref. [5]) in 1997 by using the correlated nonrelativistic wave functions discussed

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the detail in Ref. [37]. Until 2011, Rooij *et al.* obtained the transition rate as $9.1 \times 10^{-8}\,\mathrm{s}^{-1}$ using the 1557 nm laser to directly measure the forbidden transition 2^1S_0–2^3S_1 of He I.[8] The experimental result is different from the earlier theoretical results,[5,35] which arouses the interest to further calculate the transition rate.

In the present investigation, the transitions energies, E1 transitional oscillator strengths of the spin-allowed as well as the spin-forbidden and the corresponding transition rates, and complete M1, E2, M2 forbidden transition rates for He I $1s^2$, $1s2s$ and $1s2p$ states are investigated using the GRASP2K program[38–40] based on the multi-configuration Dirac–Hartree–Fock (MCDHF) method. Particularly, the influences of Breit interaction and QED effects are discussed in detail for each result. The results show that the effect of Breit interaction on the oscillator strengths and transition rates for E1 transition of spin-forbidden are greater than that of the spin-allowed E1 transition. In addition, its effect also plays a significant role in the M1, E2 transitions of $1s2p$ ($^3P_{2,1,0,}$ and $^1P_1$) states.

Theoretical Method. In the MCDHF method, the atomic energy level and wave functions are obtained by solving the Dirac equation. The Dirac–Coulomb (DC) Hamiltonian is

$$H^{\mathrm{DC}} = \sum_{i=1}^{N}\Big[c\boldsymbol{\alpha}_i\cdot\boldsymbol{p}_i + (\beta_i-1)c^2 + V_{\mathrm{nuc}}(r_i)\Big] + \sum_{i>j}\frac{1}{r_{ij}}, \tag{1}$$

where $V_{\mathrm{nuc}}(r_i)$ represents the interaction between the nucleus and electrons, $\boldsymbol{\alpha}_i$ and β_i are the Dirac matrices. The wave function of the atom is denoted by an atomic state wave function (ASF) $|\Gamma PJM\rangle$, which is obtained by the linear combination of the configuration wave functions (CSFs) $|\gamma_i PJM\rangle$ with the same parity P and angular quantum numbers J, M:

$$|\Gamma PJM\rangle = \sum_{i=1}^{n_c} c_{i\Gamma}|\gamma_i PJM\rangle. \tag{2}$$

Here $c_{i\Gamma}$ indicates the configuration mixing coefficients meaning the correlation effect of the CSFs. The CSF is antisymmetric functions obtained by the product of all one-electron orbitals. To minimize the energy eigenvalue, the radial parts of the orbitals and the mixing coefficients are optimized at the same time.[41–43]

In the relativistic configuration interaction (RCI), the electronic correlation effects are further considered. Moreover, the Breit interaction in the low-frequency approximation reads

$$H_{ij}^{\mathrm{B}} = -\frac{1}{2r_{ij}}\Big[\boldsymbol{\alpha_i}\cdot\boldsymbol{\alpha_j} + \frac{(\boldsymbol{\alpha_i}\cdot\boldsymbol{r_{ij}})(\boldsymbol{\alpha_i}\cdot\boldsymbol{r_{ij}})}{r_{ij}^2}\Big], \tag{3}$$

and low-order QED effects (self-energy and vacuum polarization) are considered as the perturbation.[38,40] Therefore, they do not influence the calculated orbit, only affect the mixing coefficient of different states. In addition, the first term of Breit interaction represents the magnetic interaction between electrons, the second term indicates the retarded effect of exchanging virtual photons in Eq. (3).

After determining the wave function, the atomic parameters can be calculated by the corresponding reduced matrix elements

$$\Big\langle\psi(f)\Big\|T^{(k)}\Big\|\psi(i)\Big\rangle, \tag{4}$$

where $T^{(\mathrm{k})}$ is the multipole radiation interaction tensor operator.[44–47] The standard Racah algebra requires that the wave functions of the initial and final states correspond to the same orbital set.[48] In actual calculations, this restriction can be relaxed through the biorthogonal transform technology.[49] Therefore, the reduced matrix elements can be calculated by using independently optimized orbital sets. According to the reduced matrix elements, the line strength is given by

$$S = \Big|\Big\langle\psi(f)\Big\|T^{(k)}\Big\|\psi(i)\Big\rangle\Big|^2, \tag{5}$$

where $k = 0$ and $k = 1$ indicate the magnetic and electric multiples, respectively.[50–52] The transition rate (A_{ji}) and absorption oscillator strength (f_{ij}) from the initial $|\psi(i)\rangle$ to final states $|\psi(f)\rangle$ are related to the line strength S,[51,52] and the relation is shown by the following equation. For the E1 transitions, the transition rates and absorption oscillator strength can be expressed as

$$\begin{aligned} A_{ji} &= 2.0261\times10^{18}\frac{1}{\lambda_{ji}^3[J_i]}S^{\mathrm{E1}},\\ f_{ij} &= 303.75\frac{1}{\lambda_{ji}[J_i]}S^{\mathrm{E1}}, \end{aligned} \tag{6}$$

then the M1 and E2 transitions are expressed, respectively, as

$$\begin{aligned} A_{ji} &= 2.6974\times10^{13}\frac{1}{\lambda_{ji}^3[J_i]}S^{\mathrm{M1}},\\ f_{ij} &= 4.044\times10^{-3}\frac{1}{\lambda_{ji}[J_i]}S^{\mathrm{M1}}, \end{aligned} \tag{7}$$

$$\begin{aligned} A_{ji} &= 1.1199\times10^{18}\frac{1}{\lambda_{ji}^5[J_i]}S^{\mathrm{E2}},\\ f_{ij} &= 167.89\frac{1}{\lambda_{ji}^3[J_i]}S^{\mathrm{E2}}. \end{aligned} \tag{8}$$

For the M2 transitions

$$\begin{aligned} A_{ji} &= 1.4910\times10^{13}\frac{1}{\lambda_{ji}^5[J_i]}S^{\mathrm{M2}},\\ f_{ij} &= 2.236\times10^{-3}\frac{1}{\lambda_{ji}^3[J_i]}S^{\mathrm{M2}}. \end{aligned} \tag{9}$$

Here, $[J_a] = \frac{1}{2J_a+1}$ is the statistical weights of J_a levels, λ_{ij} is transition energy in Å.[45]

Results and Discussion—Transition Energies. In the MCDHF calculation, the even parity states $1s^2$ ($^1S_0$),$1s2s$ ($^3S_1$,$^1S_0$) and odd parity states $1s2p$ ($^3P_{2,1,0}$ and $^1P_1$) are optimized, respectively, to obtain occupied orbitals. Based on the active space approach, CSF expansions are obtained by single and double (SD) excitation to an active set of orbitals with principal quantum number $n = 7$ and orbital angular momenta $l = i$, followed by SD excitation to orbital sets with higher $n < 10$ ($l < g$), from multi-reference $1s^2$,$1s2s$ for the even states and $1s2p$ for the odd states. In order to ensure the convergence of the calculation, the active orbital sets were systematically added layer by layer, and only the outermost layers of orbitals were optimized by using relativistic self-consistent field procedure each time.[46,47] For the RCI calculation, based on the obtained orbitals, the Breit interaction and QED effects (self-energy and vacuum polarization) are considered using the perturbation theory. Before calculating the transition rates, the wave function generated above is biorthonormal using the BIOTRA component of the GRASP2K package.[38–40]

Table 1. The transition energies (cm^{-1}) for $1s2s$ ($^3S_1$,$^1S_0$) and $1s2p$ ($^3P_{2,1,0}$ and $^1P_1$) of He I with the different correlation models.

Model	2^1S_0–2^3S_1	$2^1P_1^o$–$2^3P_0^o$	$2^3P_0^o$–$2^3P_1^o$	$2^3P_1^o$–$2^3P_2^o$
DF	9010.58	2277.25	0.17	0.35
DF + QED	9010.66	2277.25	0.17	0.35
DF + Breit	9012.88	2275.54	0.91	0.08
DF + QED + Breit	9012.96	2275.53	0.91	0.07
3SD	7049.40	2070.63	0.19	0.38
3SD + QED	7049.48	2070.63	0.19	0.38
3SD + Breit	7050.31	2068.79	0.96	0.09
3SD + QED + Breit	7050.39	2068.79	0.96	0.09
4SD	6539.27	2048.69	0.20	0.40
4SD + QED	6539.40	2048.69	0.20	0.40
4SD + Breit	6540.01	2046.77	1.00	0.09
4SD + QED + Breit	6540.14	2046.77	1.00	0.09
5SD	6481.86	2051.54	0.21	0.41
5SD + QED	6481.98	2051.54	0.21	0.41
5SD + Breit	6482.52	2049.63	0.99	0.08
5SD + QED + Breit	6482.64	2049.63	0.99	0.08
6SD	6430.69	2049.65	0.21	0.41
6SD + QED	6430.83	2049.65	0.21	0.41
6SD + Breit	6431.32	2047.74	0.99	0.08
6SD + QED + Breit	6431.46	2047.74	0.99	0.08
7SD	6426.33	2048.17	0.20	0.41
7SD + QED	6426.46	2048.17	0.20	0.41
7SD + Breit	6426.93	2046.25	0.99	0.08
7SD + QED + Breit	6427.07	2046.25	0.99	0.08
8fSD	6426.32	2048.03	0.20	0.41
8fSD + QED	6426.46	2048.03	0.20	0.41
8fSD + Breit	6426.93	2046.12	0.99	0.08
8fSD + QED + Breit	6427.06	2046.12	0.99	0.08
9fSD	6426.32	2047.96	0.20	0.41
9fSD + QED	6426.45	2047.96	0.20	0.41
9fSD + Breit	6426.93	2046.04	0.99	0.08
9fSD + QED + Breit	6427.06	2046.04	0.99	0.08
NIST[53]	6421.47	2047.07	0.99	0.08

The transition energies of $1s2s$ ($^3S_1$,$^1S_0$) and $1s2p$ ($^3P_{2,1,0}$ and $^1P_1$) states with the different correlation models are listed in Table 1, compared with the experimental results.[53] The different models show that the n and l represent the maximum principal and orbital angular momentum quantum numbers in the active orbital space for the nlSD model, respectively. The Breit interaction and QED effects (marked as + Breit, + QED or + QED + Breit) are also considered. The results of the DF model represent the calculations of the single configuration, which differ from that of other models. Taking the results of the DF + QED + Breit model as an example, the difference in transition energies of 2^1S_0–2^3S_1 and $2^1P_1^o$–$2^3P_0^o$ are greater than 2000 cm^{-1} and 200 cm^{-1} compared with other models, respectively. Therefore, the consideration of electronic correlation effects is important. The differences of transition energies for $1s2s$ ($^3S_1$,$^1S_0$) and $1s2p$ ($^3P_{2,1,0}$ and $^1P_1$) are reduced with the expanding active orbital space, which indicates that the good convergence has been reached in our calculations. For example, the transition energies hardly change

from the 7SD to the $9f$SD model, and the largest difference of excitation energy is only $0.21\,\mathrm{cm}^{-1}$ corresponding to the transition $2^1P_1^o$–$2^3P_0^o$, which shows the convergence of the calculated results. The Breit interaction plays an important role in the transitions among $1s2p$ ($^3P_{2,1,0}$ and $^1P_1$) states of He I. It should be noted that the contribution of QED effect to transition energies are offset, but its contribution to the energy level cannot be ignored.[54]

The comparison of excitation energy for principal quantum numbers $n = 8$ and $n = 9$ is made. We give that the present calculations for energy levels are converged less than $0.10\,\mathrm{cm}^{-1}$ for $1s2s$ ($^3S_1$,$^1S_0$) and $1s2p$ ($^3P_{2,1,0}$ and $^1P_1$). For the convergent $9f$SD model, the influence of the Breit interaction is around $7.10\,\mathrm{cm}^{-1}$, while the greatest contribution of QED effect is $3.37\,\mathrm{cm}^{-1}$ for $1s2s$ ($^3S_1$) state. In addition, the maximum difference of excitation energy is $21.18\,\mathrm{cm}^{-1}$ between ours and the experimental results[53] of the $1s2s$ ($^3S_1$) state. Although the results of the MCDHF method for few-electron atoms are not as accurate as that of the Hylleraas method,[55–57] it should be mentioned that the MCDHF method is used not only to calculate the atomic structure but also to obtain high-precision transition probability and oscillator strength. In addition, the MCDHF method is still effective for complex atomic systems.[58] For example, it can be used to calculate excitation energies and oscillator strengths of Sm I ($Z = 62$).[46]

E1 Transition. As discussed above, the $9f$SD models are adopted in the subsequent transition calculation. From Tables 2–5, the oscillator strengths gf in Babushkin, as well as Coulomb gauges and the corresponding transition rates A are listed for E1 transitions of $1s^2$, $1s2s$ and $1s2p$ states. These two gauges correspond to the length and velocity form of the non-relativistic transition amplitude, respectively, in which the consistency of oscillator strengths gf further verify the calculated model. For the initial and final states with the spin-allowed transition, the oscillator strengths gf of the two kinds of gauges are in good agreement with the other theoretical results,[4,6,7,27,29,30] while the spin-forbidden transitions are inconsistent between two gauges even with the above energy converged model ($9f$SD). The similar problems have also appeared in the calculated $2s2p$ $^3P_1^o$–$2s^2\,^1S_0$ transition of Be-like C ion by Ynnerman *et al.*[59] and Jö nsson *et al.*[60] Furthermore, Chen *et al.*[61] pointed that this phenomenon is caused by the neglect of negative energy states, which significantly influences the results of the Coulomb gauge. Therefore, it is considered that the differences of the oscillator strengths gf between Babushkin and Coulomb gauges for those transitions may be due to the same reason. Since the Babushkin gauge is hardly affected by the negative energy state, its result is usually recommended.

As shown in Tables 2–5, the contribution from the Breit interaction to the oscillator strengths is indeed significant through comparison of the $9f$SD and $9f$SD + Breit models. In addition, by comparing the $9f$SD and $9f$SD + QED models, although the QED effect has a greater impact on the E1 transitions of spin-forbidden than the E1 transitions of the spin-allowed, the largest impact on oscillator strengths gf is only 0.93%.

Table 2. The oscillator strengths gf in (B) the Babushkin gauge and (C) the Coulomb gauge, and the transition rates A (s^{-1}) in (B) the Babushkin gauge for the He I $1s2p(^{3,1}P_1)$–$1s^2(^1S_0)$ E1 transitions.

Model	$2^3P_1^o$–1^1S_0			$2^1P_1^o$–1^1S_0		
	gf ($\times10^{-8}$)		A ($\times10^2$)	gf ($\times10^{-1}$)		A ($\times10^9$)
	B	C	B	B	C	B
$9f$SD	0.6420	0.6766	0.4081	2.7629	2.7620	1.7989
$9f$SD + QED	0.6480	0.6730	0.4278	2.7629	2.7621	1.7989
$9f$SD + Breit	2.7803	3.2656	1.7670	2.7631	2.7624	1.7989
$9f$SD + QED + Breit	2.7928	3.2578	1.7750	2.7632	2.7624	1.7989
Głowacki *et al.*[30]	2.789			2.761		
Morton *et al.*[29]	2.794		1.776			
Morton *et al.*[27]	2.7925		1.7758	2.7616		
Johnson *et al.*[4]	2.774		1.757	2.762		1.798
Drake *et al.*[7]	2.7935			2.7616		
Łach *et al.*[6]			1.7758			

By considering Breit correlation and QED effects, the complete results of E1 transitions are obtained in Tables 2–5, which agree with other theoretical calculations[4,6,7,27,29,30] within 1%. For $2^3P_1^o$–1^1S_0 transition, the present oscillator strength differs from the results of Głowacki *et al.*,[30] Morton *et al.*,[27,29] Johnson *et al.*[4] and Drake *et al.*[7] about 0.14%, 0.04%, 0.01%, 0.68%, and 0.03%, respectively. The result 2.774×10^{-8} was calculated by Johnson *et al.* in Ref. [4] for this transition using RMBPT theory, which differs slightly from other results. As pointed out by Łach *et al.*,[6] the difference may be due to the strong numerical cancelation and inadequate electronic correlation. Moreover, the source of difference for Ref. [30] is the use of non-relativistic electron-electron interaction in the calculation using the CIDF method compared with other results. In addition, our result is in better agreement with the result obtained by Morton *et al.*[27,29] and Drake *et al.*[7] using the Hylleraas theory.

Table 3. The oscillator strengths gf in (B) the Babushkin gauge and (C) the Coulomb gauge, and the transition rates A (s^{-1}) in (B) the Babushkin gauge for the He I $1s2p(^{3,1}P_1)$–$1s2s(^1S_0)$ E1 transitions.

Model	$2^3P_1^o$–2^1S_0			$2^1P_1^o$–2^1S_0		
	gf ($\times10^{-8}$)		A ($\times10^{-2}$)	gf ($\times10^{-1}$)		A ($\times10^{6}$)
	B	C	B	B	C	B
$9f$SD	0.3253	1.5487	0.5692	3.7516	3.7497	1.9647
$9f$SD + QED	0.3266	1.5566	0.5715	3.7517	3.7517	1.9648
$9f$SD + Breit	1.5066	4.2385	2.6363	3.7511	3.7489	1.9637
$9f$SD + QED + Breit	1.5094	4.2516	2.6415	3.7511	3.7489	1.9639
Morton *et al.*[29]	1.5294		2.683			
Morton *et al.*[27]	1.5305			3.7644		
Drake *et al.*[7]	1.6899					

Table 4. The oscillator strengths gf in (B) the Babushkin gauge and (C) the Coulomb gauge, and the transition rates A (s^{-1}) in (B) the Babushkin gauge for the He I $1s2p(^3P_{0,1})$–$1s2s(^3S_1)$ E1 transitions.

Model	$2^3P_0^o$–2^3S_1			$2^3P_1^o$–2^3S_1		
	gf ($\times10^{-1}$)		A ($\times10^{7}$)	gf ($\times10^{-1}$)		A ($\times10^{7}$)
	B	C	B	B	C	B
$9f$SD	1.7972	1.7969	1.0216	5.3917	5.3906	1.0217
$9f$SD + QED	1.7972	1.7968	1.0217	5.3919	5.3904	1.0218
$9f$SD + Breit	1.7976	1.7972	1.0222	5.3922	5.3911	1.0219
$9f$SD + QED + Breit	1.7977	1.7972	1.0223	5.3923	5.3909	1.0220
Głowacki *et al.*[30]	1.796			5.988		
Drake *et al.*[7]	1.7969			5.989		

Table 5. The oscillator strengths gf in (B) the Babushkin gauge and (C) the Coulomb gauge, and the transition rates A (s^{-1}) in (B) the Babushkin gauge for the He I $1s2p(^1P_1, {}^3P_2)$–$1s2s(^3S_1)$ E1 transitions.

Model	$2^1P_1^o$–2^3S_1			$2^3P_2^o$–2^3S_1		
	gf ($\times10^{-8}$)		A ($\times10^{-1}$)	gf ($\times10^{-1}$)		A ($\times10^{7}$)
	B	C	B	B	C	B
$9f$SD	0.4096	0.2395	3.4762	2.9956	2.9947	1.0218
$9f$SD + QED	0.4111	0.2398	3.4888	2.9957	2.9946	1.0219
$9f$SD + Breit	1.8349	1.1704	1.5571	2.9956	2.9947	1.0219
$9f$SD + QED + Breit	1.8380	1.1711	1.5598	2.9957	2.9946	1.0220
Głowacki *et al.*[30]	1.817			2.994		
Morton *et al.*[29]	1.8262		1.5493			
Morton *et al.*[27]	1.8257		1.5489			
Drake *et al.*[7]				2.9948		
Łach *et al.*[6]			1.5489			

M1, E2, M2 Transitions. The forbidden M1, E2, M2 transition rates for $1s^2$, $1s2s$, and $1s2p(^3P_{0,1,2}$ and $^1P_1)$ states are listed from Tables 6–9. In Table 6, the results show that the contributions of Breit correlation and QED effects are all less than 0.01% for 2^3S_1–1^1S_0 and 2^3S_1–2^1S_0 M1 transitions. However, as shown in Table 7, the Breit interaction has a great influence on the M1 transition rates of the $1s2p(^3P_{0,1,2}$ and $^1P_1)$ states. Similar to the M1 transition, the Breit interaction also plays an important role in E2 transitions in Table 8.

As can be seen from Table 6, the $2^3S_1 - 1^1S_0$ transition was measured by Woodworth and Moos with the results of $1.10(33) \times 10^{-4}\,s^{-1}$.[3] The theoretical result $1.73 \times 10^{-4}\,s^{-1}$ was obtained by Lin *et al.* using the RRPA method,[35] which is different from the results of experiment[3] because of the inadequate description for electronic correlations. Later, the rate $1.266 \times 10^{-4}\,s^{-1}$ was obtained by Johnson *et al.* using the RMBPT theory.[4] However, Łach *et al.* pointed that the electron correlations were not well considered by Johnson *et al.*,[4] and they obtained the rate $1.272426 \times 10^{-4}\,s^{-1}$ using the QED theory.[6] Moreover, the role of the negative continuum effects and electron-electron correlation are considered by Indelicato using the MCDF method for this M1 transition,[36] his result $0.782 \times 10^{-4}\,s^{-1}$ is more consistent with our result $1.0641 \times 10^{-4}\,s^{-1}$.

For the 2^3S_1–2^1S_0 M1 transition, our result $8.1268 \times 10^{-8}\,s^{-1}$ is better in agreement with the experimental result $9.1 \times 10^{-8}\,s^{-1}$ (Ref.[8]) than the theoretical results $15.1 \times 10^{-8}\,s^{-1}$ of Lin *et al.*[35] and $6.1 \times 10^{-8}\,s^{-1}$ of Baklanov *et al.*[5] The result of Baklanov *et al.*[5] differs from the results of experiment,[8] because the correlated nonrelativistic wave functions were used.

The M2 transition rates are presented in Table 9, which are greatly consistent with the results of Morton *et al.*,[29] Johnson *et al.*,[4] and Łach *et al.*[6] By comparing the calculated models, the contributions of Breit correlation and QED effects are extremely small, ranging from 0.003% to 0.07%. As shown in Table 9,

the $2^3P_2^o$–1^1S_0 M2 transition rate of 3.2684×10^{-1} s^{-1} is much larger than that of other M2 transitions, and it differs from other theoretical values of 0.26%,[29] 0.08%[4] and 0.05%,[6] respectively. In addition, for $2^3P_2^o$–2^1S_0 and $2^1P_1^o$–$2^3S_1^o$ M2 transitions, the differences between our calculated rates and the results of Morton *et al.*[29] are all less than 1%. Moreover, the M2 transition rates of $2^3P_1^o$–2^3S_1 and $2^3P_2^o$–2^3S_1 are also obtained. It should be mentioned that the comparison of line strengths for principal quantum numbers $n = 8$ and $n = 9$ is made. We give that the present calculations for line strengths of the E1, M1, E2, M2 transitions are all converged less than 3×10^{-4} a.u.

Table 6. The transition rates A (s^{-1}) of the $1s2s$ ($^3S_1$)–$1s^2$($^1S_0$) and $1s2s$ ($^1S_0$)–$1s2s$($^3S_1$) M1 transitions for He I.

Model	2^3S_1–1^1S_0	2^1S_0–2^3S_1
	A ($\times 10^{-4}$)	A ($\times 10^{-8}$)
$9f$SD	1.0643	8.1241
$9f$SD + QED	1.0642	8.1246
$9f$SD + Breit	1.0642	8.1263
$9f$SD + QED + Breit	1.0641	8.1268
Woodworth *et al.*[3] (Expt.)	1.10	
Johnson *et al.*[4]	1.266	
Indelicato[36]	0.782	
Łach *et al.*[6]	1.272426	
Rooij *et al.*[8] (Expt.)		9.1
Lin *et al.*[35]	1.73	15.1
Baklanov *et al.*[5]		6.11

Table 7. The transition rates A (s^{-1}) of the $1s2p$ ($^3P_{0,1,2}$ and $^1P_1$) M1 transitions for He I.

Model	$2^1P_1^o$–$2^3P_0^o$	$2^3P_0^o$–$2^3P_1^o$	$2^1P_1^o$–$2^3P_1^o$	$2^3P_1^o$–$2^3P_2^o$	$2^1P_1^o$–$2^3P_2^o$
	A ($\times 10^{-9}$)	A ($\times 10^{-13}$)	A ($\times 10^{-9}$)	A ($\times 10^{-13}$)	A ($\times 10^{-10}$)
$9f$SD	4.7238	1.5326	5.8323	9.2004	2.7932
$9f$SD + QED	4.7386	1.5440	5.8466	9.2715	2.8334
$9f$SD + Breit	16.017	519.674	16.009	10.744	66.454
$9f$SD + QED + Breit	16.044	518.839	16.033	10.325	66.649

Table 8. The transition rates A (s^{-1}) of the $1s2p$($^3P_{0,1,2}$ and $^1P_1$) E2 transitions for He I.

Model	$2^1P_1^o$–$2^3P_1^o$	$2^3P_0^o$–$2^3P_2^o$	$2^3P_1^o$–$2^3P_2^o$	$2^1P_1^o$–$2^3P_2^o$
	A			
$9f$SD	4.8549×10^{-11}	5.1060×10^{-22}	1.5134×10^{-22}	1.6540×10^{-11}
$9f$SD + QED	4.8711×10^{-11}	5.1711×10^{-22}	1.5329×10^{-22}	1.6584×10^{-11}
$9f$SD + Breit	2.2182×10^{-10}	4.0613×10^{-20}	6.4715×10^{-26}	7.3366×10^{-11}
$9f$SD + QED + Breit	2.2217×10^{-10}	4.0317×10^{-20}	6.0565×10^{-26}	7.3458×10^{-11}

Table 9. The transition rates A (s^{-1}) for the M2 transitions among $1s2p$, $1s2s$, and $1s^2$ states for He I.

Model	$2^3P_2^o$–1^1S_0	$2^3P_2^o$–2^1S_0	$2^3P_1^o$–2^3S_1	$2^1P_1^o$–2^3S_1	$2^3P_2^o$–2^3S_1
	A ($\times 10^{-1}$)	A ($\times 10^{-8}$)	A ($\times 10^{-6}$)	A ($\times 10^{-5}$)	A ($\times 10^{-6}$)
$9f$SD	3.2690	1.1811	3.2025	1.9047	5.7681
$9f$SD + QED	3.2688	1.1814	3.2030	1.9050	5.7689
$9f$SD + Breit	3.2686	1.1800	3.2021	1.9050	5.7683
$9f$SD + QED + Breit	3.2684	1.1803	3.2026	1.9052	5.7691
Morton *et al.*[29]	3.2768	1.1927		1.9072	
Johnson *et al.*[4]	3.271				
Łach *et al.*[6]	3.270				

In summary, the transition energies, the E1 transitional oscillator strengths of the spin-allowed as well as spin-forbidden and the corresponding transition rates, and forbidden M1, E2, M2 transition rates for $1s^2$, $1s2s$, and $1s2p$ states of He I, are investigated using the MCDHF method. Based on the active space approach, the electronic correlation effects are considered systematically. In subsequent RCI computations, the Breit interaction and the QED effects are considered separately. Our results are in good agreement with other theoretical[4,6,7,27,29,30] and experimental results.[3,8]

For E1 transition, the contribution from the Breit interaction to the oscillator strengths is indeed significant. It should be pointed out that the Babushkin gauge is insensitive to negative energy state, therefore its result is usually recommended, especially the spin-forbidden E1 transitions. For forbidden 2^3S_1–

1^1S_0 and 2^3S_1–2^1S_0 M1 transitions, the results show that the contributions of Breit correlation are all less than 0.01%. However, its effects have a great influence on the M1 transition rates of the $1s2p(^3P_{0,1,2}$ and $^1P_1)$ states. In addition, our result $8.1268 \times 10^{-8}\,\mathrm{s}^{-1}$ is in better agreement with the experimental result $9.1 \times 10^{-8}\,\mathrm{s}^{-1}$ (Ref. [8]) than the theoretical results $15.1 \times 10^{-8}\,\mathrm{s}^{-1}$ of Lin *et al.*[35] and $6.1 \times 10^{-8}\,\mathrm{s}^{-1}$ of Baklanov *et al.*[5] for 2^3S_1–1^1S_0 M1 transition. Similar to the M1 transition of $1s2p(^3P_{0,1,2}$ and $^1P_1)$ states, the Breit interaction also plays an important role in E2 transitions. For the M2 transition, the complete transition rates are obtained, and it is found that the contributions of Breit correlation are extremely small, ranging from 0.003% to 0.07%. It is worth noting that the contribution of QED effect to the fine structure of the $1s2p$ state is offset, so its effect cannot be ignored.[54]

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