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Prediction of an $\Omega_{bbb}\Omega_{bbb}$ Dibaryon in the Extended One-Boson Exchange Model

Ming-Zhu Liu(刘明珠)[1,2] and Li-Sheng Geng(耿立升)[2,3,4*]

[1]*School of Space and Environment, Beihang University, Beijing 102206, China*
[2]*School of Physics, Beihang University, Beijing 102206, China*
[3]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[4]*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou 450001, China*

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Since Yukawa proposed that the pion is responsible for mediating the nucleon-nucleon interaction, meson exchanges have been widely used in understanding hadron-hadron interactions. The most studied mesons are the σ, π, ρ, and ω, while other heavier mesons are often argued to be less relevant because they lead to short range interactions. However, whether the range of interactions is short or long should be judged with respect to the size of the system studied. We propose that one charmonium exchange is responsible for the formation of the $\Omega_{ccc}\Omega_{ccc}$ dibaryon, recently predicted by lattice QCD simulations. The same approach can be extended to the strangeness and bottom sectors, leading to the prediction on the existence of $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ dibaryons, while the former is consistent with the existing lattice QCD results, the latter remains to checked. In addition, we show that the Coulomb interaction may break up the $\Omega_{ccc}\Omega_{ccc}$ pair but not the $\Omega_{bbb}\Omega_{bbb}$ and $\Omega\Omega$ dibaryons.

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In 1935, Yukawa proposed the one-pion exchange theory to explain the strong nuclear force.[1] Then, Taketani *et al.* employed one-pion and two-pion exchange potentials to explain the then available nucleon-nucleon (NN) scattering data.[2,3] Later, it was realized that the exchange of heavier mesons, such as σ, ρ, and ω, was needed to explain the rich experimental data, leading to the so-called one-boson exchange (OBE) model,[4] based on which the Bonn group[5] and Nijmegen group[6] constructed high-precision nuclear forces. Motivated by the success of the OBE model in describing the NN interaction, Voloshin and Okun applied the OBE model to study the interaction between charmed hadrons forty years ago.[7] After $X(3872)$ was discovered by the Belle collaboration in 2003,[8] Swanson utilized the one-pion exchange potential to interpret it as a $\bar{D}D^*$ bound state.[9] Later, Liu *et al.* studied $X(3872)$ with the OBE model by considering σ, ρ, and ω exchanges.[10,11] In Ref. [12] the likely existence of bound states of $\bar{D}^{(*)}$ and Σ_c was predicted in the OBE model (also in the unitary approach[13] and quark model[14]), which may correspond to the pentaquark states discovered by the LHCb collaboration.[15,16] Clearly, the OBE model has been an invaluable tool in describing hadron-hadron interactions containing not only light quarks but also heavy quarks.

In the conventional OBE model, the interaction between two hadrons is mainly generated by exchanging mesons consisting of light up and down quarks. It is natural to expect that the exchange of mesons containing strange quarks may also play a role in certain systems, such as K, K^*, η, and η'. In Ref. [17], it was shown that the one-kaon exchange force is able to bind the $DD_{s0}(2317)$ system. Compared with the one-pion exchange, η and η' exchanges contribute little to systems containing only light quarks due to their large masses and small isospin factor,[18] but they could contribute more to hadrons containing strange quarks.[19–21] In recent years, to understand the nature of $Z_c(3900)$,[22,23] the one charmonium exchange has been explored to investigate the $\bar{D}D^*$ interaction.[24,25] For such systems, however, since the exchange of light mesons is also allowed, the one charmonium exchange plays a minor role because of the large masses of charmonia. To confirm the relevance of one charmonium exchange, one needs to study other systems where light meson exchanges are not allowed.

Recently, the HAL QCD collaboration found one $\Omega_{ccc}\Omega_{ccc}$ dibaryon with a binding energy of $B = 5.7\,\text{MeV}$ and an rms radius of $R = 1.1\,\text{fm}$,[26] which indicates that there exist strong attractive interactions between the $\Omega_{ccc}\Omega_{ccc}$ pair (we note that the quark model study[27] did not support the existence of an $\Omega_{ccc}\Omega_{ccc}$ dibaryon). In this work, we extend the conventional OBE model by allowing for exchanges of charmonia. We show indeed that the one charmonium exchange can generate attractive interactions strong enough to bind the $\Omega_{ccc}\Omega_{ccc}$ pair. Replacing charm quarks with strange quarks, we find that the one strangeonium exchange leads to the existence of an $\Omega\Omega$ bound state, consistent with the HAL QCD

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*Corresponding author. Email: lisheng.geng@buaa.edu.cn
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result.[28] Furthermore, we predict the existence of an $\Omega_{bbb}\Omega_{bbb}$ dibaryon in the extended OBE model with the exchange of bottomonia.

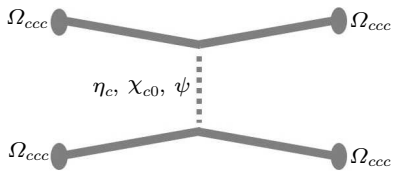

Fig. 1. One charmonium exchange for the $\Omega_{ccc}\Omega_{ccc}$ system.

Theoretical Framework. In the OBE model, the interaction between the $\Omega_{ccc}\Omega_{ccc}$ pair can only be generated by exchanging charmonia, for which we choose the ground-state η_c, J/ψ, and $\chi_{c0}(1P)$ mesons. As shown in Fig. 1, they play the same role as the π, (ρ,ω), and σ mesons in the conventional OBE model for the NN interaction. To derive the one charmonium exchange potential of Fig. 1, we need the following Lagrangians:[29]

$$\mathcal{L}_{\Omega_{ccc}\Omega_{ccc}\eta_c} = g_{\eta_c}\boldsymbol{\Omega}^{\dagger}_{ccc}\cdot(\eta_c\nabla\times\boldsymbol{\Omega}_{ccc}), \tag{1}$$

$$\mathcal{L}_{\Omega_{ccc}\Omega_{ccc}\chi_{c0}} = g_{\chi_{c0}}\boldsymbol{\Omega}^{\dagger}_{ccc}\cdot\chi_{c0}\boldsymbol{\Omega}_{ccc}, \tag{2}$$

$$\mathcal{L}_{\Omega_{ccc}\Omega_{ccc}\psi} = g_{\psi}\boldsymbol{\Omega}_{ccc}{}^{\dagger}\cdot\psi\boldsymbol{\Omega}_{ccc} - \frac{f_{\psi}}{4M}\boldsymbol{\Omega}^{i\dagger}_{ccc}(\partial_i\boldsymbol{\psi}_j-\partial_j\boldsymbol{\psi}_i)\boldsymbol{\Omega}^{j}_{ccc}, \tag{3}$$

where $\boldsymbol{\Omega}_{ccc}$ denotes the non-relativistic field of Ω_{ccc}; η_c, χ_{c0}, and ψ are the fields of exchanged mesons; g_{η_c}, $g_{\chi_{c0}}$, g_{ψ}, and f_{ψ} are the corresponding couplings, and the mass scale M renders the coupling f_{ψ} dimensionless. The Ω_{ccc} couplings to the charmonia are unknown experimentally, but they can be estimated with reasonable confidence as explained in the following.

In the OBE NN interaction,[5,30,31] one can reproduce the binding energy and rms radius of the deuteron with the following nucleon couplings to π, σ, and ω: $g_{\pi}=6.84$, $g_{\sigma}=8.46$, $g_{v}=3.25$, $f_{v}=19.82$ together with $M=0.94\,\mathrm{GeV}$, and a cutoff $\Lambda=0.86\,\mathrm{GeV}$. Assuming that the OBE $\Omega_{ccc}\Omega_{ccc}$ interaction is similar to the NN interaction by replacing light quarks with charm quarks, it is reasonable to expect that the couplings g_{η_c}, $g_{\chi_{c0}}$, and g_{ψ} are proportional to g_{π}, g_{σ} and g_{v}, respectively. For the sake of simplicity and without loss of generality, we assume that the ratios are the same, i.e., $g_{\eta_c}/g_{\pi}=g_{\chi_{c0}}/g_{\sigma}=g_{\psi}/g_{v}=f_{\psi}/f_{v}=r$, where r is a free parameter in our model, less than or equal to 1.

As in the conventional OBE model, we must take into account the finite size of exchanged mesons by supplementing the vertices of Fig. 1 with a monopolar form factor

$$F(\Lambda,q)=\frac{\Lambda^2-m^2}{\Lambda^2-q^2}. \tag{4}$$

In the OBE model with only $\pi,\sigma,\rho/\omega$ exchanges, the cutoff Λ is the only unknown parameter, which can be determined by reproducing the binding energies of some molecular candidates. From the physical perspective, the cutoff should be larger than the masses of exchanged bosons (if we replace the monopolar form factor by $\Lambda^2/(\Lambda^2+\boldsymbol{q}^2)$, to reproduce the binding energies of $\Omega\Omega$ and $\Omega_{ccc}\Omega_{ccc}$ dibaryons obtained in lattice QCD simulations, we need a cutoff smaller than the mass of the lightest meson exchanged, in agreement with Ref. [32]). Thus, for the one charmonium exchange model, the cutoff should be larger than the mass of $\chi_{c0}(1P)$. In addition, the cutoff should not be too large to justify the neglect of exchange of heavier mesons.

Using the above Lagrangians, the one-charmonium exchange potentials of the $\Omega_{ccc}\Omega_{ccc}$ system in coordinate space read

$$V_{\eta_c}(\boldsymbol{r}) = g^2_{\eta_c}\Big[-\boldsymbol{a}_1\cdot\boldsymbol{a}_2\delta(\boldsymbol{r})+\boldsymbol{a}_1\cdot\boldsymbol{a}_2 m^3_{\eta_c}W_Y(m_{\eta_c}r) + S_{12}(\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{r})m^3_{\eta_c}W_T(m_{\eta_c}r)\Big], \tag{5}$$

$$V_{\chi_{c0}}(\boldsymbol{r}) = -g^2_{\chi_{c0}}m_{\chi_{c0}}W_Y(m_{\chi_{c0}}r), \tag{6}$$

$$V_{\psi}(\boldsymbol{r}) = g^2_{\psi}m_{\psi}W_Y(m_{\psi}r)+\frac{f^2_{\psi}}{4M^2}\Big[-\frac{2}{3}\boldsymbol{a}_1\cdot\boldsymbol{a}_2\delta(\boldsymbol{r}) + \frac{2}{3}\boldsymbol{a}_1\cdot\boldsymbol{a}_2 m^3_{\psi}W_Y(m_{\psi}r) + \frac{1}{3}S_{12}(\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{r})\cdot m^3_{\psi}W_T(m_{\psi}r)\Big], \tag{7}$$

where $\boldsymbol{a}_1\cdot\boldsymbol{a}_2$ and $S_{12}(\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{r})$ denote the spin-spin and tensor operators, respectively; $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ denote the spin operator of a spin-3/2 particle, which is the product of spin-1/2 and spin-1 operators. The functions $W_Y(mr)$, $\delta(r)$, and $W_T(mr)$ are

$$W_Y(mr) = \frac{e^{-mr}-e^{-\Lambda r}}{4\pi mr}-\frac{\Lambda^2-m^2}{8\pi\Lambda m}e^{-\Lambda r}, \tag{8}$$

$$\delta(r) = \frac{(\Lambda^2-m^2)^2}{8\pi\Lambda}e^{-\Lambda r},$$

$$W_T(mr) = \frac{3}{4}\frac{e^{-mr}-e^{-\Lambda r}}{\pi m^3r^3}-\frac{3}{4}\frac{\Lambda e^{-\Lambda r}-me^{-mr}}{\pi m^3r^2} +\frac{1}{8\pi m^3 r}\Big[(m^2-3\Lambda^2)e^{-\Lambda r}+2m^2e^{-mr}\Big] -\frac{\Lambda^2-m^2}{8\pi m^3}\Lambda e^{-\Lambda r}.$$

It is well known that the S–D coupling plays an important role in the description of the deuteron, thus we also consider the S–D coupling in the extended OBE model (as all the lattice QCD simulations only considered S-wave interactions,[26,28] in the supplementary material, we discuss the results obtained in the extended OBE model with the S–D coupling turned off). For $J=0$, the allowed orbital angular momenta and spins for the $\Omega_{ccc}\Omega_{ccc}$ system are $(L=0,S=0)$ and $(L=2,S=2)$, and the relevant matrix elements of the spin-spin and tensor operators are given in Table 2. The Ω_{ccc} mass is taken from lattice QCD

simulations.[26] The masses of other relevant charmonia from PDG[33] are collected in Table 1.

Table 1. Hadron masses (in units of GeV) of relevance in this work.

Hadron	Ω_{ccc}[26]	η_c	$\chi_{c0}(1P)$	J/ψ
Mass	4.7956	2.9839	3.414	3.096
Hadron	Ω	η	$f_0(980)$	ϕ
Mass	1.672	0.958	0.99	1.019
Hadron	Ω_{bbb}[34]	η_b	$\chi_{b0}(1P)$	$\Upsilon(1S)$
Mass	14.371	9.3987	9.859	9.460

Table 2. Relevant spin-spin and tensor matrix elements for the $\Omega_{ccc}\Omega_{ccc}$ system.

State	J^P	Partial wave	$\langle \boldsymbol{a}_1 \cdot \boldsymbol{a}_2 \rangle$	$S_{12}(\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{r})$
$\Omega_{ccc}\Omega_{ccc}$	$J=0$	${}^1S_0$–${}^5D_0$	$\begin{pmatrix} -15/4 & 0 \\ 0 & -3/4 \end{pmatrix}$	$\begin{pmatrix} 0 & -3 \\ -3 & -3 \end{pmatrix}$

In the same way, the $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ potentials can also be constructed in the extended OBE model. The relevant particles and their masses[33] are given in Table 1. The mass of Ω_{bbb} is also taken from the lattice QCD simulations.[34]

As explained above, although we do not know the exact values of the couplings and cutoff in the OBE potential due to lack of experimental data, it is reasonable to assume that the couplings are not larger than the couplings of the nucleon, simultaneously the cutoff is not smaller than the mass of the heaviest meson exchanged and is not too much larger either. With these two constraints, we check whether one can reproduce the binding energies of the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega\Omega$ dibaryons obtained in lattice QCD simulations.

With the OBE potentials, we solve the Schrödinger equation to search for bound states. Assuming that the ratio r is 1 and with a cutoff $\Lambda = 3.78\,\text{GeV}$, the binding energy and rms radius for the $J^P = 0^+\,\Omega_{ccc}\Omega_{ccc}$ dibaryon are found to be $B = 5.1\,\text{MeV}$ and $R = 1.1\,\text{fm}$, respectively. Decreasing the ratio to 1/2 and with a larger cutoff of $\Lambda = 4.60\,\text{GeV}$, the corresponding binding energy and rms radius become $B = 5.8\,\text{MeV}$ and $R = 1.0\,\text{fm}$, respectively. Referring to the cutoff in the traditional OBE model containing only light quarks, we expect that the cutoff in the $\Omega_{ccc}\Omega_{ccc}$ system should be larger than $m_{\chi_{c0}(1P)} = 3.414\,\text{GeV}$, but not too much larger. Therefore, the coupling ratio of $r = 1$ is preferred.

The results listed in Table 3 imply that an $\Omega_{ccc}\Omega_{ccc}$ bound state with charm number $C = 6$ is likely to exist, corresponding to the dibaryon state discovered by the HAL QCD collaboration.[26] Using the same couplings and cutoff we did not find an $\Omega_{ccc}\Omega_{ccc}$ bound state with $J^P = 2^+$ (for more discussions about $J^P = 2^+$ configurations, see the Supplementary Material), consistent with the quark model study.[35]

Because Ω_{ccc} has an electric charge of $+2e$, the repulsive Coulomb interaction between the $\Omega_{ccc}\Omega_{ccc}$ pair should be taken into account. On top of the one charmonium exchange potential, we add the $\Omega_{ccc}\Omega_{ccc}$ Coulomb interaction $\frac{4\alpha}{r}$. The updated results show that the $\Omega_{ccc}\Omega_{ccc}$ system is no longer bound in the case of $r = 1$, which is in agreement with the HAL QCD study.[26] However, varying the cutoff by only 0.1 GeV, we find that the $\Omega_{ccc}\Omega_{ccc}$ system binds with a binding energy of $B = 13.8\,\text{MeV}$. As the reasonable range of the cutoff in the OBE model is at least a few hundreds of MeV, we can not determine whether the $\Omega_{ccc}\Omega_{ccc}$ system is bound or not once the Coulomb interaction is taken into account. The bottom line is that the Coulomb interaction is important for the $\Omega_{ccc}\Omega_{ccc}$ system.

Table 3. Binding energy B and rms radius R of the $J^P = 0^+\,\Omega_{ccc}\Omega_{ccc}$ bound states obtained with different ratios r and cutoffs Λ.

Molecule	J^P	Ratio r	Λ (GeV)	B (MeV)	R (fm)
$\Omega_{ccc}\Omega_{ccc}$	0^+	1	3.78	5.1	1.1
		1/2	4.60	5.8	1.0

We can test the above approach in the $\Omega\Omega$ system, which can exchange η', $f_0(980)$, and ϕ. The couplings between the triply strange baryon Ω and the exchanged mesons are assumed to be proportional to those nucleon couplings as in the one-charmonium exchange potential. The cutoff for the $\Omega\Omega$ system should be greater than the mass $m_\phi = 1.019\,\text{GeV}$. With the ratio $r = 1$ and cutoff $\Lambda = 1.62\,\text{GeV}$, we obtain one $J^P = 0^+\,\Omega\Omega$ bound state with a binding energy of $B = 1.6\,\text{MeV}$ and an rms radius $r = 3.1\,\text{fm}$. The HAL QCD collaboration have investigated the S-wave interaction of the $\Omega\Omega$ system, and found a bound state with a binding energy of $B = 1.6\,\text{MeV}$ and an rms radius $r = 3$–$4\,\text{fm}$.[28] If we decrease the ratio r to 1/2, we need a cutoff $\Lambda = 2.83\,\text{GeV}$ to reproduce the binding energy and rms radius of the lattice QCD study.[28] In this case, the cutoff value is quite close to $2m_\phi$, which does not seem very natural. This then implies that the ratio in the strangeness sector should be larger than 1/2 as in the charm sector. We note that allowing for mixing of light quarks with strange quarks, a larger coupling ratio than that of the charm sector is expected. Compared with the $\Omega_{ccc}\Omega_{ccc}$ dibaryon, the $\Omega\Omega$ dibaryon has a larger size, which can be easily understood because on the one hand the size of Ω is larger than that of Ω_{ccc} and on the other hand the exchanged mesons are lighter and therefore the interaction range is longer.

From the above study, it is clear that the extended OBE model can be trusted to study baryon-baryon interactions containing only heavy quarks such as charm and strangeness. As a result, it is natural to ask how about the $\Omega_{bbb}\Omega_{bbb}$ system, which can exchange η_b, $\chi_{b0}(1P)$, and $\Upsilon(1S)$. The couplings of the triply bottom baryon Ω_{bbb} and bottomo-

nia can also be estimated from the couplings of the nucleon with light mesons. A reasonable cutoff for the $\Omega_{bbb}\Omega_{bbb}$ system is expected to be larger than the mass of $m\chi_{b0}(1P) = 9.859\,\text{GeV}$. With a cutoff $\Lambda = 10.073\,\text{GeV}$ and the corresponding ratio $r = 1$ we obtain one $J^P = 0^+$ bound state with a binding energy $B = 5.7\,\text{MeV}$ and an rms radius $R = 0.55\,\text{fm}$. If we decrease the ratio to 1/2 and increase the cutoff to $\Lambda = 10.718\,\text{GeV}$, the corresponding binding energy and rms radius become $B = 5.7\,\text{MeV}$ and $R = 0.55\,\text{fm}$, respectively. Compared with the sizes of the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega\Omega$ dibaryons, the size of the $\Omega_{bbb}\Omega_{bbb}$ dibaryon is small. Nonetheless, one can see that the ratio of the size of the $\Omega_{bbb}\Omega_{bbb}$ dibaryon to the size of Ω_{bbb} is similar to the ratio of their charm counterparts, $R_{\Omega_{ccc}\Omega_{ccc}}/R_{\Omega_{ccc}}$.

Table 4. Binding energies B and rms radii R of $J^P = 0^+\,\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratio r and cutoff Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account.

Molecule	J^P	Ratio r	Λ (GeV)	B (MeV)	R (fm)
$\Omega\Omega$	0^+	1	1.62	1.6(0.7)	3.1(4.0)
		1/2	2.83	1.6(0.6)	3.0(4.2)
$\Omega_{bbb}\Omega_{bbb}$	0^+	1	10.073	5.7(0.5)	0.55(1.12)
		1/2	10.718	5.7(0.4)	0.55(1.17)

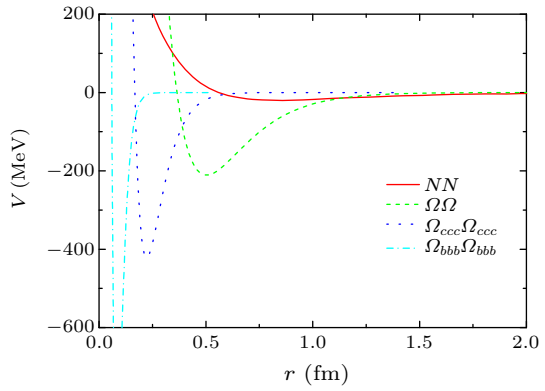

Fig. 2. One boson exchange potential for the NN, $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ systems as functions of the distance in the baryon-baryon pair.

Both Ω and Ω_{bbb} contain one negative charge $-e$, which leads to a repulsive potential $\frac{\alpha}{r}$ between the $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ pairs. With the Coulomb interaction taken into account we update the results in Table 4. The numbers in the parentheses show that the $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ systems still bind, though less bound than in the case where only the OBE potentials are considered. In addition, we find that the impact of the Coulomb interaction on the $\Omega\Omega$ system is smaller than that on the $\Omega_{bbb}\Omega_{bbb}$ system, mainly because the $\Omega\Omega$ dibaryon is more spatially extended than the $\Omega_{bbb}\Omega_{bbb}$ dibaryon.

With the coupling ratio $r = 1$ and the relevant cutoffs given in Tables 3 and 4, we show the S-wave potential of NN, $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ in Fig. 2. The line shapes of these potentials are similar, i.e., they all contain a short-range repulsive core and a medium- and long-range attractive tail. As the baryon mass becomes heavier, the baryon-baryon potential becomes shorter ranged and more attractive, which would generate a delta-like potential in the limit of infinite heavy baryon masses. As r approaches 0, all the potentials become strongly repulsive, which is qualitatively similar to the lattice QCD results.[26,28] However, the magnitude of the repulsion is larger than that of the lattice QCD potential. This is understandable because for extremely short ranges, the OBE potential is not expected to work. We note that the $^3S_1\,NN$ potential is qualitatively consistent with those of more refined phenomenological models[6,30] as well as lattice QCD simulations.[36]

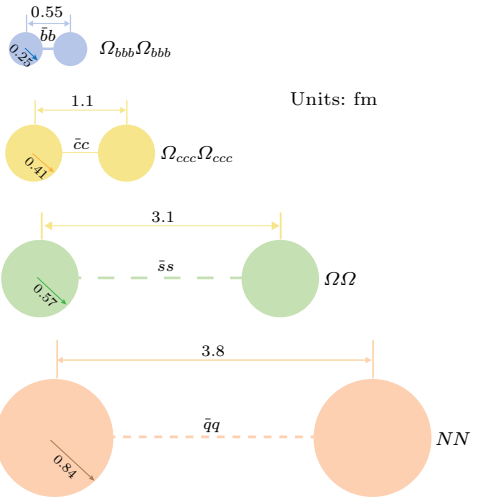

Fig. 3. The rms radii of the dibaryons and their components. Those of the nucleon, Ω, Ω_{ccc}, and Ω_{bbb} are 0.84 fm,[33] 0.57 fm,[37] 0.41 fm,[37] 0.25 fm,[34] while those of the NN, $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryons calculated in the extended OBE model are 3.8 fm, 3.1 fm, 1.1 fm, and 0.55 fm, respectively, with the ratio $r = 1$ and the cutoff $\Lambda = 0.86$, 1.62, 3.78, 10.073 GeV, respectively.

In Fig. 3, we show the rms radii of deuteron, $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryons calculated in the extended OBE model as well as those of their components (we do not distinguish charge radius from matter radius, which are assumed to be equal or similar for baryons/dibaryons studied here). It is clear that for all the dibaryons studied, the radius of the dibaryon is larger than the sum of those of their components. As a result, it is reasonable to refer to these dibaryons as molecular states. Moreover, the rms radius of the dibaryon state becomes smaller as the baryon mass increases, because the exchanged mesons become heavier. The ratio of the size of the dibaryon to its component R_{BB}/R_B is 4.5, 5.4, 2.7, 2.2, for the NN, $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryons, respectively. These results exhibit approximate $SU(3)$ symmetry and heavy quark flavor symmetry, as is expected.

It should be noted that the results obtained in the extended OBE model are cutoff dependent, as in the conventional OBE model. In fixing the cutoffs for the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega\Omega$ systems, we have used the lattice QCD simulations as guidance. However, for the $\Omega_{bbb}\Omega_{bbb}$ system, we have used the binding energy of the $\Omega_{ccc}\Omega_{ccc}$ dibaryon as a reference. The resulting cutoffs all seem very reasonable, which strongly support the existence of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryons. In particular, for the $\Omega_{bbb}\Omega_{bbb}$ system, a naive application of heavy quark flavor symmetry to connect $\Omega_{ccc}\Omega_{ccc}$ to $\Omega_{bbb}\Omega_{bbb}$ would result in a binding energy of a few tens of MeV for the latter, which implies that the Coulomb interaction will not likely break up the $\Omega_{bbb}\Omega_{bbb}$ pair, further supporting our conclusion.

Lastly, we briefly discuss about the $B\bar{B}$ system using the $\Omega_{ccc}\bar{\Omega}_{ccc}$ system as an example. If only the Coulomb interaction is considered, we obtain two bound states with binding energies $B = 1.0\,\text{MeV}$ and $B = 0.2\,\text{MeV}$. The one charmonium exchange potential for the $\Omega_{ccc}\bar{\Omega}_{ccc}$ system is related to that for the $\Omega_{ccc}\Omega_{ccc}$ system via C-parity, analogous with the G-parity in the NN system.[38] Therefore, the only difference between the $\Omega_{ccc}\bar{\Omega}_{ccc}$ potential and the $\Omega_{ccc}\Omega_{ccc}$ potential comes from the one J/ψ exchange potential. With the same parameters as those used for the $\Omega_{ccc}\Omega_{ccc}$ system, we obtain a deeply bound $\Omega_{ccc}\bar{\Omega}_{ccc}$ state, because the $\Omega_{ccc}\bar{\Omega}_{ccc}$ potential is more attractive, similar to the nucleon-antinucleon case.[38] It is natural to expect that the $\Omega_{ccc}\bar{\Omega}_{ccc}$ system may generate a bound state after considering both the Coulomb potential and the one charmonium exchange potential.

Summary and Outlook. Replacing light u/d quarks with heavy (strangeness, charm, and bottom) quarks, we extended the conventional OBE model by allowing for exchanges of heavy $s\bar{s}$, $c\bar{c}$, $b\bar{b}$ ground-state mesons. Fixing the nucleon couplings to light mesons by reproducing the binding energy and rms radius of the deuteron, and assuming that the Ω, Ω_{ccc}, and Ω_{bbb} couplings to their relevant exchanged mesons are equivalent to or less than the nucleon ones, we have investigated the possible existence of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryons via the extended OBE model. Our results show that the existence of $J^P = 0^+\,\Omega\Omega$ and $\Omega_{ccc}\Omega_{ccc}$ bound states is supported by the OBE model. Furthermore, we predict the existence of one $J^P = 0^+\,\Omega_{bbb}\Omega_{bbb}$ bound state. In addition, taking into account the Coulomb interaction, we find that the $\Omega_{ccc}\Omega_{ccc}$ system is at the verge of binding, while the $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ systems still bind. Our studies show that the spin-parities of such dibaryons favor $J^P = 0^+$ rather than $J^P = 2^+$.

Given the lattice QCD simulations and the results presented in this work, future experimental searches for the $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ dibaryons are strongly encouraged. We note that recently Zhang *et al.* estimated the production of the $\Omega\Omega$ dibaryon and pointed out that it could be searched for by the STAR and ALICE experiments.[39] In Ref. [40], Zhuang *et al.* showed that the Ω_{ccc} production in Pb–Pb collisions is much larger (about two orders of magnitude) than in pp collisions. Open charm hadron productions in high-energy nuclear collisions are also discussed in the multiphase transport model by Chen *et al.*[41] In Ref. [42] Qiu *et al.* studied the J/ψ production mechanism at the future Electron-Ion Collider. Similar studies on the production of the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ dibaryons should be pursued in the future. Experimental discovery of these dibaryons, together with recent observations of other exotic hadrons, such as $X_{0,1}(2900)$,[43] which may be a $D^*\bar{K}^*$ molecule,[44–46] could shed more light on the nature of the nonperturbative strong interaction, in addition to, e.g., the progress made in understanding the more conventional hadrons in the Dyson–Schwinger equations.[47]

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Supplementary material: Prediction of an $\Omega_{bbb}\Omega_{bbb}$ dibaryon in the extended one-boson exchange model

Ming-Zhu Liu(刘明珠)[1,2] and Li-Sheng Geng(耿立升)[2,3,4,5,*]

[1] *School of Space and Environment, Beihang University, Beijing 102206, China*
[2] *School of Physics, Beihang University, Beijing 102206, China*
[3] *Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[4] *School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China*
[5] *Beijing Advanced Innovation Center for Big Data-Based Precision Medicine, School of Medicine and Engineering, Beihang University, Beijing, 100191*

I. ONLY S-WAVE INTERACTION

To directly compare with the lattice QCD studies [1, 2], we show the results obtained with only S-wave interactions in the extended OBE model. Turning off the S-D mixing, but with the same cutoffs Λ and coupling ratios r as given in Tables III and IV of the main text, we repeat the OBE study of the $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$, and $\Omega_{bbb}\Omega_{bbb}$ systems and search for bound states. The results are shown in Table I. It is clear that without the S-D mixing, all the systems become less bound. Adding the Coulomb interaction, some of the dibaryons even become unbound. On the other hand, with a slight fine-tuning (increase) of the cutoff or coupling ratio, the systems can become bound as shown in Table 1. Therefore, as concluded in the main text, the extended OBE model supports the main results of the lattice QCD studies [1, 2].

TABLE S1. Binding energy B and RMS radius R of the $J^P = 0^+$ $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratios r and cutoffs Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account. The "−" denotes that there exists no bound state.

Molecule	J^P	Ratio r	Λ(GeV)	B (MeV)	R(fm)
$\Omega_{ccc}\Omega_{ccc}$	0^+	1	3.78	2.6(−)	1.4(−)
		1/2	4.60	1.9(−)	1.6(−)
$\Omega\Omega$	0^+	1	1.62	0.9(**0.2**)	4.0(**6.1**)
		1/2	2.83	0.3(−)	5.9(−)
$\Omega_{bbb}\Omega_{bbb}$	0^+	1	10.073	1.1(−)	1.2(−)
		1/2	10.718	0.5(−)	1.7(−)

TABLE S2. Binding energy B and RMS radius R of the $J^P = 0^+$ $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratios r and cutoffs Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account. The "−" denotes that there exists no bound state.

Molecule	J^P	Ratio r	Λ(GeV)	B (MeV)	R(fm)
$\Omega_{ccc}\Omega_{ccc}$	0^+	1	3.8075	5.7(−)	1.0(−)
		1/2	4.681	5.7(-)	1.0(-)
$\Omega\Omega$	0^+	1	1.64	1.6(**0.7**)	3.2(**4.1**)
		1/2	2.98	1.6(**0.6**)	3.0(**4.1**)
$\Omega_{bbb}\Omega_{bbb}$	0^+	1	10.10	5.3(**0.3**)	0.57(**1.3**)
		1/2	10.79	5.7(**0.5**)	0.55(**1.1**)

II. $J = 2$ CONFIGURATIONS

In the following, we discuss about the $J = 2$ configurations for the $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ systems. For $J = 2$, the following combinations of angular momentum and spin are allowed: $(L = 0, S = 2)$, $(L = 2, S = 0)$, and

* Corresponding author: lisheng.geng@buaa.edu.cn

$(L = 2, S = 2)$. With these partial waves the relevant matrix elements of the spin-spin and tensor operators of the extended OBE potential are shown in Table III.

TABLE S3. Matrix elements of the spin-spin and tensor operators for the $\Omega_{ccc}\Omega_{ccc}$ system.

State	J^P	Partial wave	$\langle\vec{a}_1\cdot\vec{a}_2\rangle$	$S_{12}(\vec{a}_1,\vec{a}_2,\vec{r})$
$\Omega_{ccc}\Omega_{ccc}$	$J=2$	${}^5S_2$-${}^1D_2$-${}^5D_2$	$\begin{pmatrix} -\frac{3}{4} & 0 & 0 \\ 0 & -\frac{15}{4} & 0 \\ 0 & 0 & -\frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} 0 & -\frac{3}{\sqrt{5}} & 3\sqrt{\frac{7}{10}} \\ -\frac{3}{\sqrt{5}} & 0 & 3\sqrt{\frac{2}{7}} \\ 3\sqrt{\frac{7}{10}} & 3\sqrt{\frac{2}{7}} & \frac{9}{14} \end{pmatrix}$

First, we notice that with the same parameters as those used for the $J = 0$ configurations studied in the main text, none of these systems bind, which indicates that in the extended OBE model, the $J = 0$ potential is more attractive than that of $J = 2$, in agreement with the quark model [3].

Next we investigate whether these systems can bind with reasonable cutoffs. For the $\Omega_{ccc}\Omega_{ccc}$ system, with a cutoff of 4.18 GeV and the coupling ratio $r = 1$, we obtain one bound state. If we change the coupling ratio to 1/2, we obtain a bound state for a cutoff of 5.45 GeV. Both cutoffs are much larger than those for $J = 0$ by about 0.4-0.9 GeV, which do not seem very natural. With the same approach, we obtain the results for the $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ as shown in Table IV. Judging from the cutoffs needed to obtain bound states, we conclude that although a $J = 2$ $\Omega_{bbb}\Omega_{bbb}$ dibaryon is unlikely, a $J = 2$ $\Omega\Omega$ dibaryon cannot be ruled out. We show the binding energy of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ as a function of the cutoff with S-D mixing turned on for the cases of $r = 1$ and $r = 1/2$ in Figs. [1-6].

TABLE S4. Binding energy B and RMS radius R of the $J^P = 2^+$ $\Omega_{ccc}\Omega_{ccc}$, $\Omega\Omega$ and $\Omega_{bbb}\Omega_{bbb}$ bound states obtained with different ratios r and cutoffs Λ. The numbers in the brackets represent the results obtained with the Coulomb interaction taken into account. The " $-$" denotes that there exists no bound state.

Molecule J^P	Ratio r	Λ(GeV)	B (MeV)	R(fm)
$\Omega_{ccc}\Omega_{ccc}$ 2^+	1	4.18	7.6($-$)	0.9($-$)
	1/2	5.45	6.6($-$)	0.9($-$)
$\Omega\Omega$ 2^+	1	1.80	5.4(**3.9**)	1.8(**2.0**)
	1/2	3.83	2.5(**1.1**)	2.4(**3.1**)
$\Omega_{bbb}\Omega_{bbb}$ 2^+	1	10.393	3.5($-$)	0.67($-$)
	1/2	11.378	5.3($-$)	0.55($-$)

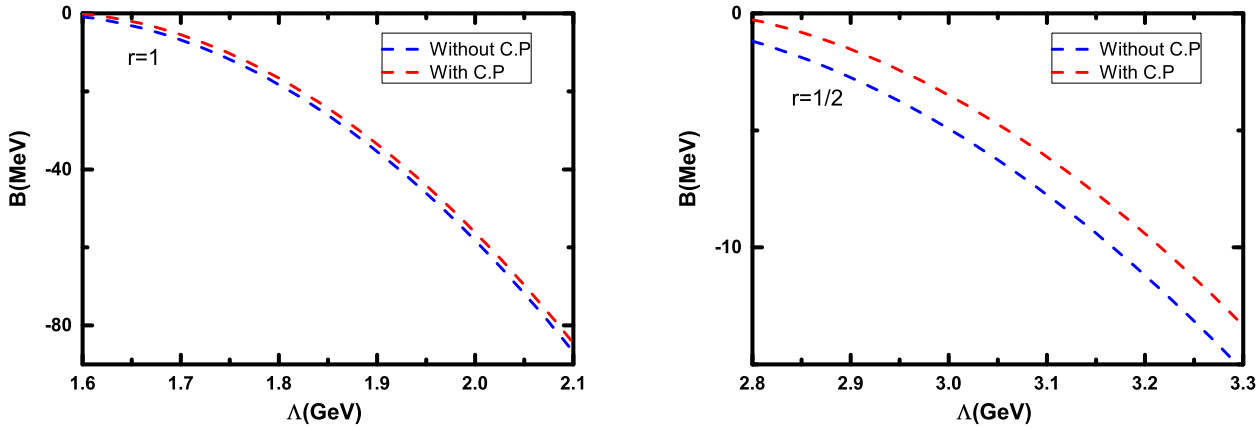

FIG S1. With and without the Coulomb interaction, the binding energy of the $J = 0$ $\Omega\Omega$ dibaryon as a function of the cutoff for the cases of $r = 1$ and 1/2.

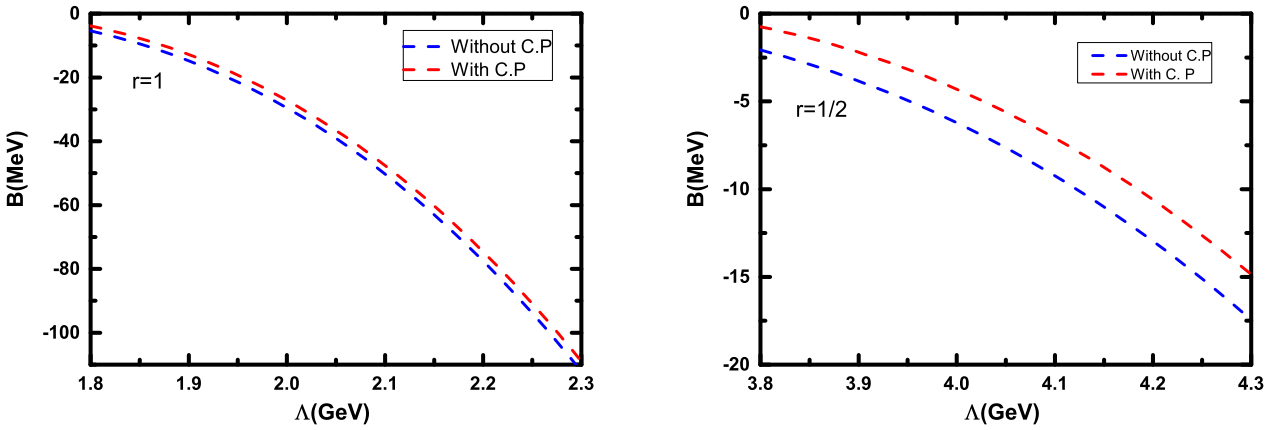

FIG S2. With and without the Coulomb interaction, the binding energy of the $J = 2$ $\Omega\Omega$ dibaryon as a function of the cutoff for the cases of $r = 1$ and $1/2$.

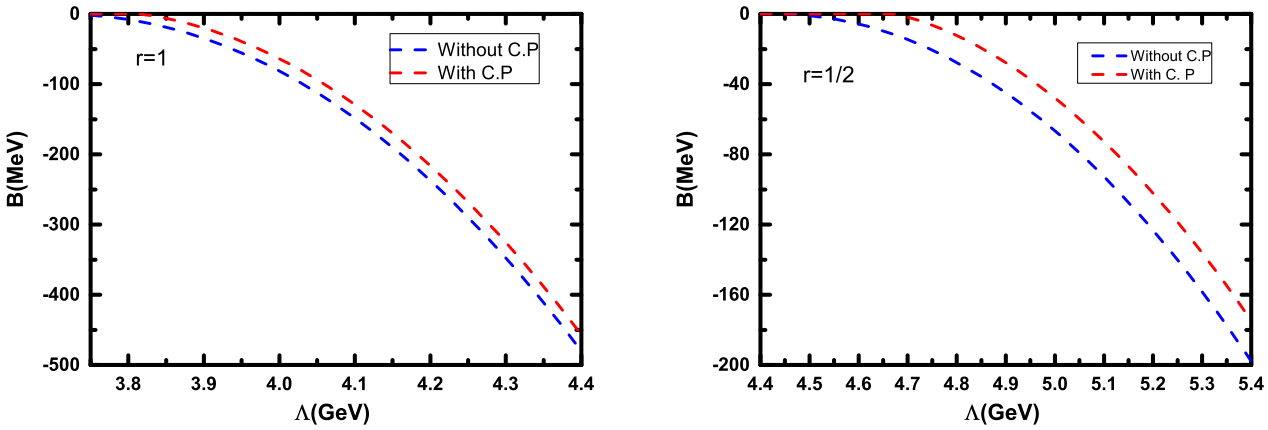

FIG S3. With and without the Coulomb potential, the binding energy of the $J = 0$ $\Omega_{ccc}\Omega_{ccc}$ dibaryon as a function of the cutoff for the case of r=1 and $1/2$.

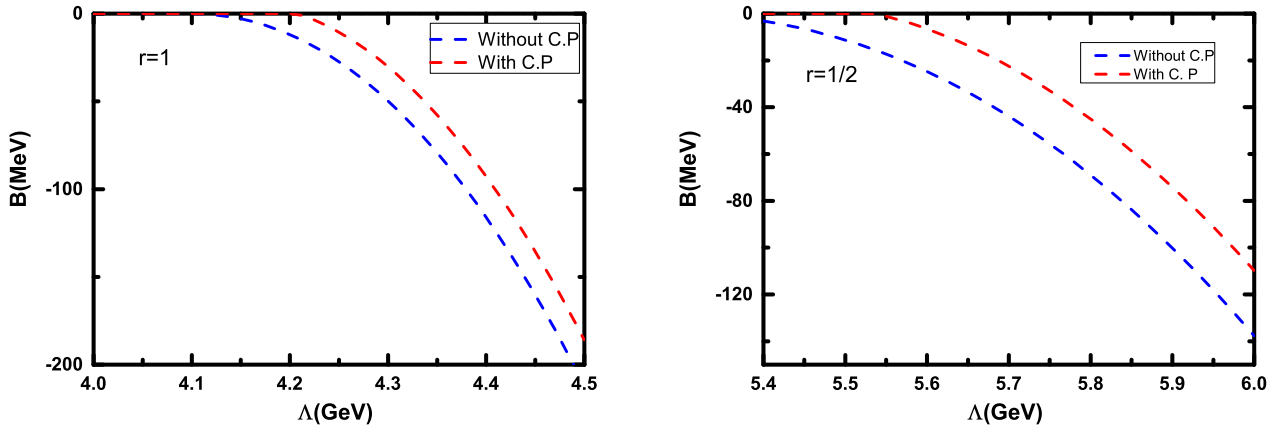

FIG S4. With and without the Coulomb potential, the binding energy of the $J=2$ $\Omega_{ccc}\Omega_{ccc}$ dibaryon as a function of the cutoff for the case of r=1 and 1/2.

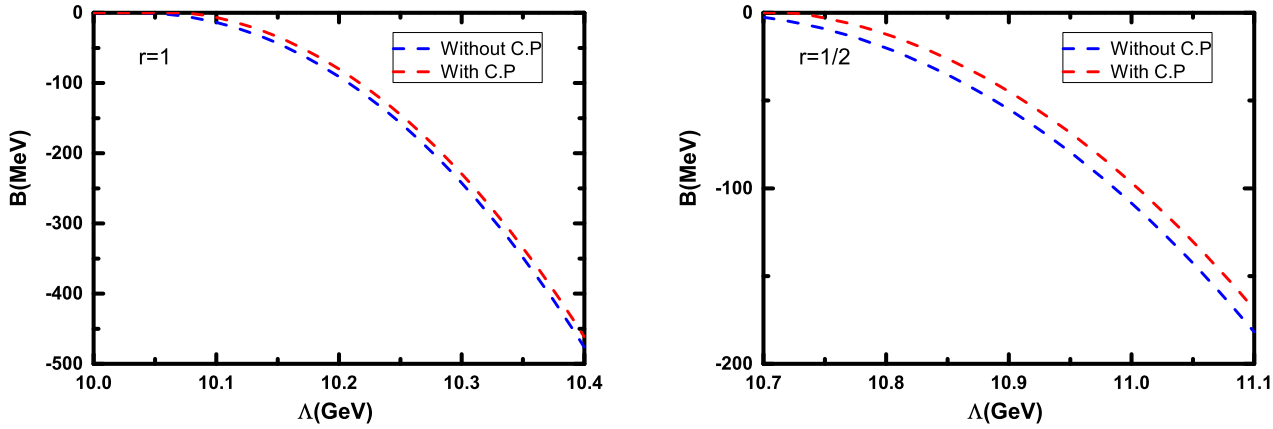

FIG S5. With and without the Coulomb potential, the binding energy of the $J=0$ $\Omega_{bbb}\Omega_{bbb}$ dibaryon as a function of the cutoff for the case of r=1 and 1/2.

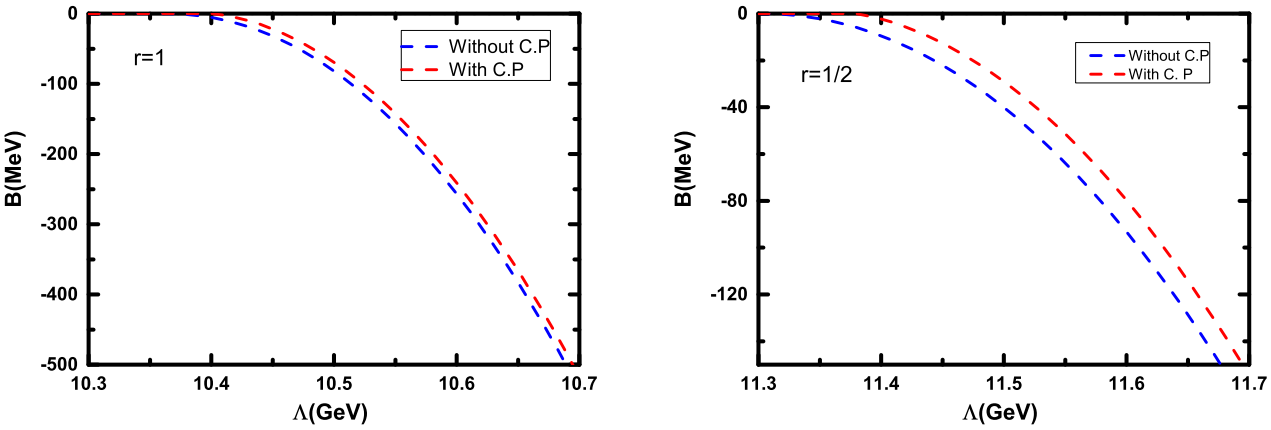

FIG S6. With and without the Coulomb potential, the binding energy of the $J = 2$ $\Omega_{bbb}\Omega_{bbb}$ dibaryon as a function of the cutoff for the case of r=1 and 1/2.

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