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# Direct Strong Measurement of a High-Dimensional Quantum State

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It is of great importance to determine an unknown quantum state for fundamental studies of quantum mechanics, yet it is still difficult to characterize systems of large dimensions in practice. Although the scan-free direct measurement approach based on a weak measurement scheme was proposed to measure a high-dimensional photonic state, how *weak* the interaction should be to give a correct estimation remains unclear. Here we propose and experimentally demonstrate a technique that measures a high-dimensional quantum state with the combination of scan-free measurement and direct strong measurement. The procedure involves sequential strong measurement, in which case no approximation is made similarly to the conventional direct weak measurement. We use this method to measure a transverse state of a photon with effective dimensionality of 65000 without the time-consumed scanning process. Furthermore, the high fidelity of the result and the simplicity of the experimental apparatus show that our approach can be readily used to measure the complex field of a beam in diverse applications such as wavefront sensing and quantitative phase imaging.

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Measuring the quantum state is of crucial importance in quantum science and technology. However, exact determination of a quantum state from a single measurement is prohibited by the quantum no-cloning theorem.[1,2] A method known as quantum state tomography (QST) is developed to reconstruct the state of a large ensemble of identically prepared quantum systems.[3−8] The experimental measurements and computational complexity of data process that QST needs to reconstruct the state grow rapidly with the increasing dimensions of the quantum system.[9,10] This makes it a challenge to reconstruct high-dimensional states via QST.

In recent years, as a typical high-dimensional quantum state, the transverse wave function of a photon attracted a large amount of research interest.[11−13] The transverse wave function of a pure state of a photon can be described by a set of complex state vectors expanded in Hilbert space. A promising method known as direct measurement provides an alternative approach for measuring the wave function of a pure state through the use of weak value.[11,14,15] Unlike a conventional measurement, the result of direct measurement, i.e., the weak value, can be a complex quantity, thus the wave function can be obtained directly since the real and imaginary parts of the wave function are proportional to the weak value. The direct measurement technique reduces the experimental complexity and needs no further data process, thus it has been regarded as an efficient method to characterize high-dimensional quantum states, even mixed states.[15,16] Yet the implementations of direct measurement need to scan through the complete bases of the state vector, thus the time required to characterize the quantum state scales with the dimension of the system. A scan-free direct measurement approach was proposed, which enabled us to measure the entire state vector at the same time. This makes it possible to measure a state with large dimensionality.[13]

However, to get the weak value, the weak interaction between the pointer (measurement apparatus) and the system is regarded to be necessary in the measurement, therefore such a weak measurement shows an approximation result.[17−19] Nevertheless, the quantum wave function can be obtained in an exact formula by direct strong measurement (DSM) based on the similar scheme in direct weak measurement but using strong measurement.[20] We here refer to measurements characterized by a strong coupling between the system and the pointer. It has been explained that a strong measurement does not always coincide with a project measurement on the system,

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and it is proved that weak value can be measured effectively using strong coupling.[21] A strong measurement leads to a larger entanglement between the system and the pointer than weak measurement, but it does not inevitably lead the wavefunction collapse to its eigenstates. Note that the DSM method extracts only a small part of the information from a single measurement, but the complete information of the wave function can be obtained by measuring a large ensemble of the identically prepared states. Thus the question of how *weak* the interaction should be to hold the approximation formula in the weak measurement is solved. It indicates that weak measurement is unnecessary for direct measurement, and DSM gives a better direct measurement result of the wave function.

Here we demonstrate a method to measure the transverse wave function of a photon using direct strong measurement. Since we implement projective measurement in the momentum space rather than in the position space, we do not need to scan through the basis of the state vector to obtain the entire wave function, thus we eliminate the time-consuming scanning process. We show that our approach combines the benefit of DSM and scan-free measurement, using a very small number of measurements to reconstruct the high-dimensional state accurately and efficiently.

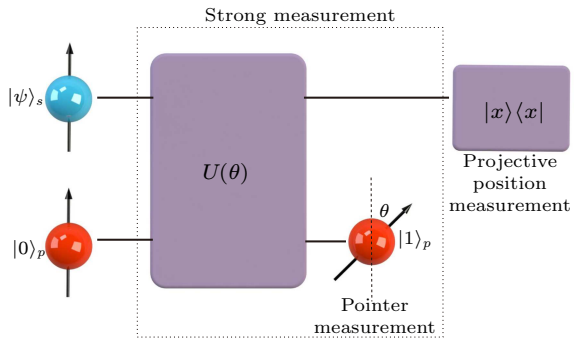


**Fig. 1.** Scheme of direct strong measurement of the quantum state. The system couples with the pointer and evolves according to the unitary operator $U(\theta) = e^{-i\theta\hat{\pi}_{p_0}\otimes\hat{\sigma}_y}$. The system state is measured in the final projective measurement.

We consider a pure quantum state $|\psi\rangle_s$ in an $N$-dimensional Hilbert space spanned in the position representation with basis $\{|x\rangle\}$,

$$|\psi\rangle_s = \sum_{i=1}^{N} \psi_i(x)|x_i\rangle, \tag{1}$$

where $\psi_i(x) = \langle x_i|\psi\rangle_s$ is the probability amplitude of state $|x\rangle$. The state $|\psi\rangle_s$ can alternatively be expressed in the momentum space $\{|p\rangle\}$,

$$|\psi\rangle_s = \sum_{i=1}^{N} \widetilde{\psi}_i(p)|p_i\rangle, \tag{2}$$

where $\{\widetilde{\psi}_i(p)\}$ is the set of probability amplitude of the state spanned in the momentum space, and $\{\widetilde{\psi}_i(p)\}$ relates to $\{\psi_i(x)\}$ through a Fourier transform. The system is coupling with the initial pointer state $|0\rangle_p$. We use a polarization qubit as the pointer in our case, which is a two-dimensional state spanned by the states $\{|0\rangle_p, |1\rangle_p\}$, hence the composite initial state is written as

$$|\Psi\rangle_i = |\psi\rangle_s \otimes |0\rangle_p. \tag{3}$$

We perform first measurement $\hat{\pi}_{p_0} = |p_0\rangle\langle p_0|$ in $|\Psi\rangle_i$, the state then evolves according to the unitary operator

$$U_{\rm sp} = e^{-i\theta\hat{\pi}_{p_0}\otimes\hat{\sigma}_y} = \hat{I} - |p_0\rangle\langle p_0|[(1-\cos\theta)\hat{I} + i\hat{\sigma}_y \sin\theta], \tag{4}$$

where $\theta$ is the coupling strength, and $\hat{\sigma}_y = -i(|0\rangle\langle 1| - |1\rangle\langle 0|)$ is a Pauli operator defined in the $\{|0\rangle, |1\rangle\}$ basis. Therefore we have the system observables in the position basis after consequent strong measurement of position

$$\langle x|\Psi\rangle = \langle x|U_{\rm sp}|\psi\rangle_s \otimes |0\rangle_p = \langle x|\psi\rangle_s|\varphi\rangle_p, \tag{5}$$

where

$$|\varphi\rangle_p = [1 - (1-\cos\theta)\langle\hat{\pi}_{p_0}\rangle_w + i\sin\theta\langle\hat{\pi}_{p_0}\rangle_w\hat{\sigma}_y]|0\rangle \tag{6}$$

is the final pointer polarization state, and the weak value is given by $\langle\hat{\pi}_{p_0}\rangle_w = \frac{\langle x|\hat{\pi}_{p_0}|\psi\rangle_s}{\langle x|\psi\rangle_s} = \frac{e^{-ip_0x/\hbar}\widetilde{\psi}_s(p_0)}{\psi_s(x)}$. Since we apply the measurement in the zero-momentum state, i.e., $p_0 = 0$, the weak value is written as $\langle\hat{\pi}_{p_0}\rangle_w = \frac{\widetilde{\psi}_s(p_0)}{\psi_s(x)}$ with the constant $\widetilde{\psi}_s(p_0)$ to be normalized. The probability amplitude of photons in the position basis is related to the weak value $\psi_s(x) = \frac{\widetilde{\psi}_s(p_0)}{\langle\hat{\pi}_{p_0}\rangle_w}$, and the weak value shows up in the above expression of the final polarization state in Eq. (6), therefore we measure the expectation value of the Pauli operator $\sigma_x, \sigma_y$, and $|1\rangle\langle 1|$ in the final polarization state, we have

$$\begin{aligned}
&\langle\varphi_p|\sigma_x|\varphi_p\rangle = -2\sin\theta[{\rm Re}\langle\hat{\pi}_{p_0}\rangle_w - (1-\cos\theta)|\langle\hat{\pi}_{p_0}\rangle_w|^2],\\
&\langle\varphi_p|\sigma_y|\varphi_p\rangle = -2\sin\theta{\rm Im}\langle\hat{\pi}_{p_0}\rangle_w,\\
&|\langle 1|\varphi_p\rangle|^2 = \sin^2\theta|\langle\hat{\pi}_{p_0}\rangle_w|^2.
\end{aligned} \tag{7}$$

The complex amplitude of the photons in position state $|x\rangle$ is then given by

$$\begin{aligned}
\psi_s(x) &= \frac{\widetilde{\psi}_s(p_0)}{\langle\hat{\pi}_{p_0}\rangle_w}\\
&= c\Big[\langle\varphi_p|\sigma_x|\varphi_p\rangle - \frac{2(1-\cos\theta)}{\sin\theta}|\langle 1|\varphi_p\rangle|^2\\
&\quad + i\langle\varphi_p|\sigma_y|\varphi_p\rangle\Big]^{-1},
\end{aligned} \tag{8}$$

where $c$ is a constant determined through normalization.

Note that there is no approximation in the above equation, hence the wave function of photons can be

measured in exact form by our method. The entire state vector is measured without the need for scanning, since the final polarization state $|\varphi\rangle_p$ at position $x$ is measured through two complementary polarization projection operators using a CMOS camera. Our experimental setup is shown in Fig. 2.

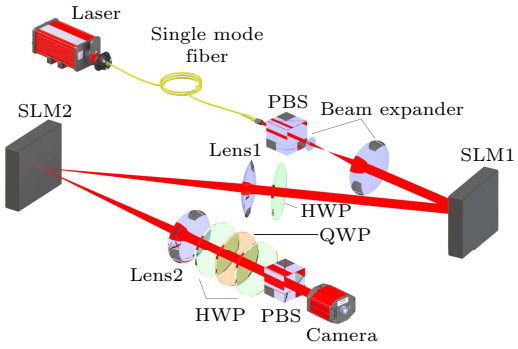


**Fig. 2.** The experimental setup for the direct strong measurement of the transverse wave function of photons. The setup consists of three modules: (a) the initial state of photons are prepared using a spatial light modulator (SLM) with the pointer state $|0\rangle$; (b) strong measurement is performed in the momentum space via a phase-only spatial light modulator (Hamamatsu X10468) placed on the focal plane of the $4f$ system. The SLM works in reflection mode, and it changes the phase of the horizontal polarization of the reflected light field by $\pi$, only the polarization of photons near zero momentum of the reflected light is rotated by an arbitrary angle $\theta$ into $|1\rangle$; (c) projective measurement with a polarization analyzer to extract real and imaginary parts of the wave function directly in the position state $|x\rangle$. PBS: polarizing beam splitter; SLM: spatial light modulator; HWP: half wave plate; QWP: quarter wave plate.

A stream of horizontal polarized Gaussian distributed photons is prepared by filtering a 780 nm laser beam with a single mode fiber and passing it through a polarized beam splitter (PBS). The beam is then expanded and the initial transverse state of photons $\psi(x)$ is prepared using a phase-only spatial light modulator (SLM).[22] The identically prepared photons of the desired complex wave function then pass through a $4f$ imaging system, and the sequential measurement are applied in the momentum space and the position space at the mutual focal plane of two lenses and the image plane of the $4f$ imaging system, respectively. The first measurement is performed by an SLM, which rotates the polarization of photons in zero-momentum state $|p\rangle = 0$ at an angle $\theta$. To implement the measurement, the polarization of photons is rotated from $|0\rangle$ to $(|0\rangle + |1\rangle)/\sqrt{2}$ by a 22.5° HWP before SLM, where $|0\rangle$ and $|1\rangle$ denote the horizontal and vertical polarization, respectively. The birefringent response of the SLM changes the horizontal polarization of the reflected light. In our experiment, the phase shift of $\pi$ is set in an area of $3 \times 3$ pixels on the SLM near the zero-momentum state, in which case $\theta = 90°$, achieving the maximum coupling strength in Eq. (4). Such an area of zero momentum is comparable to the diffraction limited spot size of the incident light with the imaging system. A CMOS camera is placed at the imaging plane of the $4f$ system to perform the strong measurement of the final polarization state at position $x$. The photons go through a QWP, an HWP and a PBS, the QWP and HWP are adjusted so that the photons are projected in the $D, A, R, L$, and $|1\rangle$ bases, thus the expectation value of Pauli observables are given by

$$\begin{aligned} \langle\varphi_p|\sigma_x|\varphi_p\rangle &= P_{|+\rangle} - P_{|-\rangle}, \\ \langle\varphi_p|\sigma_y|\varphi_p\rangle &= P_{|L\rangle} - P_{|R\rangle}, \end{aligned} \tag{9}$$

where $P_{|\pm\rangle} = |\langle\pm|\varphi_p\rangle|^2$, $P_{|L\rangle} = |\langle L|\varphi_p\rangle|^2$, $P_{|R\rangle} = |\langle R|\varphi_p\rangle|^2$, with $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$, $|L\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$, and $|R\rangle = (|0\rangle - i|1\rangle)/\sqrt{2}$. The probability amplitude of photons in position state $|x\rangle$ is given according to Eq. (7).

In our demonstration, we first measured the wave function of photons carrying orbital angular momentum (OAM), which has been the subject of much scientific research recently.[16,23,24,25] Photons with different values of OAM quantum number $l$ are generated by imposing a computer-generated hologram (CGH) on the SLM, and the photons in the first diffraction order are identically prepared in the desired state.

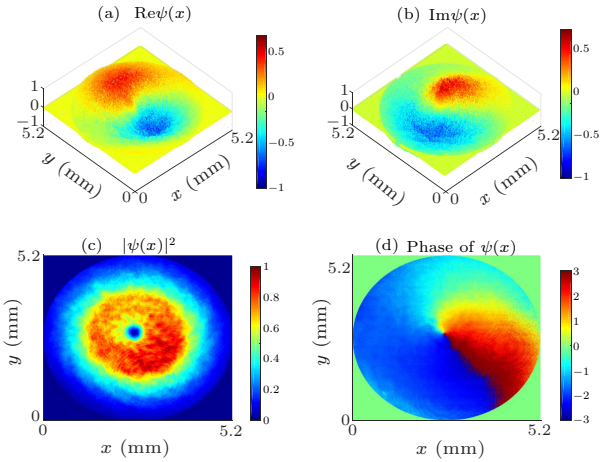


**Fig. 3.** Complex amplitude of photons carrying orbital angular momentum of $l = 1$. (a) Real and (b) imaginary parts of the wavefunction $\psi(x)$. (c) Probability amplitude and (d) phase of the wavefunction.

The real and imaginary parts of the wave function of photons carrying OAM is measured directly with the technique described above. The measured complex amplitude profile of photons with $l = 1$ is shown in Figs. 3(a) and 3(b), the corresponding probability amplitude and phase of the wave function are shown in Figs. 3(c) and 3(d). The quality of the measured state is assessed by fidelity defined as $F \equiv |\langle\psi_{\text{measured}}|\psi_{\text{ideal}}\rangle|$, where $\psi_{\text{measured}}$ and $\psi_{\text{ideal}}$ denote measured and ideal states, respectively. The

fidelity of the OAM state with $l = 1$ shown in Fig. 3 is 0.93. The less than unity fidelity is mainly due to the non-ideal preparation of the initial state. Photons carrying OAM with other quantum numbers $l$ ranging from $-2$ to 3 are measured and the phase profiles are shown in Fig. 3.

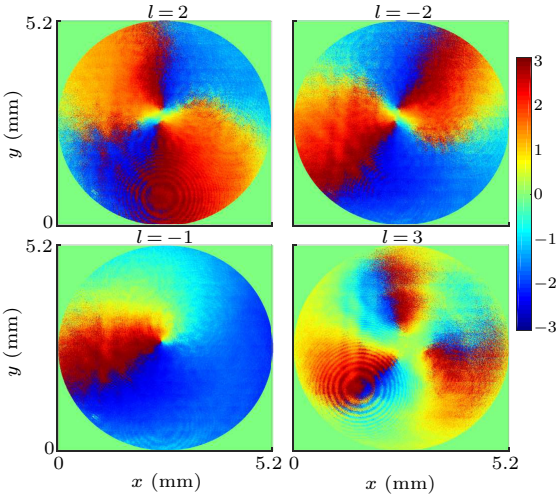


**Fig. 4.** Phase profiles of photons carrying OAM with $l = -2, -1, 2, 3$, respectively.

The position basis is a continuous Hilbert space, but since the image detector in our experiment is discrete in pixels, the dimension of the measured state is approximately 5 million ($2448 \times 2048$ with a pixel size of $3.45\,\mu$m). The number of effective dimensions of the measured state is further constrained due to the diffraction limit of the practical optical system. The diffraction limited spot size is given according to Fraunhofer diffraction theory, which is $\Delta x = 1.22\lambda/\mathrm{NA} \approx 19\,\mu$m in our case, the image size of our detector is $8.4\,\mathrm{mm} \times 7\,\mathrm{mm}$, thus the effective dimension of the measured state is approximately 65000.

It should be emphasized that an arbitrary transverse spatial state profile could be measured through our method. To further demonstrate that our proposed method is applicable for practical phase imaging, we used our approach to image a phase object. The sample is an etched photoresist pattern with four letters "USTC" at 300 nm etching depth on the glass. The phase reconstruction result is shown in Fig. 5, and the height of the sample letters is estimated from the phase map,

$$d(x, y) = \frac{\phi(x, y)}{2\pi\Delta n} \cdot \lambda, \tag{10}$$

where $\phi(x, y)$ denotes the phase difference on the sample, $\lambda$ is the wavelength of the 780 nm laser beam in the experiment, and $\Delta n$ is the difference of the refractive index between the photoresist (SPR955) and air. The phase map of the etched letter pattern and the corresponding depth along the dashed line is shown in Figs. 5(a) and 5(b). The estimated height is about 300 nm, which agrees with the fabrication result. Note that the background phase is due to the Gaussian lighting on the raw sample.

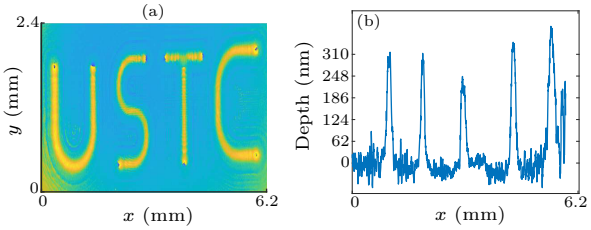


**Fig. 5.** Phase image of the raw sample with the letter pattern "USTC" fabricated by lithography and etching process. (a) Phase reconstruction of the etched letter pattern and (b) estimated etched depth along the dashed line in (a).

Our main result demonstrates that it is postselection that enables us to directly measure the quantum wave function rather than weak coupling strength in the weak measurement process. We clarified this issue theoretically and experimentally, revealing that strong measurement is a more generalized scheme for the direct measurement method. More significantly, we used our direct strong measurement method to reconstruct a high-dimensional quantum state, fulfilling a task that QST can not achieve. Since the conventional optical components are used in the experiment, an alternative classical description is available to describe our method. The measured wave function is classically analog to the transverse electric field profile of a beam of light.[26] However, it does not mean our method is classical, it only indicates that a quantum description provides a more general language in most systems. Meanwhile, our experiment procedure is ready for measuring identically prepared single photons using detector arrays with high quantum efficiency, such as electron multiplying CCD (EMCCD) cameras, or an intensified CCD cameras.

In conclusion, we have experimentally demonstrated that one can reconstruct a quantum state through direct strong measurement without the time-consuming scanning process. Moreover, the strong measurement eliminates the approximation in the direct weak measurement, which has been shown giving a better result in accuracy and precision in a two-level qubit system.[27,28] As an example to show our ability to characterize a high-dimensional quantum system efficiently and accurately, we measured the transverse state profile of a photon of an extremely high dimension with high fidelity. The simplicity of the theory and the feasibility of our experiment technique can extend to be applicable for practical use in quantum information process, wavefront sensing and quantum imaging. We anticipate that our method can inspire the direct measurement of high-dimensional quantum state in other systems such as atoms, ions, and superconducters.

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