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Quantum Deletion of Copies of Two Non-orthogonal Quantum States via Weak Measurement *

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We propose a scenario to increase the probability of probabilistic quantum deletion and to enhance the fidelity of approximate quantum deletion for two non-orthogonal states via weak measurement. More interestingly, by pretreating the given non-orthogonal states, the probability of probabilistic quantum deletion and fidelity of approximate quantum deletion can reach 1. Since outcomes of the weak measurement that we required are probabilistic, we perform the subsequent deleting process only when the outcome of weak measurement is "yes". Remarkably, we find that our scenario has better performance in quantum information process; for example, it costs less quantum resources and time.

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In recent years, information theory and its practical applications have developed rapidly. The research of quantum information theory has also made remarkable progress. A core concept in quantum theory is the linear superposition principle, which is due to the linearity of the equations of motion.[1] This property leads that many operations implemented in classical information are infeasible in quantum information theory.[2−6] The no-go theories caused by the linear superposition principle have increasingly attracted attention because they play an important role in the quantum information process.[7−12] It is easy to copy or delete a piece of information in the classical information process; that is, to copy or delete a classical bit. However, in the research of quantum information process theory, the linearity of quantum theory prohibits us from performing the accurate clone and deletion of unknown quantum states. This property leads to the essential difference between quantum information and classical information.

Classically, erasure is a primitive operation that irreversibly resets a system into a standard state. In the quantum information process, the process of erasure can be considered as swapping the unknown state with standard state and then throwing it into environment. The Landauer erasure theories tell us that this process is irreversible and will cost some energy.[13,14] However, is there a machine different from Landauer erasure? To answer this question, Pati and Braunstein first proposed the quantum no-deleting theorem, they proved that there exists no such a machine to delete an unknown state either reversibly or irreversibly. Via a unitary evolution, they defined[7] a unitary evolution as

$$|\psi\rangle_1|\psi\rangle_2|A\rangle_3 \to |\psi\rangle_1|\Sigma\rangle_2|A_\psi\rangle_3, \tag{1}$$

where the input state $|\psi\rangle|\psi\rangle$ belongs to the composite system 1 and 2, $|A\rangle$ is the initial state of the auxiliary systems, $|\Sigma\rangle$ represents the blank state after deleting $|\psi\rangle$, and $|A_\psi\rangle$ is the final state of the ancilla and usually depends on $|\psi\rangle$. It can be proved that the deletion of two copies of an unknown quantum state is available, which would lead to faster-than-light communication.[15] All these proofs indicate that the machine used to delete an unknown quantum state does not exist in a quantum information process. It is important to emphasize that the quantum deletion we are discussing about is different from erasure.

It is impossible to perform the deletion of two copies of an unknown quantum state. A natural question is whether there is a machine which can delete two copies of an unknown quantum state with a certain probability. The answer is positive. One scheme was proposed to perform the probabilistic deletion of two copies of two non-orthogonal states.[16] They can accomplish the quantum deletion of two non-orthogonal states randomly with a certain probability by a unitary evolution together with a post-selection measurement process. We give a brief review about this result in this study later. Pati has proposed another scheme to implement deletion of a qubit in an approximate manner. This scheme uses the global fidelity to characterize how similar the output state and the input state.[17] The 2–1 state-dependent conditional approximate deleting scheme[18] takes the following form:

$$\begin{aligned} &|0\rangle|0\rangle|A\rangle \to |0\rangle|\Sigma\rangle|A_0\rangle, \ |1\rangle|1\rangle|A\rangle \to |0\rangle|\Sigma\rangle|A_1\rangle, \\ &|0\rangle|1\rangle|A\rangle \to |0\rangle|1\rangle|A\rangle, \quad |1\rangle|0\rangle|A\rangle \to |1\rangle|0\rangle|A\rangle, \end{aligned} \tag{2}$$

where $|A\rangle$ is the initial state, $|A_0\rangle, |A_1\rangle$ (mutual orthogonal) are the final states of ancilla. When the input states of system 1 and 2 are identical, the deleting machine will work. Otherwise, the machine will allow the input states to pass through without any change. In this Letter, we shall improve the above two schemes by pretreating the initial states in the sense of increas-

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ing the probability of probabilistic quantum deletion and enhancing the fidelity of approximate quantum deletion.

For the general projective measurement, the initial state will collapse to the eigenstates of the quantum system and this evolution process is irreversible. However the situation for weak (partial collapse) measurement is different.[19] By using weak measurement, we can avoid the measured state randomly collapsing towards the eigenstates of the quantum system due to its unsharpness. Recently, a scheme that enhances the fidelity of a symmetric quantum cloning machine via weak measurement was proposed.[20] Inspired by this work, we shall propose a scheme in which the initial states are pretreated via weak measurement firstly, and then we study the quantum deleting process.

First, we make a brief review of probabilistic deletion of two copies of two non-orthogonal states. Then, we show that our scheme has higher probability for probabilistic quantum deletion. Moreover we obtain higher fidelity for approximate quantum deletion process via weak measurement. The way that we propose two conjectures about the balance between $P_{\rm yes}$ (the success probability of weak measurement, whose outcome is what we required) and $P_{\rm suc}$ (quantum deletion probability) allows even $P_{\rm yes}$ and F (fidelity) to be realized by adjusting the weak measurement parameter p according to the practical application. Lastly, a concise summary is given.

It is impossible to delete two copies of an unknown quantum state. However, the scheme that deleting two random non-orthogonal quantum states with a certain probability has been proposed.[16] For a certain set $\{|\psi_1\rangle, |\psi_2\rangle\}$ in a quantum system N, where $|\psi_1\rangle$ and $|\psi_2\rangle$ are non-orthogonal states and each of two copies of them randomly selected from the set can be probabilistically and exactly deleted by a unitary evolution and a post-selection measurement process. It has been proven that there always exists a unitary operator U satisfying the following evolution

$$U(|\psi_i\rangle|\psi_i\rangle|P_0\rangle) = c_{i1}|\psi_i\rangle|\Sigma\rangle|P_i\rangle + c_{i2}|\Phi_N\rangle|P_0\rangle, \quad (3)$$

where c_{ij} is a real number, $|P_0\rangle, |P_i\rangle$ are orthogonal sates of the probe system P, $i, j = 1, 2$.$|\Sigma\rangle$ is the normalized standard blank state in the quantum system N and $|\Phi_N\rangle$ is a state in the copies of quantum system $N \otimes N$. After the unitary evolution, the output states are reserved if and only if the measurement result is $|P_i\rangle$ when we perform projection measurement[21] on P and this measurement is post-selection of the outcomes. By this time, the system N has been projected onto the state $|\psi_i\rangle|\Sigma\rangle$, $i = 1, 2$. It is clear that the quantum deletion of two copies of two non-orthogonal states can be realized with the probability $|c_{i1}|^2 = 1 - \langle\psi_1|\psi_2\rangle^2$, $(i = 1, 2)$. If two states are orthogonal, then $|c_{11}|^2 = |c_{21}|^2 = 1$, which means that the success probability of the deletion can achieve 100%. Translating Eq. (3) into a generalized form

$$\begin{aligned} U(|\psi_1\rangle|\psi_1\rangle|P_0\rangle) &= \sqrt{v_1}|\psi_1\rangle|\Sigma\rangle|P_1\rangle + \sqrt{1-v_1}|\Phi_{\rm NP}^{(1)}\rangle, \\ U(|\psi_2\rangle|\psi_2\rangle|P_0\rangle) &= \sqrt{v_2}|\psi_2\rangle|\Sigma\rangle|P_2\rangle + \sqrt{1-v_2}|\Phi_{\rm NP}^{(2)}\rangle, \end{aligned} \quad (4)$$

where $|\Phi_{\rm NP}^{(i)}\rangle (i = 1, 2)$ are two normalized orthogonal states in NP system. To obtain the state $|\psi\rangle|\Sigma\rangle$ by the post-selection measurement after the unitary evolution we need $\langle P_i|\Phi_{\rm NP}^{(i)}\rangle = 0$. According to Eq. (4), we have the result $\frac{v_1+v_2}{2} \leq 1 - \langle\psi_1|\psi_2\rangle^2$,[16] which means that v_1 and v_2 could not achieve 100% at the same time, and the maximum deletion probability

$$v_1^{\max} = v_2^{\max} = 1 - \langle\psi_1|\psi_2\rangle^2. \quad (5)$$

Without loss of generality, we can suppose two non-orthogonal quantum state as

$$\begin{aligned} |\psi_1\rangle &= \cos\theta|0\rangle + \sin\theta|1\rangle, \\ |\psi_2\rangle &= \sin\theta|0\rangle + \cos\theta|1\rangle, \end{aligned} \quad (6)$$

where $\theta \in (0, \frac{\pi}{4})$.

The weak measurement operators are given as

$$\hat{M}_{\rm yes} = \sqrt{p}\hat{P}_1 + \hat{P}_2, \quad \hat{M}_{\rm no} = \sqrt{1-p}\hat{P}_1, \quad (7)$$

where $p \in [0, 1]$, $\hat{P}_1 = |+\rangle\langle+|$, $\hat{P}_2 = |-\rangle\langle-|$ and $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. Under the new set of basis $\{|+\rangle, |-\rangle\}$, the state $|\psi_i\rangle$ in Eq. (6) becomes

$$|\psi_{1,2}\rangle = \frac{\sqrt{2}}{2}(\cos\theta + \sin\theta)|+\rangle \pm \frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)|-\rangle. \quad (8)$$

Now, we perform the weak measurement on the system N. The intermediate states with the "yes" outcome after weak measurement shall have the form

$$|\psi'_{1,2}\rangle = k[\sqrt{p}(\sin\theta+\cos\theta)|+\rangle \pm (\cos\theta-\sin\theta)|-\rangle], \quad (9)$$

where $k = \frac{1}{\sqrt{(p-1)\sin(2\theta)+p+1}}$.

And the probability of obtaining "yes" is

$$P_{\rm yes} = \frac{1}{2}[(p-1)\sin(2\theta) + p + 1]. \quad (10)$$

For simplicity, we define

$$\begin{aligned} \cos\theta' &= \frac{[(\sqrt{p}+1)\cos\theta + (\sqrt{p}-1)\sin\theta]k}{\sqrt{2}}, \\ \sin\theta' &= \frac{[(\sqrt{p}+1)\sin\theta + (\sqrt{p}-1)\cos\theta]k}{\sqrt{2}}, \end{aligned} \quad (11)$$

an important relationship between θ and θ' is

$$\sin(2\theta') = \frac{(p+1)\sin(2\theta) + p - 1}{(p-1)\sin(2\theta) + p + 1}. \quad (12)$$

Then, Eq. (9) can be rewritten as

$$\begin{aligned} |\psi'_1\rangle &= \cos\theta'|0\rangle + \sin\theta'|1\rangle, \\ |\psi'_2\rangle &= \sin\theta'|0\rangle + \cos\theta'|1\rangle, \end{aligned} \quad (13)$$

and the inner product of $|\psi'_1\rangle$ and $|\psi'_2\rangle$ is

$$\langle\psi'_1|\psi'_2\rangle = \frac{(p+1)\sin(2\theta) + p - 1}{(p-1)\sin(2\theta) + p + 1}. \quad (14)$$

This means that the intermediate states can be non-orthogonal by choosing the proper value p.[20]

Now we obtain the maximum deletion probability of two intermediate non-orthogonal quantum states according to Eq. (5),

$$\begin{aligned} P_{\rm suc} &= 1-\langle\psi_1'|\psi_2'\rangle^2 \\ &= 1-\left[\frac{(p+1)\sin(2\theta)+p-1}{(p-1)\sin(2\theta)+p+1}\right]^2. \end{aligned} \tag{15}$$

For any given set of non-orthogonal qubit states as Eq. (6), we can always perform the deletion process as Eq. (1) with success probability 100% via weak measurement, as shown in Fig. 1.

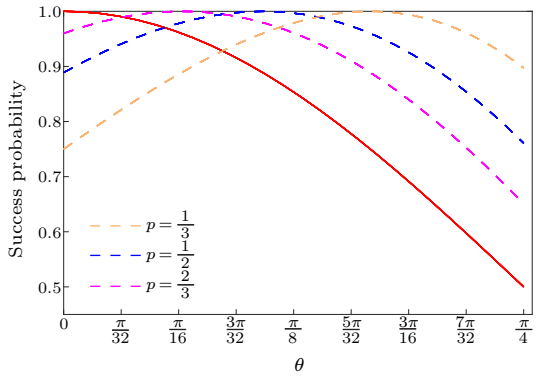

Fig. 1. The solid curve represents the tendency of $P_{\rm suc}$ without performing weak measurement and three dotted curves represent the tendency of $P_{\rm suc}$ corresponding to $p=\frac{1}{3},\ \frac{1}{2},\ \frac{2}{3}$ via weak measurement.

Remark 1. For any given two non-orthogonal qubit states as Eq. (6) (i.e., θ is fixed) when we adjust the parameter p, the deletion probability $P_{\rm suc}$ of the quantum deletion process can always achieve 100%. From Eq. (15), we obtain the optimal p as

$$p_{\rm opt}=\frac{1-\sin(2\theta)}{1+\sin(2\theta)}. \tag{16}$$

The corresponding $P_{\rm yes}=\frac{\cos^2(2\theta)}{1+\sin(2\theta)}$. It is notable that $P_{\rm yes}$ and $P_{\rm suc}$ are functions about θ and p, when θ is fixed while p is controlled by us.

Conjecture 1. We can seek a balance between $P_{\rm yes}$ and $P_{\rm suc}$.

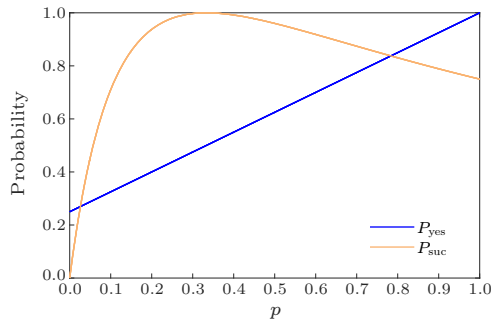

Fig. 2. The deletion probability $P_{\rm suc}$ (yellow curve) and the weak measurement probability $P_{\rm yes}$ (blue curve) versus p (with $\theta=\frac{\pi}{12}$). $P_{\rm yes}$ is monotonically increasing with p, while $P_{\rm suc}=1$ when $p_{\rm opt}=\frac{1}{3}$.

For instance, the case of $\theta=\frac{\pi}{12}$, we can investigate the dependence of the $P_{\rm yes}$ and $P_{\rm suc}$ on p as shown in Fig. 2. The maximum deletion probability $P_{\rm suc}^{\rm max}=1$ of the yellow curve can be reached at $p_{\rm opt}=\frac{1}{3}$, while $P_{\rm yes}$ is monotonically increasing with p. They cannot reach the maximum at the same time. Similarly for other θ.

We study the fidelity for approximate quantum deletion of two non-orthogonal states via weak measurement in the following. Taking an arbitrary qubit state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$ from the set of non-orthogonal states given as Eq. (6) as the input state, according to the deletion scheme Eq. (2), we have

$$\begin{aligned} &|\psi\rangle|\psi\rangle|A\rangle=[\alpha^2|00\rangle+\beta^2|11\rangle+\alpha\beta(|01\rangle+|10\rangle)]|A\rangle \\ &\to\alpha^2|0\rangle|\Sigma\rangle|A_0\rangle+\beta^2|0\rangle|\Sigma\rangle|A_1\rangle+\alpha\beta(|01\rangle+|10\rangle)]|A\rangle \\ &=|\psi_{\rm out}\rangle. \end{aligned} \tag{17}$$

Because of the linearity of quantum theory, we cannot delete an unknown state perfectly. The ideal process is $|\psi\rangle|\psi\rangle|A\rangle\to|\psi\rangle|\Sigma\rangle|A_\psi\rangle$.[7] The fidelity was defined as

$$F=\langle\Phi_{\rm ideal}|\rho_{\rm out}|\Phi_{\rm ideal}\rangle, \tag{18}$$

where $|\Phi_{\rm ideal}\rangle$ represents the ideal state and $\rho_{\rm out}$ represents the final state after evolution. F characterizes how well it is to realize the operation of Eq. (17). For the strong (standard) measurement, the maximum fidelity is 0.85 for deleting unknown states.[22] Now, we perform weak measurement on $|\psi_i\rangle$. Since the two non-orthogonal states are selected with equal probability, we can take an arbitrary intermediate state $|\psi_i'\rangle$ as the input state. For instance, the $|\psi_1'\rangle$ is selected as the input state for deletion process. The deletion operation will create the following state

$$\begin{aligned} |\psi_1'\rangle|\psi_1'\rangle|A\rangle &= [\cos^2\theta'|00\rangle+\sin^2\theta'|11\rangle \\ &\quad+\cos\theta'\sin\theta'(|01\rangle+|10\rangle)]|A\rangle \\ &\to\cos^2\theta'|0\rangle|\Sigma\rangle|A_0\rangle+\sin^2\theta'|0\rangle|\Sigma\rangle|A_1\rangle \\ &\quad+\cos\theta'\sin\theta'(|01\rangle+|10\rangle)]|A\rangle \\ &=|\psi_1''\rangle. \end{aligned} \tag{19}$$

By tracing the ancilla system, we can obtain the reduced density matrix as

$$\begin{aligned} \rho_{12} &= {\rm tr}_3(|\psi_1''\rangle\langle\psi_1''|)=(\cos^4\theta')|0\rangle\langle0|\otimes|\Sigma\rangle\langle\Sigma| \\ &\quad+(\sin^4\theta')|1\rangle\langle1|\otimes|\Sigma\rangle\langle\Sigma|+\sin(2\theta')|\varphi\rangle\langle\varphi|, \end{aligned} \tag{20}$$

where $|\varphi\rangle=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$ is the maximally entangled state.

The reduced density matrix for the second system is

$$\rho_2={\rm tr}_1(\rho_{12})=(1-\sin(2\theta'))|\Sigma\rangle\langle\Sigma|+\frac{1}{2}\sin(2\theta')I, \tag{21}$$

where I is the identity matrix in two-dimensional Hilbert space. Due to the imperfect deletion process

of Eq. (17), we will inevitably include some errors in ρ_2. The fidelity of this deletion process is

$$F_2 = \langle\Sigma|\rho_2|\Sigma\rangle = 1 - \frac{1}{4}\sin^2(2\theta') = 1 - \frac{1}{4}\left[\frac{(p+1)\sin(2\theta)+p-1}{(p-1)\sin(2\theta)+p+1}\right]^2. \tag{22}$$

The result shows that: (i) $\frac{3}{4} < F_2 \leq 1$ due to $\sin^2(2\theta') < 1$, (ii) for the strong (standard) measurement, the maximum fidelity is 0.85,[22] obviously, our fidelity is higher.

Remark 2. For any given state, we can always obtain higher fidelity by adjusting the weak measurement parameter p. The maximum fidelity $F_2^{\max} = 1$ can be obtained when parameters p and θ satisfy the relationship of Eq. (16). It corresponds to the perfect deletion. For instance, $\theta = \frac{\pi}{12}$, the parameter $p_{\text{opt}} = \frac{1}{3}$ is determined and then $\theta' = \frac{\pi}{4}$. Now the given initial states are as follows

$$|\psi_1\rangle = \frac{\sqrt{2-\sqrt{3}}}{2}|0\rangle + \frac{\sqrt{2+\sqrt{3}}}{2}|1\rangle, \quad |\psi_2\rangle = \frac{\sqrt{2+\sqrt{3}}}{2}|0\rangle + \frac{\sqrt{2-\sqrt{3}}}{2}|1\rangle. \tag{23}$$

Then, we confirm weak measurement operators

$$\hat{M}_{\text{yes}} = \frac{\sqrt{3}}{3}\hat{P}_1 + \hat{P}_2, \quad \hat{M}_{\text{no}} = \frac{\sqrt{6}}{3}\hat{P}_1, \tag{24}$$

and the intermediate states would be

$$|\psi_1'\rangle = \frac{\sqrt{2}}{2}|+\rangle + \frac{\sqrt{2}}{2}|-\rangle, \quad |\psi_2'\rangle = \frac{\sqrt{2}}{2}|+\rangle - \frac{\sqrt{2}}{2}|-\rangle. \tag{25}$$

We can implement perfect deletion of $|\psi_i'\rangle$ because $|\psi_1'\rangle$ and $|\psi_2'\rangle$ are orthogonal. Since the qubits of the first system only partially collapse via weak measurement, we can translate $|\psi_i'\rangle$ to the initial states $|\psi_i\rangle$. The final states would be

$$|\psi_1\rangle = \left(\frac{\sqrt{2-\sqrt{3}}}{2}|0\rangle + \frac{\sqrt{2+\sqrt{3}}}{2}|1\rangle\right)\otimes|\Sigma\rangle, \quad |\psi_2\rangle = \left(\frac{\sqrt{2+\sqrt{3}}}{2}|0\rangle + \frac{\sqrt{2-\sqrt{3}}}{2}|1\rangle\right)\otimes|\Sigma\rangle. \tag{26}$$

In this situation, P_{suc} and F_2 reach 1, respectively, according to Eqs. (15) and (22), which means that we obtain the perfect deletion of the initial states.

In the following, we discuss the relationship of the success probability P_{yes} of weak measurement expressed by Eq. (10) and the fidelity F_2 by Eq. (22) in conditional approximate deletion process. Both of them are the functions of θ and p, where $\theta \in (0, \frac{\pi}{4})$ and $p \in (0, 1)$.

On the one hand, higher successful probability for the deletion process does not mean we can implement the deletion more perfectly; on the other hand, higher fidelity may lead to lower probability. As is shown in Fig. 3 ($\theta = \frac{\pi}{12}$), the success probability increases monotonically with p while the fidelity increases before $p_{\text{opt}} = \frac{1}{3}$ and then decreases afterwards. The fidelity reaches 1 at $p_{\text{opt}} = \frac{1}{3}$, meanwhile the success probability is $P_{\text{yes}} = \frac{\cos^2(2\theta)}{1+\sin(2\theta)}$. The fidelity and the success probability can not reach the maximum at the same time.

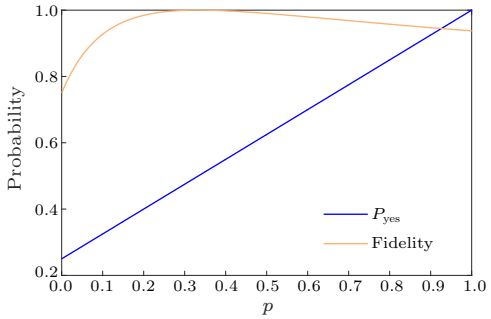

Fig. 3. The success probability P_{yes} (blue curve) and the fidelity F (yellow curve) versus p ($\theta = \frac{\pi}{12}$). The success probability is monotonically increasing with p while the fidelity increases before $p_{\text{opt}} = \frac{1}{3}$ and then decreases.

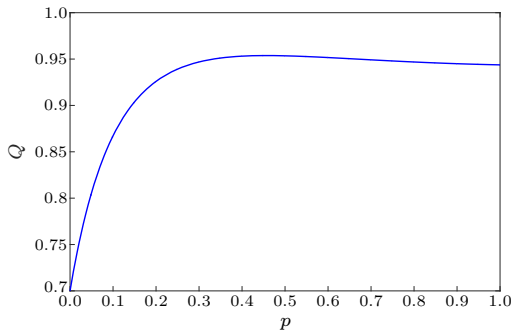

Fig. 4. The curve of $Q = 0.1P_{\text{yes}} + 0.9F_2$ versus p ($\theta = \frac{\pi}{12}$). Q is maximum when $p \approx 0.46$.

Conjecture 2. We can seek a balance between P_{yes} and F_2.

For example, we can add weights to P_{yes} and F_2 to meet the practical requirements as follows:

$$Q = f(P_{\text{yes}}, F_2), \tag{27}$$

where the function $f(x, y)$ quantifies the practical factors. In Eq. (27) we can keep a balance between the success probability and the fidelity according to the deletion application. For instance, one denotes $f(P_{\text{yes}}, F_2) = (1-x)P_{\text{yes}} + xF_2$, where x is the weight coefficient ranging from 0 to 1. If the fidelity is urgently required then we have to sacrifice some success probability according to the practical application, for example, we can let $x = 0.9$. We can know how well can we balance P_{yes} and F_2 by adjusting value p as shown in Fig. 4. For $\theta = \frac{\pi}{12}$, Q increases before $p \approx 0.46$ and then decreases. Similarly for other θ.

In summary, we have proposed a scenario to study probabilistic quantum deletion and approximate quantum deletion for two non-orthogonal states via weak measurement. By choosing the proper opera-

tors of weak measurement, our scenario shows better performance in these two processes than that which existed before. Since the success weak measurement is probabilistic, we quit the deletion process when obtain the outcome “no" via weak measurement. Once the outcome is “yes", we send the intermediate state to the quantum deletion machine. Thus, we pretreat the given qubit states to obtain higher probability of probabilistic quantum deletion and higher fidelity of approximate quantum deletion. The key thought is that we sacrifice the success probability of weak measurement in some degree to achieve our goal. It should be stressed that our scenario is valid for more than two linear independent states. This method could also be used in other quantum information processes.

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