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Dark Soliton of Polariton Condensates under Nonresonant $\mathcal{PT}$-Symmetric Pumping *

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A quantum system in complex potentials obeying parity-time ($\mathcal{PT}$) symmetry could exhibit all real spectra, starting out in non-Hermitian quantum mechanics. The key physics behind a $\mathcal{PT}$-symmetric system consists of the balanced gain and loss of the complex potential. We plan to include the nonequilibrium nature (i.e., the intrinsic kinds of gain and loss of a system) to a $\mathcal{PT}$-symmetric many-body quantum system, with an emphasis on the combined effects of non-Hermitian due to nonequilibrium nature and $\mathcal{PT}$ symmetry in determining the properties of a system. To this end, we investigate the static and dynamical properties of a dark soliton of a polariton Bose–Einstein condensate under the $\mathcal{PT}$-symmetric non-resonant pumping by solving the driven-dissipative Gross–Pitaevskii equation both analytically and numerically. We derive the equation of motion for the center of mass of the dark soliton's center analytically with the help of the Hamiltonian approach. The resulting equation captures how the combination of the open-dissipative character and $\mathcal{PT}$-symmetry affects the properties of the dark soliton; i.e., the soliton relaxes by blending with the background at a finite time. Further numerical solutions are in excellent agreement with the analytical results.

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At present, there is significant interest and ongoing efforts in investigating parity-time ($\mathcal{PT}$)-symmetric non-Hermitian quantum mechanics[1–5] in different contexts of physical systems. The motivation behind this research line is twofold. First, the introduction of $\mathcal{PT}$-symmetry has gone conceptually beyond the Hermitian quantum mechanics. Traditionally, it was well believed that any physical phenomena in quantum systems must be described by Hermitian Hamiltonians with real eigenvalues. However, non-Hermitian systems with the $\mathcal{PT}$-symmetric complex potential can have real spectra, staring out in non-Hermitian quantum mechanics.[2,6] Second, the physical systems with the $\mathcal{PT}$-symmetry can provide some enlightening applications, such as single-mode $\mathcal{PT}$ lasers[7,8] and unidirectional reflectionless $\mathcal{PT}$-symmetric meta material at optical frequencies.[9] Up to now, extension of $\mathcal{PT}$ symmetry to other branches of physics is a hot topic, such as mechanical oscillators,[10] optical waveguides,[11] and optomechanical systems.[12]

Along this line of research, there are timely efforts of extending the studies of $\mathcal{PT}$-symmetry to non-equilibrium quantum systems with emphasis on the combined effects of $\mathcal{PT}$-symmetry and the intrinsic non-equilibrium nature on a quantum system. In this end, Bose–Einstein condensates (BECs) of polaritons in quantum well semiconductor microcavities[13–16] have opened up intriguing possibilities to explore $\mathcal{PT}$-symmetric non-Hermitian quantum mechanics beyond thermal equilibrium. On the one hand, as polaritons undergo rapid radiative decay, their population in the condensate is maintained by persistent optical pumping. Hence a polariton condensate is inherently in non-equilibrium with an open-dissipative character. Its mean-field physics can be well-captured by a Gross–Pitaevskii equation (GPE) with gain and loss,[17–27] where the nonlinearity results from the strongly and repulsively interacting excitons. On the other hand, there are ongoing experimental investigations on non-Hermitian $\mathcal{PT}$-symmetric systems in the context of polariton condensates.[28,29] For example, the non-trivial topology of eigenmodes and unusual transport properties in the vicinity of exceptional points are currently under investigation.[29] Moreover, the direct observation of non-Hermitian degeneracies in a chaotic exciton-polariton billiard has been carried out in Ref. [28].

In this work, we propose and investigate an alternative route to study a non-equilibrium $\mathcal{PT}$-symmetric many-body quantum system in the context of a polariton condensate with $\mathcal{PT}$-symmetric non-resonant pumping. In more details, we focus on the dynamics of a dark soliton in a polariton condensate under the nonresonant spatial dependent pumping with $\mathcal{PT}$-symmetry by solving open-dissipative GP equation coupled to a rate equation for a density with a combination of analytical and numerical approaches. To this end, we first use the Hamiltonian approach to derive the equation of motion for the soliton parameters analytically; i.e., the velocity of the dark soliton. Then, we compare these analytical results with the numerical solutions for the trajectory of dark soli-

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tons by solving the GP equation. We find a remarkable agreement between the them as is expected. Our work would open a new perspective of including the nonequilibrium nature (i.e., the intrinsic kinds of gain and loss of a system) to a $\mathcal{PT}$-symmetric many-body quantum system.

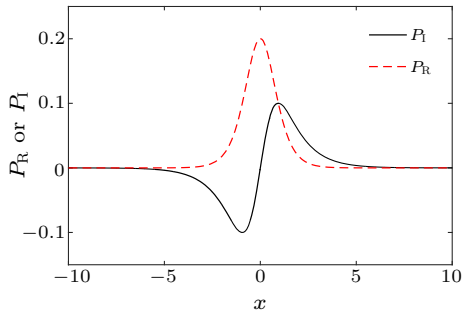

Fig. 1. The schematic profiles of the real and imaginary parts of the non-resonant PT symmetry pumping based on Eq. (3).

We are interested in the dynamics of the polariton condensate formed under uniform non-resonant pumping in a wire-shaped microcavity, as motivated by Ref. [30]. At the mean field level, the condensate can be well described by the driven-dissipative GPE characterized by the time-dependent condensate order parameter of $\psi(x,t)$, coupled to a rate equation for a density, $n_{\rm R}(x,t)$, of an uncondensed reservoir of high-energy near-excitonic polaritons as follows:[17–24]

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \Big[-\frac{\hbar^2}{2m_{\rm eff}}\nabla^2 + g_{\rm C}\left|\psi\right|^2 + g_{\rm R}n_{\rm R}(x,t) + \frac{i\hbar}{2}\left(Rn_{\rm R}(x,t) - \gamma_{\rm C}\right)\Big]\psi(x,t), \quad (1)$$

$$\frac{\partial n_{\rm R}(x,t)}{\partial t} = P\left(x\right) - \left(\gamma_{\rm R} + R\left|\psi\right|^2\right)n_{\rm R}(x,t). \quad (2)$$

In Eqs. (1) and (2), $\psi(x,t)$ and $n_{\rm R}(x,t)$ represent the order parameter of the polariton condensate and the reservoir density respectively. Here $m_{\rm eff} = 10^{-4}m_{\rm e}$ labels the effective mass of polaritons with $m_{\rm e}$ being the free electron mass; $g_{\rm C}$ and $g_{\rm R}$ characterize the strength of nonlinear interaction of polaritons and the interaction strength between reservoir and polariton, respectively. The condensed polaritons with a finite lifetime $\gamma_{\rm C}^{-1}$ are continuously replenished from reservoir polaritons at a rate R. High energy exciton-like polaritons are injected into the reservoir by laser pump $P\left(x\right)$ and relax at the reservoir loss rate $\gamma_{\rm R}$.

In this work, we consider that the $\mathcal{PT}$-symmetric pumping of $P\left(x\right)$[13–16] in Eq. (2) consist of a constant part P_0 and the spatial dependent part as $P\left(x\right) = P_0 + P_{\mathcal{PT}}\left(x\right)$. For the pumping rate P to be $\mathcal{PT}$ symmetric, $P_{\mathcal{P}}\mathcal{T}$ must be successive action of the parity ($\mathcal{P}$) and time-reversal ($\mathcal{T}$) operators: $\mathcal{PT}\{P_{\mathcal{P}}\mathcal{T}(x)\} = P_{\mathcal{P}}\mathcal{T}^*\left(-x\right) = P_{\mathcal{P}}\mathcal{T}(x)$. In more details, we are interested in $P_{\mathcal{PT}}(x)$ in the following form:

$$P_{\mathcal{PT}} = P_{\rm R} + iP_{\rm I} = V_0{\rm sech}^2\left(\frac{U\xi}{x_{\rm h}}\right) + iV_1{\rm sech}\left(\frac{U\xi}{x_{\rm h}}\right)\tanh\left(\frac{U\xi}{x_{\rm h}}\right), \quad (3)$$

with V_0 and V_1 being the amplitudes of the real and imaginary part of nonresonant $\mathcal{PT}$-symmetric pumping and $\xi = x - v_{\rm s}t$, $x_{\rm h}$ being the healing length, and $U = \sqrt{1-v_{\rm s}^2}$. As shown in Fig. 1, we plot the real part and imaginary part of $P_{\mathcal{PT}}$, corresponding to the even and odd functions respectively. Note that the imaginary part represents the gain-loss effects due to the $\mathcal{PT}$-symmetric pumping. We remark that the reservoir density $n_{\rm R}$ of Eq. (2) is supposed to be complex under the $\mathcal{PT}$-symmetric pump, which will challenge the physical explanation of $n_{\rm R}$.

The emphasis and value of this work is to include the nonequilibrium nature characterized the intrinsic kinds of gain and loss of a system, to a $\mathcal{PT}$-symmetric many-body quantum system, with the emphasis on the combined effects of non-Hermitian due to both nonequilibrium nature and $\mathcal{PT}$ symmetry in determining the properties of a system. It is clear now that the intrinsic kinds of gain and loss of a polariton condensate are described by the parameters of R and $\gamma_{\rm C}$ in Eq. (1). In contrast, the imaginary part of $\mathcal{PT}$-symmetric pumping in Eq. (3) captures the key physics of $\mathcal{PT}$ symmetry inducing gain-loss effects of the polariton condensate. Hence, the competition of non-Hermitian due to both nonequilibrium nature and $\mathcal{PT}$ symmetry in determining the properties of a system can be determined by the rich interplay among four parameters of R, $\gamma_{\rm C}$ in Eq. (1) and V_0, V_1 in Eq. (3). Next, we focus on the static and dynamical properties of a dark soliton of a polariton condensate under the $\mathcal{PT}$-symmetric non-resonant pumping by solving driven-dissipative GP description.

For simplicity, we proceed to recast Eqs. (1) and (2) into a dimensionless form by rescaling space time in units of the healing length $r_h = \hbar/\left(mc_{\rm s}\right)$ and $\tau_0 = r_h/c_{\rm s}$. Here $c_{\rm s} = \left(g_{\rm C}n_{\rm C}^*/m\right)^{1/2}$ is a local sound velocity in the condensate and $n_{\rm C}^*$ is a characteristic value of the condensate density. For a cw background, it is convenient to choose $n_{\rm C}^* = n_{\rm C}^0 = (P_0 - P_{\rm th})/\gamma_{\rm C}$ with $P_{\rm th} = \gamma_{\rm R}\gamma_{\rm C}/R$. The dimensionless equations for the normalized condensate wave function $\bar{\psi} = \psi/\sqrt{n_{\rm C}^*}$ and reservoir density $\bar{n}_{\rm R} = n_{\rm R}/n_{\rm C}^*$ take the form

$$i\frac{\partial\bar{\psi}}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \left|\bar{\psi}\right|^2 + \bar{g}_{\rm R}\bar{n}_{\rm R} + \frac{i}{2}\left(\bar{R}\bar{n}_{\rm R} - \bar{\gamma}_{\rm C}\right)\right]\bar{\psi}, \quad (4)$$

$$\frac{\partial\bar{n}_{\rm R}}{\partial t} = \bar{P}_0 + \bar{P}_{\mathcal{PT}}(x) - \left(\bar{\gamma}_{\rm R} + \bar{R}\left|\bar{\psi}\right|^2\right)\bar{n}_{\rm R}. \quad (5)$$

In the above, the dimensionless parameters are defined

as follows:

$$\bar{g}_{\rm R}=\frac{g_{\rm R}}{g_{\rm C}},\quad \bar{R}=\frac{\hbar R}{g_{\rm C}},\quad \bar{\gamma}_{\rm C}=\frac{\hbar\gamma_{\rm C}}{g_{\rm C}n_{\rm C}^{*}},\quad \bar{\gamma}_{\rm R}=\frac{\hbar\gamma_{\rm R}}{g_{\rm C}n_{\rm C}^{*}},$$
$$\bar{P}\left(r\right)=\bar{P}_{0}+\bar{P}_{\mathcal{PT}}(x)=\frac{\hbar P(x,t)}{g_{\rm C}n_{\rm C}^{*2}}.$$

The goal of this work is to investigate the dynamics of the dark soliton. To this end, we consider perturbations of the condensate wave function and the reservoir density in the following general form:[18,24]

$$\bar{\psi}(x,t)=\psi(x,t)\exp[-i(1+\bar{g}_{\rm R}\bar{\gamma}_{\rm C}/\bar{R})t], \quad (6)$$
$$\bar{n}_{\rm R}(x,t)=\bar{g}_{\rm R}\bar{\gamma}_{\rm C}/\bar{R}+m_{\rm R}(x,t). \quad (7)$$

The perturbations $\psi(x,t)$ and $m_{\rm R}(x,t)$ are governed by the dynamical equations

$$i\frac{\partial\psi}{\partial t}=\left[-\frac{1}{2}\nabla^{2}+\left|\psi\right|^{2}-1+\bar{g}_{\rm R}m_{\rm R}+\frac{i}{2}\bar{R}m_{\rm R}\right]\psi, \quad (8)$$
$$\frac{\partial m_{\rm R}}{\partial t}=\bar{P}-\left(\bar{\gamma}_{\rm R}+\bar{R}\left|\psi\right|^{2}\right)m_{\rm R}+\bar{\gamma}_{\rm C}\left(1-\left|\psi\right|^{2}\right). \quad (9)$$

Before investigating the effects of $\mathcal{PT}$-symmetric pumping on the soliton, we first briefly review some important features of a dark soliton in the nonlinear Schrödinger equation, corresponding to $m_{\rm R}(x,t)=0$ in Eq. (8); i.e., in the absence of the open-dissipative and $\mathcal{PT}$-symmetric pumping effects. There exists an exact dark-soliton solution, which takes the form

$$\psi_{\rm s}(\xi=x-\upsilon_{\rm s}t)=\sqrt{1-\upsilon_{\rm s}^{2}}\tanh(\sqrt{1-\upsilon_{\rm s}^{2}}\xi)+i\upsilon_{\rm s}, \quad (10)$$

with $\upsilon_{\rm s}$ being the velocity of the traveling dark soliton. In particular, for a moving soliton, the minimum value of density $n_{\rm C}^{\rm min}=\left|\psi_{\rm s}\left(\xi=0\right)\right|^{2}$ increases in proportion to the square of the soliton velocity $n_{\rm C}^{\rm min}=\upsilon_{\rm s}^{2}$.

Next, we proceed to include both the open-dissipative effects and $\mathcal{PT}$-symmetric pumping as captured by $m_{\rm R}(x,t)\neq 0$. Equation (10) is not the exact solution to Eq. (8) any more. Directly following Refs. [18,24,31], we plan to adopt the Hamiltonian approach for the perturbation theory of soliton. Therefore, the dark soliton's velocity becomes slow functions of time, but the functional form of the dark soliton remains unchanged; i.e., $\upsilon_{\rm s}\to\upsilon_{\rm s}(t)$ in Eq. (10). Then, the time evolution of the parameter of $\upsilon_{\rm s}$ can be routinely determined by calculating the time variation of the dark soliton's energy,

$$\frac{dE}{dt}=\upsilon_{\rm s}\int_{-\infty}^{+\infty}d\xi\left(F\left(m_{\rm R}^{0},\psi_{\rm s}\right)\frac{\partial\psi_{\rm s}^{*}}{\partial\xi}+F^{*}\left(m_{\rm R}^{0},\psi_{\rm s}\right)\frac{\partial\psi_{\rm s}}{\partial\xi}\right), \quad (11)$$

with $F\left(m_{\rm R},\psi\right)=\left(\bar{g}_{\rm R}+\frac{i}{2}\bar{R}\right)m_{\rm R}\psi$ and the energy of the dark soliton given by

$$E=\frac{1}{2}\int_{-\infty}^{+\infty}d\xi\left[\left|\frac{\partial\psi_{\rm s}}{\partial\xi}\right|^{2}+\left(1-\left|\psi_{\rm s}\right|^{2}\right)^{2}\right]$$
$$=\frac{4}{3}\left(1-\upsilon_{\rm s}^{2}\left(t\right)\right)^{3/2}. \quad (12)$$

We limit ourselves to the experiment-related fast reservoir limit; i.e., in the parameter regime of $\bar{\gamma}_{\rm C}\ll\bar{\gamma}_{\rm R}$. Directly following Refs. [18,24], we can obtain the density of reservoir as follows:

$$m_{\rm R}^{0}(\xi)=\frac{\bar{\gamma}_{\rm C}(1-\left|\psi\right|^{2})+\bar{P}_{\mathcal{PT}}(\xi)}{\bar{\gamma}_{\rm R}}, \quad (13)$$

with $\bar{P}_{\mathcal{PT}}$ being the dimensionless form $\mathcal{PT}$-symmetric pumping of Eq. (3).

Finally, by plugging Eqs. (12) and (13) into Eq. (11), one can readily arrive at the equation of motion for the dark soliton's velocity as follows:

$$\frac{d\upsilon_{\rm s}}{dt}=F_{\rm eff}, \quad (14)$$

with $F_{\rm eff}$ representing an effective force given by

$$F_{\rm eff}=\frac{1}{3}\frac{\bar{R}\bar{\gamma}_{\rm C}}{\bar{\gamma}_{\rm R}}\upsilon_{\rm s}(1-\upsilon_{\rm s}^{2})+\frac{1}{3}\frac{\bar{R}P_{1}}{\bar{\gamma}_{\rm R}}\upsilon_{\rm s}+\frac{\pi}{32}\frac{\bar{R}P_{2}}{\bar{\gamma}_{\rm R}}\sqrt{1-\upsilon_{\rm s}^{2}}. \quad (15)$$

Based on Eq. (14), we can regard the dynamics of the dark soliton as the motion of a classical particle of mass $m_{\rm eff}=1$ subjected to an external force $F_{\rm eff}$.

Based on Eq. (14), the effects of $\mathcal{PT}$-symmetric pumping in the non-equilibrium scenario on the dark soliton can be explained as follows: (i) The first term in Eq. (15) comes from the effects due to the non-equilibrium nature; the second and third terms originate from the real and imaginary part of $\mathcal{PT}$-symmetric pumping. (ii) The last two terms in Eq. (15) contain two competitive parts: when the action of the first part leads to acceleration of the dark soliton, the second part can be rewritten as $\frac{\pi}{32}\frac{\bar{R}P_{2}}{\bar{\gamma}_{\rm R}}(1-\frac{1}{2}\upsilon_{\rm s}^{2})$ in the limit of slow velocity. Then the imaginary part of $\mathcal{PT}$-symmetric pumping affects the dark soliton by giving a background fixed force of $\frac{\pi}{32}\frac{\bar{R}P_{2}}{\bar{\gamma}_{\rm R}}$ and slowing down the motion of the dark soliton with the force of $-\frac{\pi}{64}\frac{\bar{R}P_{2}}{\bar{\gamma}_{\rm R}}\upsilon_{\rm s}^{2}$. With the knowledge of $\upsilon_{\rm s}$ by solving Eq. (14), we can proceed to determine darkness of a dark soliton through the simple relation $n_{\rm C}^{\rm min}\left(t\right)=\upsilon_{\rm s}^{2}(t)$.

Below we are ready to investigate how the combined effects of the non-equilibrium nature and $\mathcal{PT}$-symmetry on a dark soliton (see Figs. 2) by numerically solving Eqs. (8) and (9) with the initial condition of Eq. (10).

As a benchmark for later analysis, we recall the case of the vanishing $\mathcal{PT}$-symmetric pumping ($P_1=P_2=0$), corresponding to the scenario of Ref. [18]. As shown in Figs. 2(a)–2(b), the non-equilibrium nature of the model system is to blend the soliton with the background at a finite time. Meanwhile, trajectory and lifetime predicted by Eq. (14) (see Fig. 2(c)) are in excellent agreement with direct numerical simulations. We begin with discussing the effect of $\mathcal{PT}$-symmetric pumping characterized by the parameters of P_1 and

P_2 on the dark soliton of a polariton condensate with the fixed nonequilibrium nature related parameters. For this purpose, we devise the following scenario: we fix the real part P_1 of $\mathcal{PT}$-symmetric pumping and investigate the role of the imaginary part P_2 on the dark soliton. In Fig. 2, we show the evolution of the condensate density distribution $n_{\rm C}(x,t) = |\psi(x,t)|^2$ and associated perturbations of the polariton reservoir density $m_{\rm R}(x,t)$ in the following figure (left and middle columns, respectively). The right column in the figure shows the time dependence of the minimum value of the condensate density associated with the dark soliton. The red line shows the darkness $n_{\rm C}^{\rm min}(t) = v_{\rm s}^2(t)$ calculated analytically using Eq. (14), and shows an excellent agreement with numerics. As shown in the left column of Fig. 2, although the lifetime of the dark soliton becomes to be shorter with the increase of the P_2, the corresponding propagation time for the dark soliton reaches $t \sim 10^2$ ps, which is much longer than the condensate and reservoir relaxation times. Note that with the further increase of P_2, it is supposed that there will exist a quantum phase transition from the $\mathcal{PT}$-symmetry phase to $\mathcal{PT}$-symmetry broken phase.[32,33] The dark soliton should be destroyed in the parameter regime of $\mathcal{PT}$-symmetry broken phase, which is beyond the scope of this work.

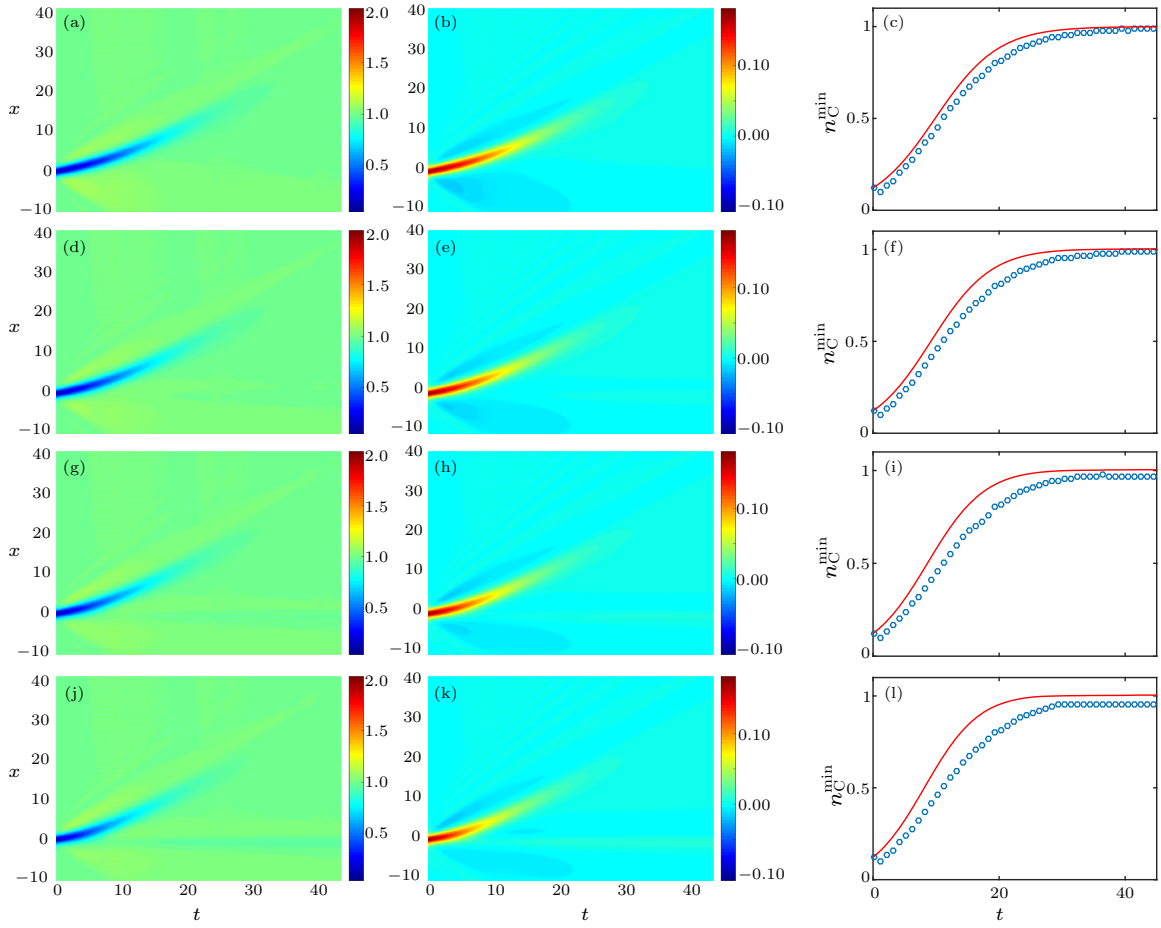

Fig. 2. Dynamics of a 1D dark soliton with the initial velocity $v_{s0} = 0.35$ in the case of weak pumping; i.e., contour plots of $n_{\rm C}(x,t)$ and the real part of $m_{\rm R}(x,t)$ and the dependence $n_{\rm C}^{\rm min}(t)$ computed using Eqs. (15) and (16). The parameters are $\bar{g}_{\rm R} = 2$, $\bar{\gamma}_{\rm C} = 3$, and $\bar{\gamma}_{\rm R} = 15$, $\bar{R} = 1.5$, for (a)–(c) $P_1 = 0$, $P_2 = 0$; (d)–(f) $P_1 = 0.01$, $P_2 = 0.3$; (g)–(i) $P_1 = 0.01$, $P_2 = 0.6$; (j)–(l) $P_1 = 0.01$, $P_2 = 0.9$.

In summary, we have investigated the dynamics of dark solitons appearing in a polariton condensates under non-resonant $\mathcal{PT}$-symmetric pumping. In particular, we have derived analytical expression of the equation of motion of the velocity of the dark soliton. Within the framework of Hamiltonian approach, our analytical results capture the essential physics as to how the combined effects of the open-dissipative and $\mathcal{PT}$ symmetry affects the lifetime of the dark solitons. We also solve the modified open-dissipative GP equation numerically. The numerical results are in agreement with the analytically ones. We also demonstrate that the dark solitons can exist for a long time in which they are influenced by the $\mathcal{PT}$-symmetric parameters.

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