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Measurement of Spin Singlet-Triplet Qubit in Quantum Dots Using Superconducting Resonator *

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The spin qubit in quantum dots is one of the leading platforms for quantum computation. A crucial requirement for scalable quantum information processing is the high efficient measurement. Here we analyze the measurement process of a quantum-dot spin qubit coupled to a superconducting transmission line resonator. Especially, the phase shift of the resonator is sensitive to the spin states and the gate operations. The response of the resonator can be used to measure the spin qubit efficiently, which can be extend to read out the multiple spin qubits in a scalable solid-state quantum processor.

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Spin qubit in quantum dots is one of the most promising candidates for realizing quantum information processing.[1−3] In recent years, there are many remarkable progresses in quantum dot spin qubits: performing single-qubit gates with high fidelity >99%,[4−7] and manipulating two-qubit with exchange interaction with fidelity >94%.[8−13] For implementing more advanced quantum information processing such as quantum error correction schemes, the measurement of spin qubit states is required to be performed within the coherence times and with high fidelity. So far, the detection of spin qubit has mostly depended on external electrometers such as quantum point contact[1] or single electron transistor.[14] However, these devices need additional electrodes and reservoirs, which precludes them from being integrated into a two-dimensional quantum dot arrays.

An alternative method for scalable architecture of semiconducting quantum dots is to use an intermediary coupler, i.e., an on-chip superconducting microwave resonator.[2] This circuit quantum electrodynamics (CQED) scheme has been achieved in coupling various quantum dot qubits to the resonator photons,[15−21] which attracts much attention due to the potential of using the spin qubits and photons as information carriers in solid state processors. As a consequence, both improving the fidelity for the control and the efficiency for readout of qubits are important topics of current research, as they are crucial for realizing quantum information processing in such systems. However, compared to the numerous studies of manipulation,[22−24] the studies of readout of the qubit in this architecture is much fewer. It is highly desirable to establish an effective measurement method for the spin qubits in such a CQED platform.

Here we demonstrate how to readout the spin states in quantum dots using the time-resolved signals of an microwave field transmitted through the resonator. Thank to the coupling between the spin qubit and the resonator, the amplitude and phase of the microwave signals of the resonator changes along with the spin qubit states. We analyze a hybrid system of electron spin qubit in double quantum dots with the superconducting resonator. Then we discuss the motion of equations describing the dynamic of the hybrid system including the dissipations. On the one hand, we simulate the stability diagram of the spin qubit, which characterizes the basic properties of the spin degrees of freedom. On the other hand, we simulate the gate operations of the spin qubit resulting in an oscillation pattern in the readout signals, which is an excellent probe for the qubit dynamics. Moreover, the CQED measurement method has the advantages that the qubit states remain unchanged after the detection.[25] These results thus pave the way towards the high efficient and non-demolition measurement of multiple spin qubits in a quantum dot processor.

As shown in Fig. 1(a), we implement a CQED scheme based on spin qubit in double quantum dots coupled to a superconducting transmission line resonator with high quality factor. In particular, we consider singlet-triplet qubit encoded in two-electron spin states.[26,27] Owing to the correlation of the spin and charge degrees of freedom, the spin-photon coupling is proportional to the charge dipole momentum and can be tuned by the voltages. As a result, a strong spin-photon coupling rate is realized to be about MHz, which paves the way to many quantum information

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applications.

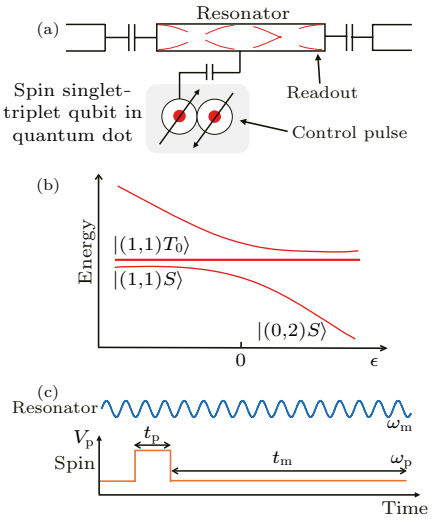

Fig. 1. (a) The schematic of the coupled system with spin qubit in double quantum dots and photons in a transmission line resonator. (b) The energy diagram of the three levels. (c) Pulse sequences used for spin qubit control and resonator readout.

The relevant two-electron states are defined as $|(1,1)T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$, and $|(1,1)S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. Here S and T represent the spin singlet and triplet state, the notation $(n_{\rm L}, n_{\rm R})$ indicates the left and right quantum dot with $n_{\rm L}$ and $n_{\rm R}$ number of electrons, respectively. Apart from this, the state $|(1,1)S\rangle$ couples to an auxiliary state $|(0,2)S\rangle$ through the coupling. Thus the quantum dots is described as a three-level Hamiltonian on the basis of $|(1,1)T_0\rangle$, $|(1,1)S\rangle$, and $|(0,2)S\rangle$:

$$H_{\rm QD} = \begin{pmatrix} 0 & dB & 0 \\ dB & 0 & t \\ 0 & t & -\epsilon \end{pmatrix}. \tag{1}$$

Here ϵ and t are the potential difference and tunneling of the two quantum dots, dB is a magnetic field gradient between the two quantum dots originating from the dynamical nuclear spin polarization[27] or integrated micromagnet.[28] The spectrum of the electrons is displayed in Fig. 1(b). We are interested in the spin singlet-triplet qubit, and the system can be expressed to an effective two-level system on the basis of $|0\rangle = |(1,1)T_0\rangle$, $|1\rangle = \cos\theta\,|(1,1)S\rangle + \sin\theta\,|(0,2)S\rangle$:

$$H_{\rm q} = \begin{pmatrix} 0 & dB \\ dB & -J \end{pmatrix}. \tag{2}$$

Here the exchange energy $J = \frac{\epsilon}{2} + \sqrt{\frac{\epsilon^2}{4} + t^2}$, and the mixing angle $\theta = \frac{1}{2}\arctan(\frac{2\,t}{\epsilon})$.

The considered superconducting transmission line resonator has the quantized voltage as $\hat{V} = \sqrt{\frac{\hbar\omega_{\rm r}}{LC_{\rm R}}}(a_k + a_k^\dagger)$.[29] Here $\omega_{\rm r} = \frac{\pi}{LZ_{\rm R}C_{\rm R}}$, L is the length of resonator, $C_{\rm R}$ and $Z_{\rm R}$ are the capacitance and impedance per unit length. Hence the transmission line resonator can be modeled by

$$H_{\rm R} = \hbar\omega_{\rm r}\left(a^\dagger a + \frac{1}{2}\right). \tag{3}$$

Here $\omega_{\rm r}$ represents the frequency of the resonator, and $a^\dagger$ and a are the creation and annihilation operators, respectively.

When coupling the quantum dots with the resonator, the energy potential contains two parts: the dc voltage ϵ_0 of the quantum dots and the voltage $\hat{V}$ of the resonator. Thus $\epsilon = \epsilon_0 + e\hat{V}\frac{C_{\rm C}}{C_{\rm QD}}$, where $C_{\rm C}$ is the capacitive coupling of the resonator to the quantum dots, and $C_{\rm QD}$ is the capacitance of the quantum dots. The interaction between the quantum dots and the resonator can be written as

$$H_{\rm I} = g(a^\dagger + a)|(0,2)S\rangle\langle(0,2)S|\,. \tag{4}$$

Here $|(0,2)S\rangle\langle(0,2)S|$ represents the charge dipole momentum of the quantum dots and the coefficient is

$$g = e\frac{C_{\rm C}}{C_{\rm QD}LC_{\rm R}}\sqrt{\frac{\hbar\pi}{Z_{\rm R}}}.$$

Putting things together, the combined system can be described by the total Hamiltonian

$$\begin{aligned} H_{\rm total} =& H_{\rm q} + H_{\rm R} + H_{\rm I} \\ =& \frac{J}{2}\widetilde{\sigma}_z + dB\widetilde{\sigma}_x + \hbar\omega_{\rm r}\left(a^\dagger a + \frac{1}{2}\right) \\ & - g\sin^2\theta(a + a^\dagger)\widetilde{\sigma}_z. \end{aligned} \tag{5}$$

Here $\widetilde{\sigma}_i$ are the Pauli matrix on the basis of $|0\rangle$ and $|1\rangle$. To obtain the electron spin eigenstates of the quantum dots, we can use the bases as $|e\rangle = \cos\phi\,|0\rangle + \sin\phi\,|1\rangle$ and $|g\rangle = \cos\phi\,|1\rangle - \sin\phi\,|0\rangle$. Here the mixing angle $\phi = \frac{1}{2}\arctan(\frac{2\,dB}{J})$ and the energy gap between the levels $\omega_{\rm q} = \sqrt{J^2 + 4dB^2}$. Now we arrive at an effective Hamiltonian of the hybrid system as follows:

$$H_{\rm eff} = \frac{\hbar\omega_{\rm q}}{2}\sigma_z + \hbar\omega_{\rm r}\left(a^\dagger a + \frac{1}{2}\right) + (a + a^\dagger)(-g_z\sigma_z + g_x\sigma_x). \tag{6}$$

Here σ_i are the Pauli matrix on the bases of $|e\rangle$ and $|g\rangle$, the coupling strengths $g_z = g\sin^2\theta\cos\phi$ and $g_x = g\sin^2\theta\sin\phi$ depending on the mixing angle θ and ϕ. Thus, the parameters g_z and g_x can be tuned by changing the spin exchange energy J and the magnetic field gradient dB.

As illustrated in Fig. 1(c), the states of spin qubit are coherently controlled by applying a pulse of amplitude $V_{\rm p}(t)$ and frequency $\omega_{\rm p}$,[11] which is resonant with the spin qubit transition frequency $\omega_{\rm q}$. Similarly, the measurement microwave field is employed through the resonator with the amplitude $V_{\rm m}(t)$ and frequency

$\omega_{\rm m}$. These external control and measurement fields are modeled by the Hamiltonian[29]

$$H_{\rm d} = V_{\rm m}(t)(a^{\dagger}e^{-i\omega_{\rm m}t} + ae^{i\omega_{\rm m}t}) + V_{\rm p}(t)(\sigma_{+}e^{-i\omega_{\rm p}t} + \sigma_{-}e^{i\omega_{\rm p}t}). \quad (7)$$

The dynamics of the hybrid system involving the dissipation and dephasing is described by the master equation[30,31]

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \kappa D[a]\rho + \gamma_l D[\sigma_-]\rho + \frac{\gamma_\phi}{2}D[\sigma_z]\rho. \quad (8)$$

Here $H = H_{\rm eff} + H_{\rm d}$ and $D[L]\rho = L\rho L^{\dagger} - \frac{1}{2}(L^{\dagger}L\rho + \rho L^{\dagger}L)$, ρ is the density matrix of the coupled system, κ is the photon decay rate of the resonator, γ_l is the qubit relaxation rate, γ_ϕ is the qubit pure dephasing rate. When the qubit transition frequency $\omega_{\rm q}$ is detuned from the resonator frequency $\omega_{\rm r}$, there is no energy exchange between the qubit and the resonator. This results in a phase shift of the microwave field depending on the qubit states. We use $\phi = \arg(i\langle\hat{a}\rangle)$ as the response signal to calculate the phase shift in different situations. When the energy detuning between the qubit and resonator $\Delta = \omega_{\rm q} - \omega_{\rm r} \gg g$, we can derive the phase shift $\phi = \arctan(2{\rm g}^2/\kappa\Delta)\sigma_{\rm z}$.

To investigate the dynamics of the hybrid system, we derive the equations of motions for the expectation values of the resonator and qubit operators $\langle a\rangle$ and $\langle\sigma_i\rangle$.[31] However, from the master Eq. (8), we obtain an infinite set of equations for these expectation values. Here we use a semiclassical factorization to truncate the infinite series to a finite set, which keeps the lower order products and factors the higher order terms.[31] For example, $\langle a^{\dagger}a\sigma_i\rangle \approx \langle a^{\dagger}\rangle\langle a\sigma_i\rangle$ and $\langle a\sigma_i\rangle \approx \langle a\rangle\langle\sigma_i\rangle$. Therefore we can acquire a complete set of four differential equations as follows:

$$\frac{d\langle a\rangle}{dt} = -i\Delta_{\rm rm}\langle a\rangle + ig_z e^{i\omega_{\rm m}t}\langle\sigma_z\rangle - ig_x e^{i\omega_{\rm m}t}\langle\sigma_x\rangle - iV_{\rm m} - \frac{\kappa}{2}\langle a\rangle, \quad (9)$$

$$\frac{d\langle\sigma_x\rangle}{dt} = -\Delta_{\rm qp}\langle\sigma_y\rangle + 2g_z(e^{-i\omega_{\rm m}t}\langle a\rangle + e^{i\omega_{\rm m}t}\langle a^{\dagger}\rangle)\langle\sigma_y\rangle - \left(\frac{\gamma_l}{2} + \gamma_\phi\right)\langle\sigma_x\rangle, \quad (10)$$

$$\frac{d\langle\sigma_y\rangle}{dt} = \Delta_{\rm qp}\langle\sigma_x\rangle - 2g_z(e^{-i\omega_{\rm m}t}\langle a\rangle + e^{i\omega_{\rm m}t}\langle a^{\dagger}\rangle)\langle\sigma_x\rangle - 2g_x(e^{-i\omega_{\rm m}t}\langle a\rangle + e^{i\omega_{\rm m}t}\langle a^{\dagger}\rangle)\langle\sigma_z\rangle - V_{\rm p}\langle\sigma_x\rangle - \left(\frac{\gamma_l}{2} + \gamma_\phi\right)\langle\sigma_y\rangle, \quad (11)$$

$$\frac{d\langle\sigma_z\rangle}{dt} = 2g_x(e^{-i\omega_{\rm m}t}\langle a\rangle + e^{i\omega_{\rm m}t}\langle a^{\dagger}\rangle)\langle\sigma_y\rangle + V_{\rm p}\langle\sigma_y\rangle - \gamma_l(1 + \langle\sigma_z\rangle). \quad (12)$$

Here $\Delta_{\rm rm} = \omega_{\rm r} - \omega_{\rm m}$ and $\Delta_{\rm qp} = \omega_{\rm q} - \omega_{\rm p}$ are the detuning of resonator from measurement frequency and qubit from pulse frequency, respectively.

To investigate the spin qubit spectroscopy with different values of parameters dB and J, we initialize the spin qubit in the ground state and let the hybrid system evolve for a long time to reach a steady state. Then we measure the phase shift of the resonator. Here we simulate the results by solving Eqs. (9)–(11) numerically with the realistic parameters as the experiments $\omega_{\rm r}/2\pi = 6.0\,{\rm GHz}$, $\kappa/2\pi = 1\,{\rm MHz}$, $g/2\pi = 50\,{\rm MHz}$, $\gamma_l = 0.5\,{\rm MHz}$, $\gamma_\phi = 100\,{\rm MHz}$.[16,32]

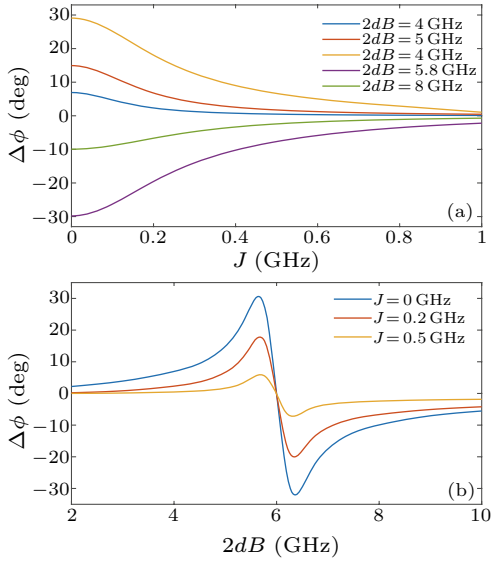

Fig. 2. (a) The phase shift $\Delta\phi$ as a function of the exchange energy J for a range of magnetic field gradient dB. (b) The phase shift as a function of dB for different J.

As shown in Fig. 2(a), we plot the phase shift $\Delta\phi$ of the resonator as a function of the exchange energy J ranging from 0 to 1 GHz with several fixed values of magnetic field gradient dB. In this case, as the value of J increases, the absolute value of the phase shift $\Delta\phi$ decreases and approaches to zero. This result can be qualitatively analyzed as follows: when the exchange energy J is large, the energy level splitting of spin qubit is also large. Therefore there is a large detuning between the spin qubit and the resonator, which induces a small phase shift of the resonator. Conversely, when $J = 0$, there appears a maximum absolute value of the phase shift.

As shown in Fig. 2(b), we can observe a sign change of the phase shift $\Delta\phi$ as dB varies. With the increasing dB, the absolute value of the phase shift increases and has a maximum value. Then there appears a sharp reduction of $\Delta\phi$ from positive to negative near $2dB = 6\,{\rm GHz}$. This behavior can be explained by the fact that when the spin qubit is tuned into resonance with the resonator, there are energy exchanges between them. From these figures, we can conclude that the spin qubit spectroscopy can be efficiently probed by the signals of the coupled resonator.

As shown in Fig. 1(c), we apply a control pulse to the spin qubit. Due to the employed pulse, the

qubit starts to evolve during the pulse time t_p. When the pulse is completed, the spin qubit stays in the superposition of the ground and excited states. Simultaneously, we record the continuous monition of the phase shift of the resonator. Here we also numerically simulate the results with Eqs. (9)–(11) and the input parameters as mentioned above.

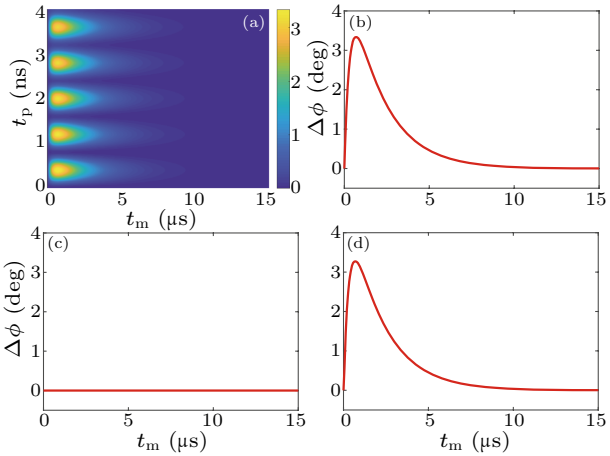

Fig. 3. (a) The phase shift $\Delta\phi$ of the resonator as functions of measurement time t_m and control pulse length t_p. A clear qubit Rabi oscillation pattern is obtained in the phase shift domain. (b)–(d) The phase shift $\Delta\phi$ as the measurement time t_m, corresponding to the π-pulse, 2π-pulse, and 3π-pulse, is applied to the spin qubit, respectively.

As shown in Fig. 3(a), we observe that the phase shift $\Delta\phi$ correlates with the measurement time t_m and pulse length t_p. When a control pulse is applied to manipulate the spin qubit, there will appear a clear periodical Rabi oscillation pattern of the phase shift. In Fig. 3(b), when $t_p = 0.4$ ns, we see a time dependence of the phase shift. Before the pulse is applied, the qubit is in the ground state and the phase shift has a minimum value which is approximately 0. Because the applied π-pulse pulls the qubit from the ground state to an excited state, the phase shift of the resonator rises rapidly to a maximum value of 3.3°. Additionally, since the qubit would decay from its excited state to the ground state exponentially within the relaxation time $T_1 = 1/\gamma$, the phase shift would also decay correspondingly. As the measurement time t_m extends above 5 μs, we can hardly observe any phase shift signal. As a result, the signal of the phase shift is a response to the competition between the control pulse and decay process of the qubit.

In Fig. 3(c), when $t_p = 0.8$ ns, no phase shift is observed since the qubit returns to its ground state under the 2π-pulse and the control time t_p is much shorter than the resonator response time. As shown in Fig. 3(d), when $t_p = 1.2$ ns (i.e., 3π-pulse), the phase shift is nearly the same as that for $t_p = 0.4$ ns. This is due to fact that the time scale of Rabi oscillation is much shorter than the coherence time and the relaxation time of the qubit. Also, we notice that the phase shift signal is not completely the same as the control time t_p extending, because the existence of decoherence of qubit which results in the decay of Rabi oscillation amplitude. Overall, we can probe the dynamics of a quantum-dot spin qubit by monitoring the phase shift of the microwave resonator.

In summary, we have studied the readout dynamics of a quantum dot spin qubit coupled to a superconducting resonator. The phase shift of the resonator can be used to measure the spectroscopy and the gate operations of the spin qubit. The present methods will contribute to the continued improvement of tools and techniques for quantum information processing with spin qubits in semiconductor nanostructures.[33] Moreover, this method is universal and can be applied to other solid state spin systems.

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