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Long-Range Interaction Enhanced Adiabatic Quantum Computers

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A quantum computer is not necessarily alone, e.g., thousands and millions of quantum computers are simultaneously working together for adiabatic quantum computers based on nuclear spins. Long-range interaction is inevitable between these nuclear spin qubits. Here we investigate the effect of long-range dipolar interaction between different adiabatic quantum computers. Our analytical and numerical results show that the dipolar interaction can enhance the final fidelity in adiabatic quantum computation for solving the factorization problem, when the overall interaction is negative. The enhancement will become more prominent if a single quantum computer encounters an extremely small energy gap which occurs more likely for larger-size systems.

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More and more powerful quantum computers have been realized in various physical systems, such as superconducting quantum computers,[1,2] trapped ions,[3−5] cold/ultracold trapped neutral atoms,[6] photonic quantum computers,[7−9] and nuclear magnetic resonance (NMR) quantum computers.[10,11] In most cases, a quantum computer is well isolated from its environment in order to suppress noises which may decohere the qubits in the quantum computer.[12] However, the NMR quantum computers, which employ many nuclear spins as the working qubits, are always in a very large number, because it is almost impossible to separate an isolate quantum computer (usually a large molecule with many nuclear spins) from other quantum computers.[13] In practice, it is exceptionally challenging for a single NMR quantum computer to prepare, control and measure its quantum state.

For these NMR quantum computers, local and short-range interaction has rather limited effect or is well controlled on the performance of the quantum computation. Once the long-range interaction such as the dipolar interaction is present, the quantum computer's performance may be significantly affected, because there are thousands and millions of other quantum computers interacting with it. Although the long-range dipolar interactions between different-type qubits in two quantum computers may be suppressed with different chemical shifts, the dipolar interactions between the same-type qubits are always inevitable. To our best knowledge, the effect of such a long-range interaction on the performance of the NMR quantum computers has not been investigated before.

In this Letter, we investigate the effect of the dipolar interaction between two adiabatic quantum computers, which carry out the algorithm to solve the number factorization problem. Originally proposed by Farhi *et al.*,[14] adiabatic quantum computation is based on the quantum adiabatic theorem: a quantum system would remain in its instantaneous eigenstate if the system Hamiltonian varies slowly enough and there is a gap between this eigenstate and the rest of the Hamiltonian eigenstate.[15] This method has been proven to be equivalent to the standard quantum computation model.[16,17] Adiabatic quantum algorithms solving problems such as 3-SAT and integer factorization have been studied both theoretically and experimentally.[18−20] While the standard quantum computation model faces difficulties like lack of robustness against dephasing and decoherence, adiabatic quantum algorithms can overcome these difficulties by choosing to evolve on the instantaneous ground state.[21] The adiabatic computation approach may fail when the energy gap becomes very small, as the system requires very long evolution time to stay adiabatic.[22−25] Many methods have been proposed to improve the performance of the algorithm. While approaches like alternating the route of the evolution and creating shortcut in the algorithm may improve the performance significantly, the requirement on the complete knowledge of the ground state renders it extremely challenging.[26−28]

In general, the long-range dipolar interaction is treated as a source of quantum noise, which may cause dephasing, decoherence and relaxation.[29−31] As a result, this noise could potentially cause relaxation from higher energy states to the ground state, which actually enhances the efficiency of an adiabatic quantum computation. Moreover, the dipolar interaction can split the near-degenerate ground states in the small-gap cases and also improve the performance of the algorithm. Without loss of generality, we investigate the effect of a constant dipolar interaction between two adiabatic quantum computers. We find that the long-range dipolar interaction indeed enhances the performance of the quantum computation in a fairly large

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parameter region. Our results point out an easy and possible way to improve the performance of NMR quantum computers. Theoretically, other types of adiabatic quantum computers can simulate the dipolar interaction to achieve the same improvement of performance in various adiabatic quantum algorithms, as our analysis is not constrained to specific configurations in our numerical result.

We consider in specific two quantum computers running adiabatic algorithm while interacting with long-range dipolar interaction. The adiabatic algorithm is

$$H_{1,2}(t) = [1 - s(t)]H_0 + s(t)H_{\rm p}, \tag{1}$$

where $H_0 = g\sum\sigma_x^i$ is a given Hamiltonian at the start with $\sigma_{x,y,z}^i$ being the Pauli matrix of the ith nuclear spin and g a constant, s is a transition function taking the form t/T, and $H_{\rm p}$ represents the question with its ground state to be the answer.[14] If the total evolution time T is large enough, the system stays on the ground state, and the answer is obtained after the evolution. In our study, $H_{\rm p}$ corresponds to the factorization problem,[13]

$$\begin{aligned} H_{\rm p} &= (\hat{N} - \hat{X}\hat{Y})^2, \\ \hat{X} &= I + \sum_{i=1}^{n_x} 2^{n_x - i}\frac{I - \sigma_z^i}{2}, \\ \hat{Y} &= I + \sum_{i=1}^{n_y} 2^{n_y - i}\frac{I - \sigma_z^{i+n_x}}{2}. \end{aligned} \tag{2}$$

The total number of qubits (nuclear spins) for a single quantum computer is $n_x + n_y$. The two adiabatic quantum computers, which are described by $H_1(t)$ and $H_2(t)$ if isolated, are interacting via the following dipolar interaction,[32] $[\gamma_i\gamma_j(3\cos^2\theta - 1)/2r^3](\boldsymbol{I}_i \cdot \boldsymbol{I}_j - 3I_z^iI_z^j)$, where γ_i and γ_j are the nuclear gyromagnetic ratio, r the distance between the two quantum computers and θ the angle between $\boldsymbol{r}$ and the external magnetic field. We assume that the distance r is much larger than the nuclei distance within an adiabatic quantum computer, so that we may neglect other deviations for different nuclei in two adiabatic quantum computers. Consequently, the interaction strength stays constant throughout the evolution. Due to the fact that the chemical shift of nuclei is much larger than the dipolar interaction strength between two adiabatic quantum computers, a dipolar decoupling scheme can eliminate the dipolar interaction between different-type qubits, the residue is quite small.[33] Here, for the purpose of simplification we assume that the dipolar interaction only exists between the same-type qubits in the two adiabatic quantum computers. The Hamiltonian of the dipolar interaction is modeled as

$$H_{\rm D} = \gamma\sum_{i=1}^{n_x+n_y}\alpha_i(\sigma_x^{1i}\sigma_x^{2i} + \sigma_y^{1i}\sigma_y^{2i} - 2\sigma_z^{1i}\sigma_z^{2i}), \tag{3}$$

where α_i is assumed to be a random positive number between 0 and 1, taking into account of different type nuclear spins, the superscript 1 or 2 indexes the quantum computer, and γ is the overall interaction strength which could be negative due to the term $3\cos^2\theta - 1$. The total Hamiltonian for the two quantum computers becomes

$$H(t) = H_1(t) + H_2(t) + H_{\rm D}. \tag{4}$$

Before we carry out numerical simulations, we analytically study the system with perturbation theory, since the dipolar interaction may be treated as a perturbation if r is large enough. The eigenstates of the single quantum computer $H_{1,2}$ without the dipolar interaction are $|\psi_{11}\rangle = |\varphi_1\rangle\otimes|\varphi_1\rangle$, $|\psi_{12}\rangle = |\varphi_1\rangle\otimes|\varphi_2\rangle$, $|\psi_{21}\rangle = |\varphi_2\rangle\otimes|\varphi_1\rangle$, and $|\psi_{22}\rangle = |\varphi_2\rangle\otimes|\varphi_2\rangle$, where $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are the ground state with energy ε_1 and the first excited state with energy ε_2 of a single adiabatic quantum computer. Without the dipolar interaction, it is clear that $|\psi_{11}\rangle$ is the ground state, and $|\psi_{12}\rangle$ and $|\psi_{21}\rangle$ are two degenerate states. Using the first order perturbation theory, the ground state's energy with the dipolar interaction becomes

$$E_1 = 2\varepsilon_1 + \langle\psi_{11}|H_{\rm D}|\psi_{11}\rangle. \tag{5}$$

For the first excited state, by applying the degenerated perturbation theory on ψ_{12} and ψ_{21}, we obtain

$$\left|\gamma\begin{pmatrix}\alpha_1 & \alpha_2\\ \alpha_2^* & \alpha_1\end{pmatrix} - (E - \varepsilon_1 - \varepsilon_2)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right| = 0, \tag{6}$$

where we have defined

$$\begin{aligned} \langle\psi_{12}|H_{\rm D}|\psi_{12}\rangle &= \langle\psi_{21}|H_{\rm D}|\psi_{21}\rangle = \gamma\alpha_1, \\ \langle\psi_{12}|H_{\rm D}|\psi_{21}\rangle &= \gamma\alpha_2, \quad \langle\psi_{21}|H_{\rm D}|\psi_{12}\rangle = \gamma\alpha_2^*. \end{aligned}$$

Solving Eq. (6), then we can obtain two eigenvalues of the degenerate states as follows:

$$\begin{aligned} E_2 &= \varepsilon_1 + \varepsilon_2 + \gamma\alpha_1 - |\gamma\alpha_2|, \\ E_3 &= \varepsilon_1 + \varepsilon_2 + \gamma\alpha_1 + |\gamma\alpha_2|. \end{aligned}$$

Thus, the change of the energy difference between the ground and the first excited state is

$$\Delta_\gamma - \Delta_0 = \gamma\alpha_1 - |\gamma\alpha_2| - \langle\psi_{11}|H_{\rm D}|\psi_{11}\rangle,$$

where $\Delta_\gamma = E_2 - E_1$ and $\Delta_0 = \varepsilon_2 - \varepsilon_1$. This result suggests that the gap changes linearly with the dipolar strength γ because $H_{\rm D}$ is proportional to γ. Therefore, there may exist a suitable region of γ, where the gap will increase with γ if $|\gamma\alpha_1 - \langle\psi_{11}|H_{\rm D}|\psi_{11}\rangle| > |\gamma\alpha_2|$. As a larger gap usually produces a higher success probability in adiabatic quantum computation, this suitable region of γ actually improves the performance of the quantum algorithm.

The effect of dipolar interaction can be viewed from another perspective. We adopt the instantaneous eigenstate picture, and transform $H(t)$ into the instantaneous eigenstate picture of $H_1(t)+H_2(t)$; $|\phi_m\rangle$ is the

eigenstates when $|\varphi_l\varphi_j\rangle$ have been sorted according to the corresponding eigenvalue, and the corresponding eigenvalue can form a diagonalize matrix $E(t)$,

$$|\psi\rangle = A|\psi'\rangle, \quad A = (|\phi_1\rangle \ \ldots |\phi_m\rangle \ldots |\phi_{2^{2(n_x+n_y)}}\rangle)\,. \tag{7}$$

We can obtain the evolution equation in the instantaneous eigenstate picture as follows:

$$i\frac{\partial}{\partial s}|\psi'\rangle = \Big(TE + TA^\dagger H_{\rm D} A - iA^\dagger \frac{\partial A}{\partial s}\Big)|\psi'\rangle, \tag{8}$$

the dipolar interaction term becomes $TA^\dagger H_{\rm D} A$. Here, we use the interaction picture, setting $TA^\dagger H_{\rm D} A$ as the interaction Hamiltonian. The evolution operator for $TE - iA^\dagger \frac{\partial A}{\partial s}$ is

$$U_0(s) = S\exp\int_0^1 -i\Big(TE - iA^\dagger \frac{\partial A}{\partial s}\Big)ds. \tag{9}$$

The Hamiltonian in the interaction picture is $U_0^\dagger(s)TA^\dagger H_{\rm D} AU_0(s)$, and the evolution operator is

$$U_I = S\exp\int_0^1 -iU_0^\dagger(s)TA^\dagger H_{\rm D} AU_0(s)ds, \tag{10}$$

under perturbation, we have

$$U_I = I - \int_0^1 iU_0^\dagger(s)TA^\dagger H_{\rm D} AU_0(s)ds, \tag{11}$$

and the final state in the interaction picture is

$$|\psi_I'(1)\rangle = \Big(I - \int_0^1 iU_0^\dagger(s)TA^\dagger H_{\rm D} AU_0(s)ds\Big)|\psi'(0)\rangle. \tag{12}$$

In the instantaneous eigenstate picture, the final state is

$$\begin{aligned}&|\psi'(1)\rangle\\ &= U_0(1)\Big(I - \int_0^1 iU_0^\dagger(s)TA^\dagger H_{\rm D} AU_0(s)ds\Big)|\psi'(0)\rangle.\end{aligned} \tag{13}$$

Assume that the final state of the evolution with no dipolar interactions is $|\psi'_{\rm sep}(1)\rangle$, then the final state with dipolar interactions is

$$\begin{aligned}&|\psi'(1)\rangle\\ &= |\psi'_{\rm sep}(1)\rangle - U_0(1)\int_0^1 iU_0^\dagger(s)TA^\dagger H_{\rm D} AU_0(s)ds|\psi'(0)\rangle.\end{aligned} \tag{14}$$

This suggests that the final fidelity f is linearly dependent on γ.

As we have two identical adiabatic quantum computers in the system, the fidelity of a single adiabatic quantum computer is $\tilde{f} = \sqrt{f}$. The success rate for a single quantum computer is $F = \tilde{f}$. This suggests that either a positive dipolar strength γ or a negative γ can increase the final fidelity. The physical implication is clear, the dipolar interaction negates part of the excitation caused by $iA^\dagger \frac{\partial A}{\partial s}$, thus increasing the final fidelity. If the Hamiltonian consists of N quantum computers, we would only obtain a different $|\phi_m\rangle$ and $E(t)$, the rest of the analysis is the same. This indicates that the conclusion holds for a general NMR system where there are N quantum computers.

Here, we note that the perturbation theory only holds when $\gamma \ll T^{-1}$, the approximation will break down if the dipolar strength becomes too great. In the large γ limit beyond the perturbation theory, $H_{\rm D}$ may destroy the final ground state and lower the success probability of the algorithm.

The analysis we provide here is a general analysis on two adiabatic quantum computers with interactions, as we did not use specified adiabatic quantum algorithm $H(t)$ and $H_{\rm D}$ can be replaced with any kind of interaction as long as it is governed by one overall interaction strength parameter γ. Theoretically this analysis can be applied to any adiabatic quantum algorithms with an interaction governed by one overall interaction strength parameter γ.

Numerically, we adopt the Chebyshev polynomial expansion method to expand the evolution operator up to the expansion order 3000 within a small time step Δt, where the Hamiltonian is treated as a constant $H_n = H(n\Delta t)$, and Δt is dependent on both the maximal energy and the smoothness of $H(t)$.[34] We set $g = 30$ in the Hamiltonian. For a single adiabatic quantum computer, we simulate the factorization process for different evolution times. We consider the factorization problem that is solved once the success probability is higher or equal to 1/8 and record the evolution time T.[35,36] Numerically, we find that the success evolution time T falls into two categories:[36] (i) one is $T < 10$ which we call the factorization problem "easy", (ii) the other is $T > 100$ (calculation stops at $T > 100$ due to insufficient computational power) which we call the factorization problem "hard". In fact, the success evolution time is estimated to be larger than 500 for all the hard problems we calculated. It is reasonable to assume that for larger systems the standard on T for "hard" problems would be larger.

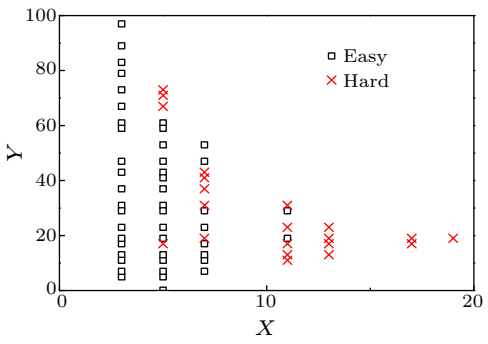

Fig. 1. Distribution of easy and hard problems. X and Y are the two factors of N.

The distribution of the easy and hard problems is present in Fig. 1. We find that the small size problems

are usually in the easy category while the large size problems are in the hard one. This trend may be due to the gap decreasing for a large size system. In addition, we also find a few hard problems in the region of easy ones, e.g., the factorization of $N = 85$ which has an exceptionally long evolution time T. Indeed, the gap in the case of $N = 85$ is extremely small, as shown in Fig. 2. Compared to the typical easy problem for $N = 39$, whose energy gap is about 10, the gap for $N = 85$ is in the order of 0.01, almost 3 orders of magnitude smaller.

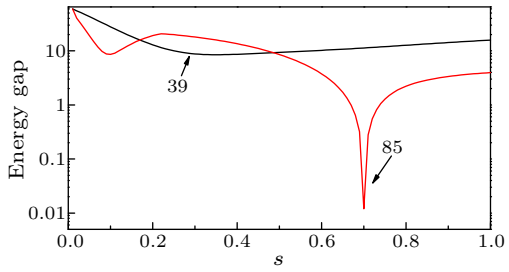

Fig. 2. Comparison of the energy gaps for the case of 85 and 39. The case of 85 has a gap of ∼0.01, while the case of 39 has a gap of ∼10.

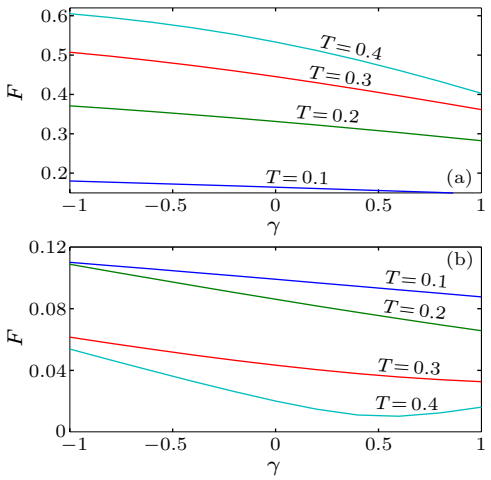

Fig. 3. The effect of the dipolar interaction between two adiabatic quantum computers for (a) a typical easy problem $N = 39$ and (b) a hard problem $N = 85$. The success probability F changes linearly with the interaction strength γ when $|\gamma|$ is small.

Now we introduce the long-range dipolar interaction between two adiabatic quantum computers. We investigate the effect of the dipolar interaction in these two problem categories, in particular the hard one. We map the relation between fidelity $\tilde{f}(\gamma, T)$ and the interaction strength γ and evolution time T for all the different N's that require 7 qubits or less. In addition, we employ an enhancement exponent k to assess the improvement of the quantum computer's performance due to the long-range dipolar interaction,

$$k = \max_{\gamma,T} \left\{ \frac{\ln[1 - \tilde{f}(\gamma, T)]}{\ln[1 - \tilde{f}(0, T)]} \right\}, \tag{15}$$

where k denotes the number of evolutions needed for non-interacting cases to match the interacting cases when the enhancement is largest for all possible T we simulate. Statistically, by repeating k times the computation with a success probability F, the success probability of having at least one successful event is $1 - (1 - F)^k$.

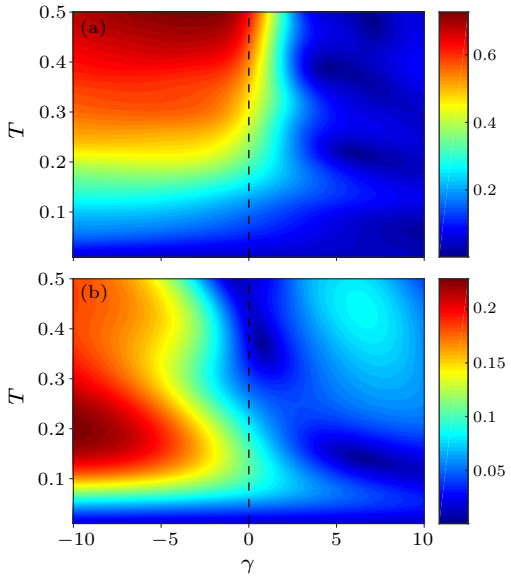

Fig. 4. Typical results for the effect of the dipolar interaction between two adiabatic quantum computers for (a) $N = 39$ and (b) $N = 85$. The color defines the final success probability after the evolution. The vertical black line in the middle marks $\gamma = 0$. One observes enhanced success probability in the region $\gamma < 0$ for both easy and hard categories: (a) $N = 39$, $k = 1.59$; (b) $N = 85$, $k = 11.7$.

Two typical results with $N = 39$ and $N = 85$ are shown in Figs. 3 and 4 (see more details in the Supplementary Information). The factorization of $N = 39$ belongs to the easy category and $N = 85$ the hard one. For both the cases, we find linear dependence of the success probability F on the dipolar interaction strength γ for different evolution times. For $N = 39$, the perturbation theory is valid so that we can find

$$F \approx F_0 + \frac{dF}{d\Delta}\Big|_{\Delta_0} (\Delta_\gamma - \Delta_0),$$

and $F \propto \gamma$ because $(\Delta_\gamma - \Delta_0) \propto \gamma$. These numerical results confirm the validity of previous analytical predictions based on the perturbation theory. It is interesting that the hard problem $N = 85$ shows similar linear dependence, except that nonlinearity appears around $\gamma \approx 0.5$ for $T = 0.4$. We note that the hard problem $N = 85$ exhibits unusual behavior, i.e., the success probability decreases as the evolution time T increases for $\gamma = 0$.

More numerical results for $N = 39$ and $N = 85$ are presented in Fig. 4. At a first glance, both the graphs show that when γ is negative, the dipolar interaction increases the success probability of the ground state significantly with k larger than 1 (indicating the improvement of the performance of the quantum computers). This confirms our analytical result again.

Also shown in the figure, the success probability exhibits a decrease when $|\gamma|$ becomes too large. This manifests the phenomenon that the strong dipolar interaction actually destroys the structure of the original Hamiltonian for the isolated quantum computers.

The case of $N = 85$ is very special due to an unexpected extremely small gap. Both negative and positive dipolar interaction strength γ could increase the fidelity of the ground states when evolution time T is around 0.4. As a result, its improvement exponent $k = 11.7$ is very large. While the gap size decreases inevitably in a quantum adiabatic algorithm when the number of qubits increases, the occasion when the gap turns unusually small is not a rarity. Such a small gap will result in an absurd amount of time required to achieve an adiabatic evolution, and that could be a serious problem in practice. For these special hard problems, one could potentially speed up the quantum algorithm significantly if the dipolar interaction affects these cases just like in the case of $N = 85$.

In summary, we have investigated the effect of long-range dipolar interaction on two quantum computers running adiabatic quantum algorithm. We analytically derive that the gap changes linearly with weak dipolar strength, as a result, the success probability changes linearly with dipolar strength. Our numerical results suggest that a negative dipolar interaction can improve the performance of the adiabatic quantum algorithm solving the number factorization problem. For the hard problems with an extremely small gap, the dipolar interaction can significantly improve the quantum computers' performance. However, the structure of the problem Hamiltonian may be destroyed if the dipolar interaction is too strong, and the success probability is actually lowered. As our analytical analysis is general, the dipolar interaction should work for other adiabatic quantum algorithms, such as scalable adiabatic quantum algorithms for solving exact cover problems[37] or factorizations.[18] Also, any adiabatic quantum computer that can introduce dipolar interaction, or establish interactions like dipolar interaction should exhibit the same enhancement described in this study, and there could be other kinds of interaction which may better enhance the computation.

References

[1] Stassi R, Cirio M and Nori F 2020 *npj Quantum Inf.* **6** 67
[2] Wang B N, Hu F, Yao H N and Wang C 2020 *Sci. Rep.* **10** 7106
[3] Bruzewicz C D, Chiaverini J, Mcconnell R and Sage J M 2019 *Appl. Phys. Rev.* **6** 021314
[4] Grzesiak N, Blümel R, Beck K, Wright K, Chaplin V, Amini J M, Pisenti N C, Debnath S, Chen J S and Nam Y 2020 *Nat. Commun.* **11** 2963
[5] Yang B Y and Yang L 2020 *Sci. Chin. Inf. Sci.* **63** 202501
[6] Sheng C, He X D, Xu P, Guo R J, Wang K P, Xiong Z Y, Liu M, Wang J and Zhan M S 2018 *Phys. Rev. Lett.* **121** 240501
[7] Qiang X G, Zhou X Q, Wang J W, Wilkes C M, Thomas L, O'Gara S, Laurent K, Marshall G D, Raffaele S and Ralph T C 2018 *Nat. Photon.* **12** 534
[8] Ji W T, Zhang L, Wang M Q, Zhang L, Guo Y H, Chai Z H, Rong X, Shi F Z, Liu X J, Wang Y and Du J F 2020 *Phys. Rev. Lett.* **125** 020504
[9] Zhang J F, Hegde S S and Suter D 2020 *Phys. Rev. Lett.* **125** 030501
[10] Cui J Y, Li J, Liu X M, Peng X H and Fu R Q 2018 *J. Magn. Reson.* **294** 83
[11] Xin T, Li Y S, Fan Y, Zhu X R, Zhang Y J, Nie X F, Li J, Liu Q H and Lu D W 2020 *Phys. Rev. Lett.* **125** 090502
[12] Ladd T D, Jelezko F, Laflamme R, Nakamura Y, Monroe C and O'Brien J L 2010 *Nature* **464** 45
[13] Peng X H, Liao Z Y, Xu N Y, Qin G, Zhou X Y, Suter D and Du J F 2008 *Phys. Rev. Lett.* **101** 220405
[14] Farhi E, Goldstone J, Gutmann S and Sipser M 2000 arXiv:quant-ph/0001106
[15] Kato T 1950 *J. Phys. Soc. Jpn.* **5** 435
[16] Mizel A, Lidar D A and Mitchell M 2007 *Phys. Rev. Lett.* **99** 070502
[17] Yu H Y, Huang Y L and Wu B 2018 *Chin. Phys. Lett.* **35** 110303
[18] Xu N Y, Zhu J, Lu D W, Zhou X Y, Peng X H and Du J F 2012 *Phys. Rev. Lett.* **108** 130501
[19] Wu B, Yu H Y and Wilczek F 2020 *Phys. Rev. A* **101** 012318
[20] Peng W C, Wang B N, Hu F, Wang Y J, Fang X J, Chen X Y and Wang C 2019 *Sci. Chin.-Phys. Mech. Astron.* **62** 1
[21] Childs A M, Farhi E and Preskill J 2001 *Phys. Rev. A* **65** 012322
[22] Amin M H S 2008 *Phys. Rev. Lett.* **100** 130503
[23] Rezakhani A T, Abasto D F, Lidar D A and Zanardi P 2010 *Phys. Rev. A* **82** 012321
[24] Choi V 2010 arXiv:1004.2226v1 [quant-ph]
[25] Kay A 2015 *Phys. Rev. A* **92** 062329
[26] Campbell S and Deffner S 2017 *Phys. Rev. Lett.* **118** 100601
[27] Berry M V 2009 *J. Phys. A* **42** 365303
[28] Mustafa D and Stuart A R 2008 *J. Chem. Phys.* **129** 154111
[29] Orioli A, A P, S, Wildhagen H, Günter G, Berges J, Whitlock S and Weidemüller M 2018 *Phys. Rev. Lett.* **120** 063601
[30] Witzel W M, Carroll M S, Cywiński L and Sarma S D 2012 *Phys. Rev. B* **86** 035452
[31] Cywiński L, Witzel W M and Sarma S D 2009 *Phys. Rev. Lett.* **102** 057601
[32] Klinowski J 2017 *NMR of Solids* 3rd edn (Oxford: Academic Press)
[33] Warren W S, Richter W, Andreotti A H and Farmer B T 1993 *Science* **262** 2005
[34] Dobrovitski V V and De Raedt H A 2003 *Phys. Rev. E* **67** 056702
[35] Farhi E, Goldstone J, Gutmann S, Lapan J, Lundgren A and Preda D 2001 *Science* **292** 472
[36] Shi A Q, Guan H Y and Zhang W X 2020 *Phys. Lett. A* **384** 126745
[37] Choi V 2011 *Proc. Natl. Acad. Sci. USA* **108** E19–E20

Supplementary Information: Long-range interaction enhanced adiabatic quantum computers

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Complete numerical result for all calculated cases

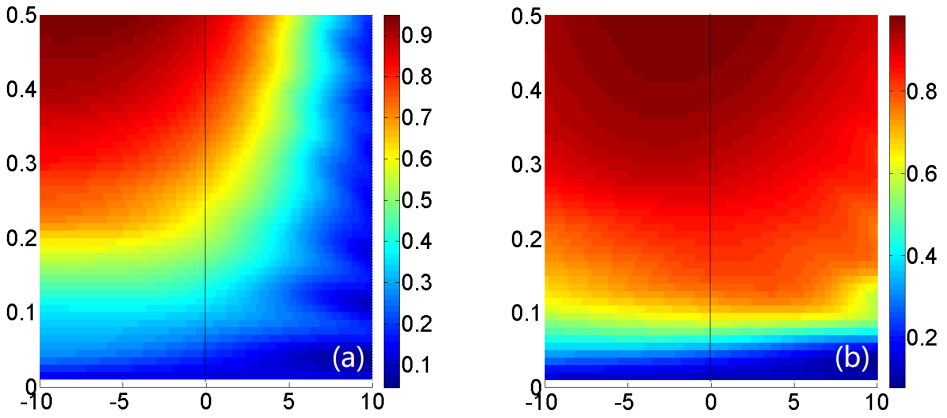

FIG. S1: Numerical results for 3 qubits, (a) $N = 15$, $k = 1.53$, (b) $N = 21$, $k = 1.36$. The horizontal axis is the strength of the dipolar interaction γ, the vertical axis is the evolution time T and the color defines the final fidelity. The vertical black line in the middle marks $\gamma = 0$.

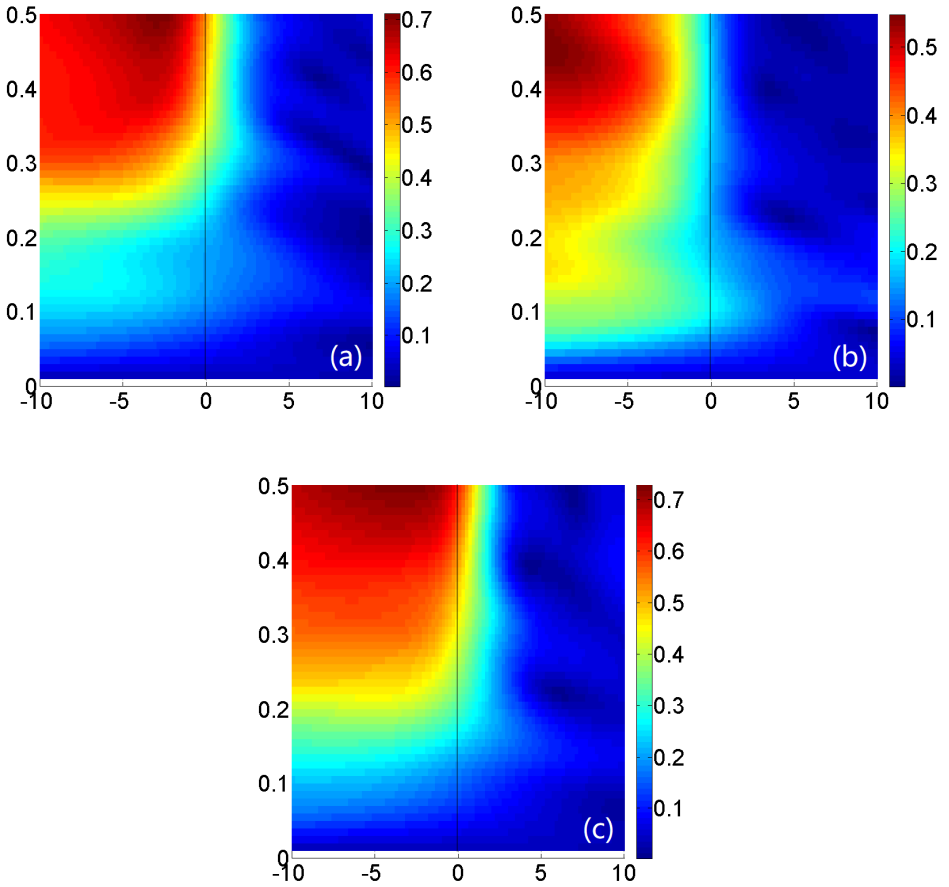

FIG. S2: Same as Fig. S1 except for 5 qubits, (a) $N = 33$, $k = 1.85$, (b) $N = 35$, $k = 4.01$, (c) $N = 39$, $k = 1.59$.

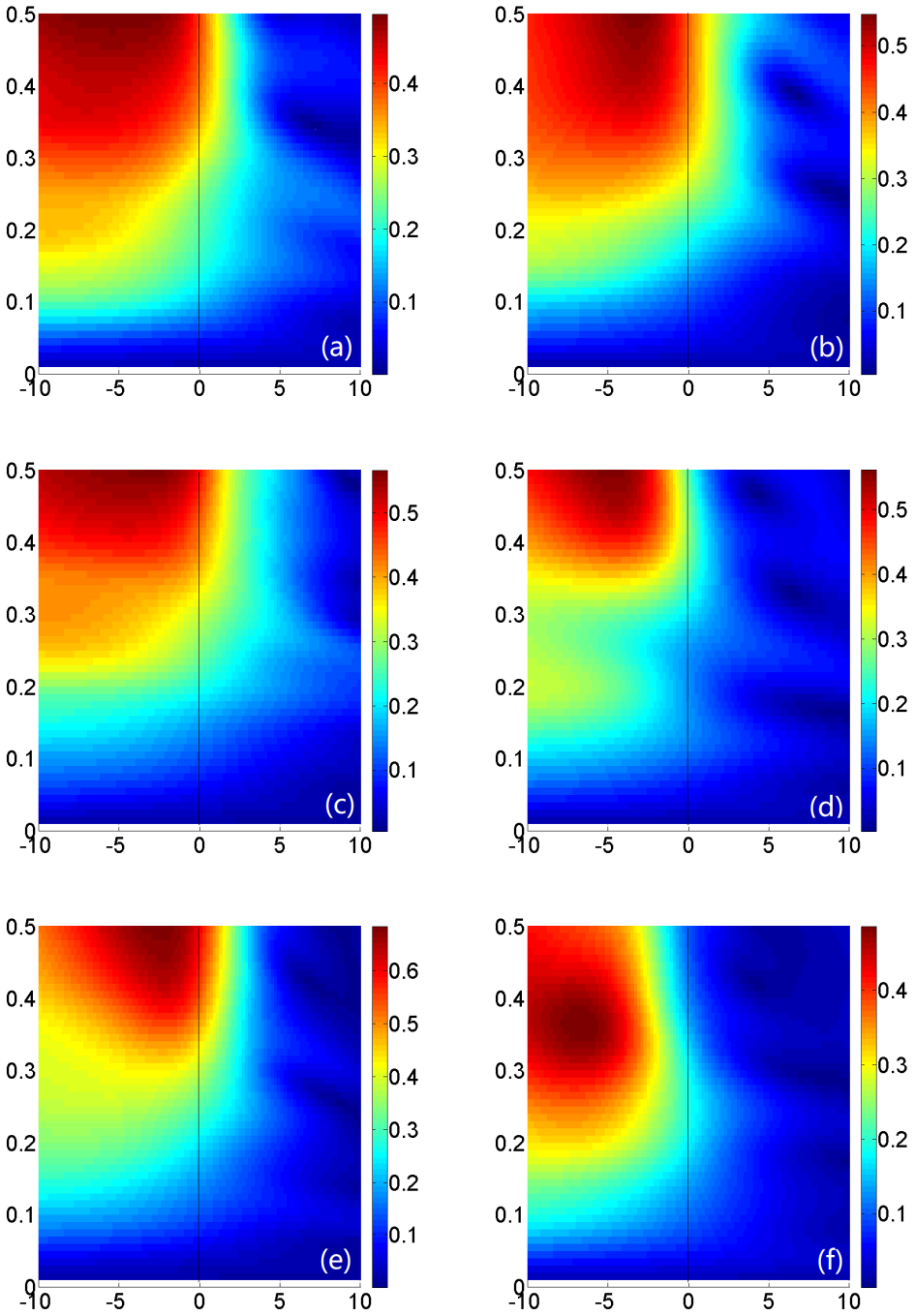

FIG. S3: Same as Fig. S1 except for 6 qubits, (a) $N = 51$, $k = 1.59$, (b) $N = 55$, $k = 1.94$, (c) $N = 57$, $k = 1.53$, (d) $N = 65$, $k = 2.73$, (e) $N = 69$, $k = 1.61$, (f) $N = 77$, $k = 4.97$.

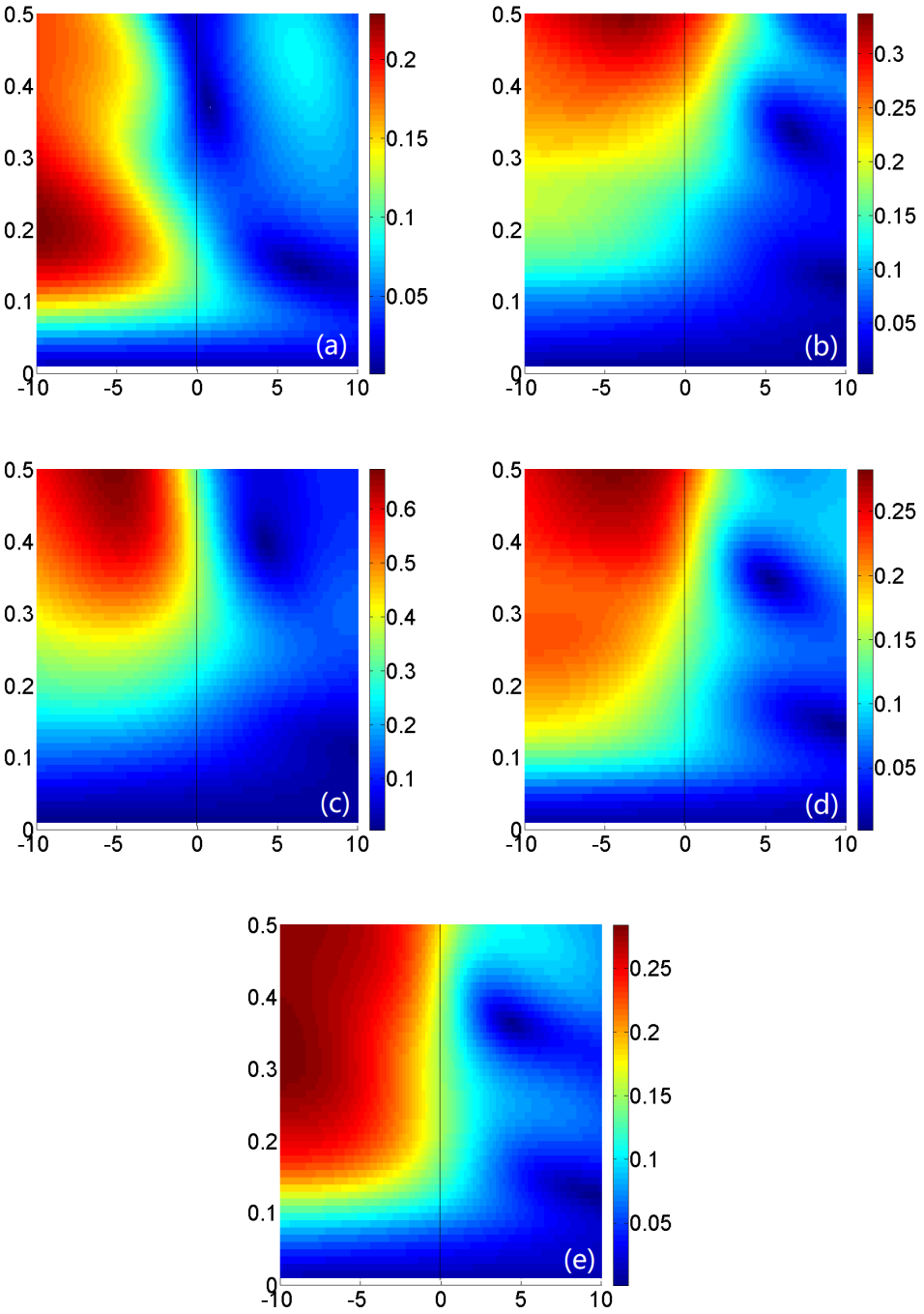

FIG. S4: Same as Fig. S1 except for 7 qubits, (a) $N = 85$, $k = 11.7$, (b) $N = 87$, $k = 1.59$, (c) $N = 91$, $k = 2.96$, (d) $N = 93$, $k = 1.69$, (e) $N = 95$, $k = 2.05$.

Complete energy landscape for 39 and 85

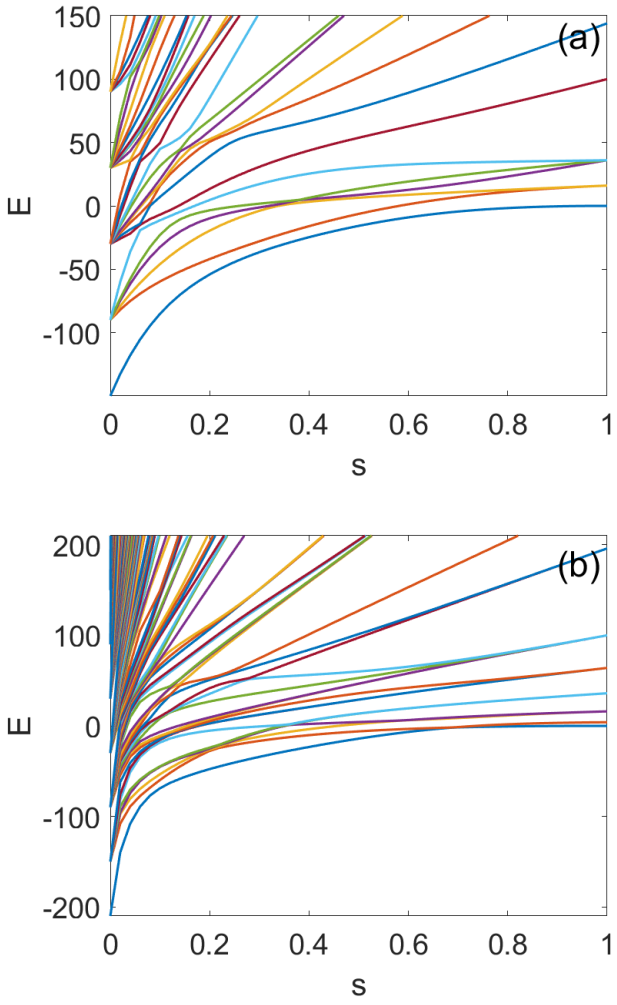

FIG. S5: (a) Energy landscape for 39, (b) Energy landscape for 85.