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# Topological Distillation by Principal Component Analysis in Disordered Fractional Quantum Hall States

Na Jiang(江娜)* and Min Lu(卢旻)

*Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou 310027, China*

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We study the behavior of two-dimensional electron gas in the fractional quantum Hall (FQH) regime in the presence of disorder potential. The principal component analysis is applied to a set of disordered Laughlin ground state model wave function to enable us to distill the model wave function of the pure Laughlin state. With increasing the disorder strength, the ground state wave function is expected to deviate from the Laughlin state and eventually leave the FQH phase. We investigate the phase transition from the Laughlin state to a topologically trivial state by analyzing the overlap between the random sample wave functions and the distilled ground state wave function. It is proposed that the cross point of the principal component amplitude and its counterpart is the critical disorder strength, which marks the collapse of the FQH regime.

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Fractional quantum Hall (FQH) phases are often represented by model wave functions, which characterize the topological order of the phases. These model wave functions can be obtained via diagonalizing pseudopotential Hamiltonians,[1] composite fermion construction,[2] calculating conformal blocks,[3] and Jack polynomials.[4] The prototypical example is the Laughlin state,[5] whose topological aspects are reflected, e.g., in the ground state degeneracy,[6] its edge mode counting,[7] the edge tunneling exponent,[8] and the structure of the entanglement spectrum.[9] When effective mass and/or interaction anisotropy is introduced, the state can have an additional geometrical degree of freedom,[10] and the corresponding anisotropic model wave function can be obtained by a unimodular transformation of the isotropic Laughlin state.[11] These states are said to belong to a single family of Laughlin states sharing exactly the same topology.[10] Meanwhile, with the effect of an in-plane magnetic field, the system also has a strong anisotropy.[12] For small anisotropy, the spectral gap remains robust.[13]

On the other hand, impurities and imperfections are always present in real samples and essential for the quantum Hall plateau. Disorder tends to suppress the *spectral gap*, but no suitable FQH wave function other than the isotropic model wave function can be used to describe the disordered states. An FQH-insulator transition occurs at sufficiently large disorder when the *mobility gap* closes, which can be detected by, e.g., Chern number fluctuations.[14] However, the characteristic disorder strengths for the two gap closures are practically the same, at least in numerically accessible systems.[15] Recently, quantum entanglement-based quantities, such as disorder-averaged entanglement entropy and level statistics of the entanglement spectrum, have also been applied successfully to detect this transition.[16–18] Compared to the gap calculation, quantum entanglement calculations involve ground states only and are, hence, less time-consuming. Ground state information can also be used to calculate fidelity[18] and tunneling matrix element,[19] both of which allow the identification of the critical point.

In this Letter, we transfer the ideas that we have learned from anisotropic FQH systems[20] to disordered states. We regard the ground states of an FQH system with varying disorder strength as a family of wave functions, topologically nontrivial below the critical disorder and trivial above. Unlike the anisotropic case, the family of wave functions is not controlled by a single geometrical parameter or, possibly, a few parameters at some filling factors. In fact, a collection of parameters exist, whose correspondences to low-energy excitations in the clean system are identified by the principal component analysis (PCA). We demonstrate that the topology of FQH wave functions, encoded in the model wave functions, can be distilled from disordered wave functions even in the insulating phase in which the spectral gap vanishes. We propose a new diagnostic for the FQH-insulator transition based on the PCA study.

Firstly, we discuss the FQH model with rotational symmetric disorder and the PCA method. Secondly,

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*Corresponding author. Email: 0617234@zju.edu.cn

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we show that model wave functions can be distilled from disordered wave function by the PCA. We compare the ability of the distillation to wave function overlaps. Thirdly, we demonstrate that the weight of the PCA can be used to identify the critical point of the FQH-insulator transition. We compare the result to that of the spectral gap. Also the origin of the principal components are presented in the disorder-free spectrum. Finally, we summarize our results.

*Model and Method.* To model an FQH state in the presence of semi-realistic disorder, we consider the following lowest Landau level projected Hamiltonian

$$H = H_{\rm int} + U_W \tag{1}$$

in the disk geometry, where the interaction among electrons is

$$H_{\rm int} = \frac{1}{2}\sum_{m_i=0}^{N_{\rm orb}-1} V_{m_1m_2m_3m_4} c^\dagger_{m_1} c^\dagger_{m_2} c_{m_3} c_{m_4}. \tag{2}$$

Here $c^\dagger_m$ creates an electron in the orbital with angular momentum $m$, whose single-particle wave function is

$$\psi_m(z) = (2\pi 2^m m!)^{-1/2} z^m e^{-|z|^2/4}, \tag{3}$$

where $z = x + iy$ is the complex coordinate of the 2D electron. For simplicity, we consider a short-range hard-core interaction with the Haldane pseudopotential $V_m = \delta_{1,m}$, such that the densest zero-energy ground state is the Laughlin state.

We consider a random disorder potential

$$U_W = \sum_m \epsilon_m c^\dagger_m c_m, \tag{4}$$

where $\epsilon_m$ are chosen randomly by uniform distribution in range $[-W, W]$. The potential can be thought of as resulting from the orbital average of a rotational-symmetry breaking potential, so the total angular momentum $M_{\rm tot}$ remains to be a good quantum number. Therefore, we can focus on the angular momentum sector of the Laughlin state.

PCA is a dimensional reduction technique,[21] such that it projects the original data into a relatively lower dimensional spaces. At the same time it extracts the dominating feathers of the original data into the fewest possible dimensions of the new spaces.[22] It has been used to explore many physical problems.[23–28] The main advantage of PCA in our study on the FQH system is that we can extract the topological characterization of Laughlin ground state efficiently without knowing its exact coefficients (weights).

In this work, we apply PCA to the disordered ground states. The ground state wavefunction with disorder strength $W$ can be expressed as

$$|\Psi_{\rm sample}\rangle = \left|\psi^{W^n_m}\right\rangle = \sum_{i=1}^{D} c_i^{W^n_m} |i\rangle, \tag{5}$$

where $m$ and $n$ are the label of disorder strengths and disorder samples respectively, while $D$ is the dimensions of the many-particle basis $|i\rangle$. Thus, we obtain $(M \times N) \times D$ matrix, $M$ is the total number of different disorder strengths and $N$ is the sample number for each disorder. We collect the coefficients into the data matrix $X$ in the form of

$$X = \begin{pmatrix} c_1^{W_1^1} & c_2^{W_1^1} & \cdots & c_D^{W_1^1} \\ \vdots & \vdots & & \vdots \\ c_1^{W_1^N} & c_2^{W_1^N} & \cdots & c_D^{W_1^N} \\ c_1^{W_2^1} & c_2^{W_2^1} & \cdots & c_D^{W_2^1} \\ \vdots & \vdots & & \vdots \\ c_1^{W_2^N} & c_2^{W_2^N} & \cdots & c_D^{W_2^N} \\ \vdots & \vdots & & \vdots \\ c_1^{W_M^N} & c_2^{W_M^N} & \cdots & c_D^{W_M^N} \end{pmatrix}. \tag{6}$$

After that, a singular value decomposition is applied,

$$X = U\Sigma V^{\rm T}, \tag{7}$$

where $V$ is the $D \times D$ orthogonal matrix containing eigenvectors of $X$ in each column, $\Sigma$ is a diagonal matrix with diagonal elements $\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots, \sqrt{\lambda_D}$. The first $d$ columns of $V$ are the first $d$ principal components, they contain most of the total variance of the matrix $X$. The eigenvalues of the covariance matrix $X^{\rm T}X$ are $\lambda_1, \lambda_2, \ldots, \lambda_D$. They are known as explained variance ratios (EVRS).

In the following, we apply PCA to study the disordered wave functions at a fixed disorder strength $W$ (disorder sector) or mix-disordered strengths $W_{\rm s}$. In the first situation, we explore topological information of Laughlin state for different disorder strengths, even in the case that the Laughlin state is destroyed by a larger disorder strength. In the latter one, PCA can locate the FQH-insulator critical point accurately.

*Results—Topological Distillation.* We demonstrate the PCA's ability to distill the topological characterization of Laughlin state from a disorder sector of ground state wave functions. We choose $N$ electrons subject to the short-range $V_1$ interaction and rotationally invariant disorder $U_W$ in $N_{\rm orb} = 3N - 2$ orbitals where the $-2$ is the topological shift for the Laughlin state, such that $\nu = 1/3$ in the thermodynamic limit, and we work in the angular momentum subspace $M_{\rm tot} = 3N(N-1)/2$ as that for the Laughlin state.

We first demonstrate the effect of disorder on the Laughlin ground state by investigating the change in the shape of distribution of disorder-to-clean state overlap. We start our observation with a relatively small disorder strength $W = 0.2$, at which we generate 1000 disordered Laughlin wave functions for $N = 6$. We plot the distribution of the overlaps with the exact Laughlin ground state in Fig. 1(a). The distribution

peaks at 0.96 with a long tail on the left side. The mean of the distribution is 0.946, and the median is 0.953. In particular, the largest overlap is 0.9913.

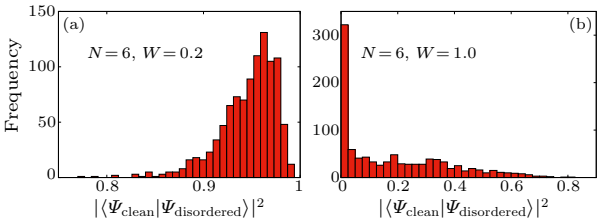


**Fig. 1.** Histograms of the overlap between the Laughlin state and the ground states at disorder strengths $W = 0.2$ (a) and 1.0 (b). The system has $N = 6$ electrons at $\upsilon = 1/3$ with $V_1$ interaction.

We further explore the case for $W = 1.0$. The distribution has one sharp peak at 0 with a long tail stretching to 0.83. The mean of the distribution is 0.184 and the median is 0.126. From the stark contrast in the shape of the projection distribution, we may conclude that for $W = 0.2$, the state is still a Laughlin-like FQH state, while for $W = 1.0$, the state has left FQH phase and entered in an insulating phase. The critical disorder strength lies somewhere in between 0.2 and 1.0. We will investigate $W_{\rm c}$ in the following.

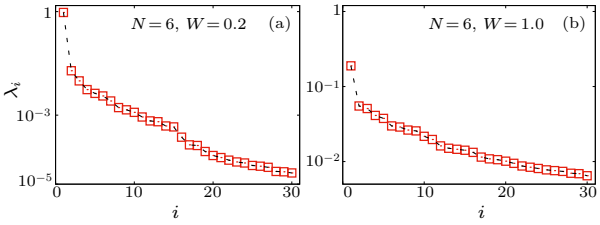


**Fig. 2.** Explained variance ratios $\lambda_i$ extracted by PCA for $N = 6, N_{\rm orb} = 16, M_{\rm tot} = 45$ at fixed disorder strength $W = 0.2$ (a) and $W = 1.0$ (b). The first leading 30 components are plotted in semi-log scale. The results are generated from 1000 samples for each disorder strength.

We plot the leading explained variance ratios (EVRs) $\gamma_i$ for $i = 1$–30 for disorder strength $W = 0.2$ and $W = 1.0$ in Fig. 2. In Fig. 2(a), the largest one $\gamma_1 = 0.946$ dominates the EVRs and, in particular, is one order of magnitude larger than $\gamma_2 = 0.019$. The desired wave function $\Psi_{\gamma_1}$, corresponding to the largest EVR, can be found in the first row of $V^{\rm T}$. It has an overlap of 0.99989 with the Laughlin state, which is significantly (two orders of magnitude) better than the disordered wave function that has the largest overlap. In Fig. 2(b), we also find that $\gamma_1$ is one order of magnitude larger than $\gamma_2$. The wave function $\Psi_{\gamma_1}$ has an overlap 0.98697 with the Laughlin state. Therefore, we demonstrate that PCA can extract a wave function that has a larger projection to the Laughlin state than the sample-Laughlin projection, hence we name the process of obtaining $\Psi_{\gamma_1}$ by PCA as topological distillation.

We continue the investigation on the distillation of the Laughlin state by increasing disorder strength $W$ for systems with $N = 4$–8 electrons. Figure 3 shows the system-size dependence of $|\langle\Psi_{\gamma_1}(N,W)|\Psi_{\rm Laughlin}\rangle|^2$ for various $W$. For $N = 4, 5$ and 6, $\Psi_1$ has above 99% overlaps. While for $W = 0.2$ and 0.4, the overlaps stay near unity for all $N$'s. For $W = 1.0$ and for $N = 7$ and 8, the overlaps drop to 98% and 96%, respectively. These results agree with the fact that topological properties of FQH systems are robust against weak disorder. More interestingly, even in a disordered system, where a significant portion of topologically trivial states exist, PCA is still able to extract its topologically nontrivial characterization.

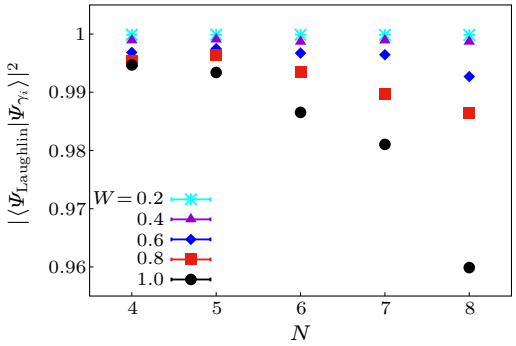


**Fig. 3.** Overlaps of the distilled wave functions and the Laughlin state $|\langle\Psi_{\rm Laughlin}|\Psi_{\gamma_i}\rangle|^2$ as a function of system sizes $N = 4$–8 with fixed disorder strengths $W = 0.2, 0.4, 0.6, 0.8, 1.0$, respectively.

*FQH–Insulator Transition.* In the above section, we have found a range that the critical disorder $W_{\rm c}$ may exist from projection distribution, which is between $W = 0.2$ and $W = 1.0$. Now, we apply PCA to the data matrix of mix-disorder ground state wave functions to calculate the exact value of $W_{\rm c}$. Generally, the ground state topological properties are robust in the presence of a weak disorder, and the system will be destroyed when the disorder strength is enhanced. We still consider $V_1$ interaction with rotationally invariant disorder $U_W$. We have generated 1000 samples for systems sizes at $N = 4, 5, 6, 7, 8$ via exact diagonalization in the angular momentum subspace $M_{\rm tot} = 3N(N-1)/2$.

We feed PCA with ground state wave functions from different disorder sectors $W_{\rm s} = 0.1$–1.5 with interval 0.1. As shown in Fig. 4, the largest EVRs $\gamma_1 = 0.418$ dominate, which is one order of magnitude larger than $\gamma_2 = 0.045$, and the first leading component $\Psi_{\gamma_1}$ contains the dominating features in the mixed disordered dataset. Physically, PCA redistributes the data into a new set of complete basis $|\Psi_{\gamma_i}\rangle$ and $\gamma_i$ are corresponding weights. We name the quantity $|\langle\Psi_{\gamma_i}|\Psi_{\rm sample}\rangle|^2$ are the amplitudes of the principal components, then the weights are averaged amplitudes over all samples. To discover the evolution

of the system with respect to the effect of increasing disorders, we investigate the amplitudes in each disorder sector separately. For small enough disorder strength, the amplitude on the first leading component is nearly one. As the disorder strength increasing, the amplitudes on the first leading one drops down. When the value of amplitude drops to 50%, it means that the ground state has been destroyed by the disorder from the first leading component to some other components. So we can propose a new diagnostic for the phase transition point by disorder strength from the crossing of the projections on the first leading one and the sum of left components.

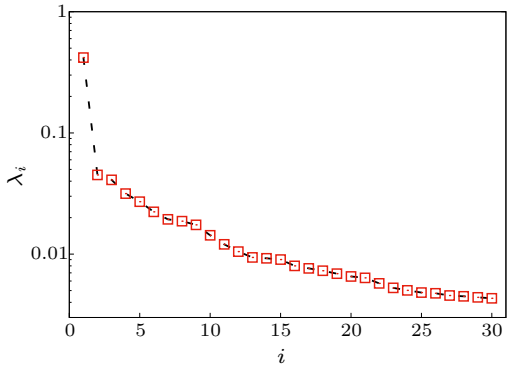


**Fig. 4.** Explained variance ratio $\lambda_i$ extracted by PCA at a mixed disorder range from $W_s = 0.1$ to 1.5 with increment of 0.1. They are obtained from raw ground state wavefunctions for $N = 6$, $N_{orb} = 16$, $M_{tot} = 45$. The leading 30 components are plotted in semi-log scale.

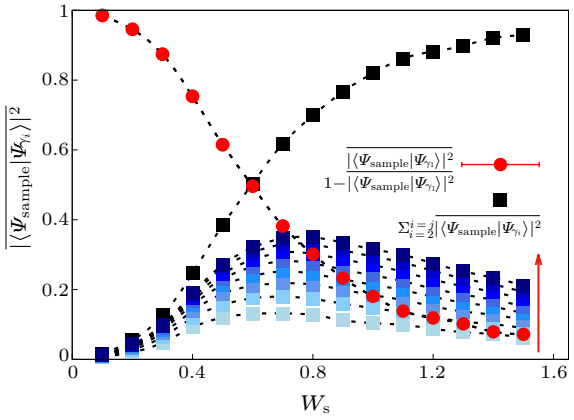


**Fig. 5.** Averaged sample projections square to the leading components. The red points are projections of the ground state wavefunctions to the first leading component $\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_1}\rangle|^2}$ as a function of disorder strength $W_s$. The black squares are sum of averaged squared projections to all components except the first leading one, e.g., $1-\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_1}\rangle|^2}$. The light-blue to dark-blue points are the sum of averaged squared projections from the second leading components to $j$th leading component, e.g., $\sum_{i=2}^{i=j}\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_i}\rangle|^2}$ respectively.

We now turn to the amplitudes of the principal components $\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_i}\rangle|^2}$ and have obtained 15 averaged amplitudes for each $\gamma_i$ in a specific system size $N$. In Fig. 5, we interpolate a curve for each set of the 15 partial sums for $N = 6$. The averaged amplitudes of the leading component $\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_1}\rangle|^2}$ is plotted in red points. In addition, the sum of averaged amplitudes from 2nd to $i$th components are plotted in blue squares. We observe that as $i$ approaches $D$, the sum of amplitudes will be exactly $1-\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_1}\rangle|^2}$. In addition, the sum of averaged amplitudes on first nine components $\overline{|\langle\Psi_{\rm sample}|\Psi_{\gamma_i}\rangle|^2}$ have been shown from light-blue to dark-blue squares. Therefore we plot the unity-counter part of the principal component amplitude in black and locate the crossing point of the two supplementary curves. $W_c(\mathrm{PCA}) = 0.6$ indicates that the state has an equal probability to enter the $\Psi_{\gamma_1}$ phase or $\Psi_{\gamma_i}$ orthogonal to it. This critical disorder strength marks the boundary of the topologically nontrivial FQH phase.

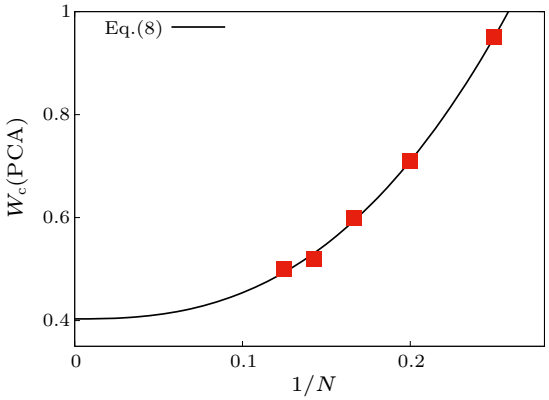


**Fig. 6.** The critical disorder strengths $W_c(\mathrm{PCA})$ of different systems are extracted by PCA and the scaling of $W_c(\mathrm{PCA})$ as a function of $1/N$, e.g., $N = 4, 5, 6, 7, 8$. We can determine $W_c(\mathrm{PCA}) \approx 0.403$ in the thermodynamic limit.

We also apply the same analysis to $N = 4, 5, 7, 8$ electron systems to obtain the critical disorder strengths $W_c(\mathrm{PCA})$. From these finite-size results, we are able to extract a scaling relation for $W_c(\mathrm{PCA})$ and approximate $W_c(\mathrm{PCA})$ in the thermodynamic limit. As soon as we have carried out the finite-size scaling, we adopt the polynomial to fit the critical disorder strength for simplification. As shown in Fig. 6 we fit $W_c(\mathrm{PCA})$ as a function of $1/N$ and obtain

$$C\left(\frac{1}{N}\right) = 19.91\left(\frac{1}{N}\right)^{2.59} + 0.40. \tag{8}$$

The $y$-intercept $W_c(\mathrm{PCA}) = 0.403 \pm 0.030$ is the critical disorder strength that marks the collapse of FQH phase.

For comparison, we perform the critical disorder calculation using PCA and some other methods, such as energy gap between the first excited state and the ground state, $\Delta E = E_1 - E_0$. We have carried out the averaged energy gap with different disorder strengths for different electron systems ($N = 5 \sim 10$),

e.g., $\overline{\Delta E_W}$. For each disorder strength, we can plot $\overline{\Delta E_W}$ against $1/N$ to extrapolate energy gap ($\Delta E_W^{\rm lim}$) in the thermodynamic limit. In Fig. 7. The values of thermodynamic limit energy gap $\Delta E_W^{\rm lim}$ for $W = 0.1, 0.2, 0.3, 0.4, 0.5$ have been obtained. Then, we extrapolate a scaling behavior between $W$ and $\Delta E_W^{\rm lim}$, and find out the critical disorder where the energy gap closed in the thermodynamic limit:

$$\Delta E_W^{\rm lim}(W) = 0.302 - 0.775W. \tag{9}$$

The critical disorder strength $W_{\rm c}$ can be obtained from energy gap by solving for the $x$-intercept of this line. In this approach, $W_{\rm c}$ comes out to be $0.390 \pm 0.002$, which agrees with the PCA result $W_{\rm c}$(PCA), $0.403 \pm 0.030$. The agreement of the critical disorder has proven that the PCA method can be successfully worked in the disorder problem.

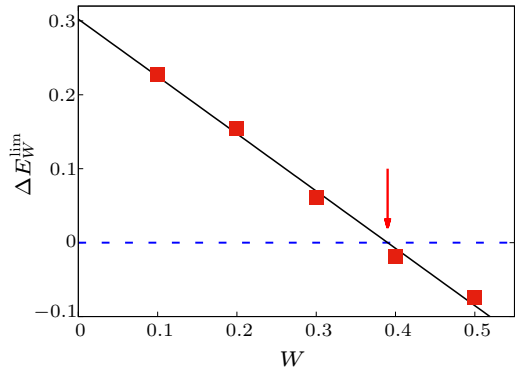


**Fig. 7.** The energy gaps in the thermodynamic limit $\Delta E_W^{\rm lim}$ can be obtained from the scaling of different electron systems ($N = 5-10$) for each disorder strength. Then the critical disorder strength, which means zero energy gap in the thermodynamic limit, is shown by the arrow, which is $W_{\rm c} \approx 0.390$.

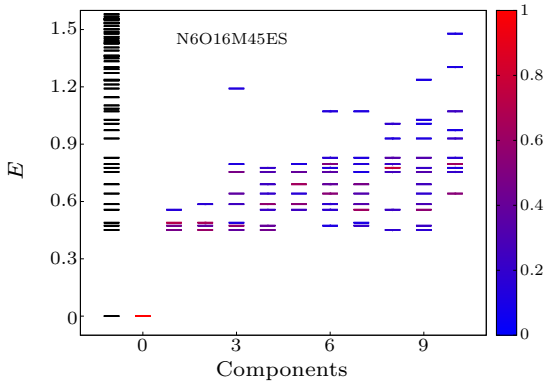


**Fig. 8.** The origin of the principal components extracted from the PCA results. The black bars are the energy spectrum of the free disorder system, such as $N = 6$, $N_{\rm orb} = 16$, $M_{\rm tot} = 45$. The colorful bars are the states which have nonzero overlaps between components and states in the free disorder spectrum. Different colors present values of overlaps from 0 to 1.

However, what is the origin of the principal components? In the rotational invariant model, we have performed exact diagonalization in individual momentum sectors of the Laughlin state, so the components all come from the same momentum sector. We have obtained the first leading component, which have a nearly unity overlap with Laughlin state. This means that the PCA extracts the topological information of this system as one character. From the analysis of Fig. 5, the main character will be destroyed by enhanced disorder strength. Therefore, what are characters that have been shown from PCA except the topological information? In Fig. 8, we have calculated the overlaps between components and the free disorder spectrums and labeled the values by colors. As with the first component (label 0 in the $x$-axis), we label the red bar, and the overlap value is presented in the palette. From the origin of the other components (components $= 1, \ldots, 9$), which contain almost the remaining information of the dataset $X$ except the first component in the FQH regime, we can find out that they almost come from the low energy excited states.

In conclusion, we have explored the disorder effect in a fractional quantum Hall system. In previous studies, the critical disorder of FQH-insulator transition was obtained from mobility gap and quasiparticle tunneling amplitudes. By using the degrees of freedom in the center of mass system, the bulk energy gap had been obtained in disk geometry for the thermodynamic limit.[29] In this study, we have explored related physics in a disordered system via a popular statistical learning method, i.e., the principal component analysis.

We have calculated the overlaps between the distilled wave functions and Laughlin state. It is demonstrated that PCA reduces the dimension of input data to extract the dominant characters, i.e., the topological information of the ground state wave function. Interestingly, the ground state can be distilled even the input data is partially in the insulating phase at a large disorder strength. The FQH-insulator transition is also explored by PCA in this study. We project the input ground state wave functions onto the principal component wave functions. The critical disorder obtained from the crossing of projections agrees well with the result obtained from the ground state energy gap results. Motivated by the thermal Hall conductance experiments,[30] understanding the effect of the disorder is very important to the $\frac{5}{2}$ state. We can also use this property in the model of Coulomb interaction with realistic impurities in future works.

We thank Xin Wan for useful discussion.

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