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Influence of Coating Layer on Acoustic Wave Propagation in a Random Complex Medium with Resonant Scatterers *

Hang Yang(杨杭)[1], Xin Zhang(张欣)[1**], Jian-hua Guo(郭建华)[2**],
Fu-gen Wu(吴福根)[3], Yuan-wei Yao(姚源卫)[1]

[1]*School of Physics and Optoelectronic Engineering, Guangdong University of Technology, Guangzhou 510006*
[2]*School of Materials Science and Engineering, South China University of Technology, Guangzhou 510640*
[3]*Department of Experiment Education, Guangdong University of Technology, Guangzhou 510006*

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We investigate the influence of coating layer on acoustic wave propagation in a dispersed random medium consisting of coated fibers. In the strong-scattering regime, the characteristics of wave scattering resonances are found to evolve regularly with the properties of the coating layer. By theoretical calculation, frequency gaps are found in acoustic excitation spectra in a random medium. The scattering cross section results present the evolution of scattering resonances with the properties of the coating layer, which offers a good explanation for the change of the frequency gaps. The velocity of the propagation quasi-mode is also shown to depend on the filling fraction of the coating layer. We use the generalized coherent potential-approximation approach to solve acoustic wave dispersion relations in a complicated random medium consisting of coating-structure scatterers. It is shown that our model reveals subtle changes in the behavior of the acoustic wave propagating quasi-modes.

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In recent years, there has been very significant interest in acoustic or elastic wave propagation and scattering in complex media.[1–7] For periodic media, this is concerned with the band gap or pass band in the dispersion relations of phononic crystals generated by Bragg scattering,[2,3,8] which strongly depends on the periodic structure of the artificial crystal. When there exists local resonance and the local resonance dominates the wave scattering, the wave propagation alters drastically and shows many intriguing properties, such as negative refraction,[9] invisibility cloaks,[10,11] super absorption,[12,13] and topological interface mode.[14] Such materials are called metamaterials. Generally, the local resonance band gap does not depend on the lattice periodicity of the composite materials,[15] but depends on the structure of the independent scattering unit.

One of the key issues for the variation of acoustic wave propagation is the variation of the scattering mechanism, which rests with the structure and composition of the scattering unit.[16–18] Especially in a random medium,[5–7,19–22] the structure and composition of the scattering unit control the enhancement effect of the resonance scattering. Based on scattering matrix T determined by the scattering unit, the generalized coherent potential-approximation (GCPA) approach successfully determines the acoustic excitation spectra in a random medium.[5,19–22] However, the scatterers in the scattering unit are limited to simple sphere or simple cylinder scatterers.[5–7,19–22] These simple structures are insufficient for investigating the diversified acoustic wave modes and the variation of the acoustic wave scattering in a random medium. Remarkably, coating scatterers gives rise to a revolution in the traditional phononic crystal and to many charming properties in metamaterials.[23–26] For a random medium, new delicate properties are also expected in acoustic excitation spectra arising from coating scatterers.

In this Letter, we use the GCPA approach for the excitation spectra in two-dimensional random media, consisting of parallel coated fibers dispersed in water. In the intermediate-frequency regime where the wavelength is comparable to the size of the coating fibers, significant phenomena are found. A number of frequency gaps in the dispersion relations are found in strongly scattering excited mode, which arise from the coherent coupling of resonances on neighboring particles. The frequency gaps are found to change regularly with the density and modulus of the coating layer. The calculated results of the cross section present the evolution of scattering resonances with the properties of the coating layer, which provide a detailed image for understanding the change of the frequency gaps. The velocity of the propagation quasi-mode is also shown to depend on the filling fraction of the coating layer. Our model solves the acoustic wave dispersion relation in a complicated random medium consisting of coating-structure scatterers, and offers a reliable way to precisely modulate the velocity of the acoustic propagation quasi-mode and gap frequency. To make this clearer, the details of the generalized GCPA approach

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**Corresponding author. Email: xinxintwinkle@163.com; psjhguo@scut.edu.cn
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theory are presented.

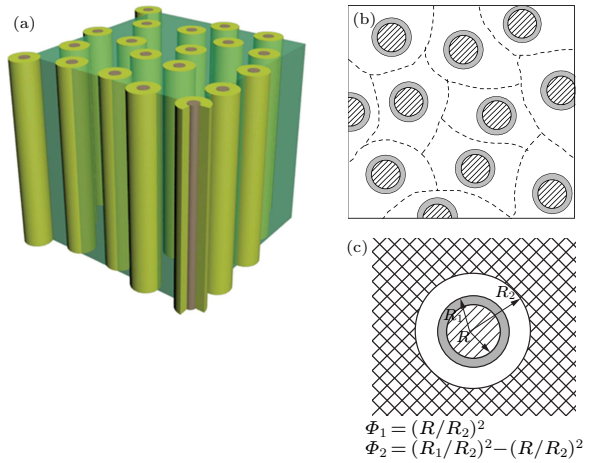

Fig. 1. (a) The configuration of the two-dimensional dispersed random system. (b) The effective medium model. (c) The unit cell of the effective medium model. The double-coating cylinder is embedded in a homogenized effective medium composed of similar units of coated cylinders.

The two-dimensional dispersed random system studied consists of solid fibers coated by fluid which are aligned parallel to one another. This dispersed medium is characterized by the dispersion microgeometry where each coating cylinder is individually enveloped by water. We discuss the acoustic waves propagating in the two-dimensional plane that is vertical to the cylinders' axis. The configuration of the system is illustrated in Fig. 1(a). To calculate the effective macroscopic properties of the system, we employ the effective medium model, which is schematically depicted in Fig. 1(b). Figure 1(c) shows the unit cell of the effective medium. The double-coated fiber is embedded in a homogenized effective medium (shaded area in Fig. 1(c)) composed of similar units of double-coated fiber. The density of the effective medium is an average value, calculated by

$$\rho_0 = \rho_3\phi_1 + \rho_2\phi_2 + \rho_1(1-\phi_1-\phi_2), \tag{1}$$

where ρ_1, ρ_2 and ρ_3 are the densities of the water background, fluid coating layer and solid fiber, respectively, and φ_1, φ_2 are the filling fractions of the solid fiber and fluid coating, respectively. The effective-medium speed of the medium is determined by some self-consistent conditions. The Green function[1,6,7,18–20] for an acoustic wave in the random system can be expressed as

$$G = G_{\rm e} + G_{\rm e}TG_{\rm e}, \tag{2}$$

where $G_{\rm e}$ denotes the Green function for the effective medium, and T is the scattering operator, which is different from that in previous works for a single coated sphere, and T is developed into the total scattering operator between double-coating fibers, and a continuous boundary condition is applied. For the GCPA condition,[19–22] it requires looking for the minima of $\langle T\rangle$, where the angular brackets denote configuration average, by adjusting the effective-medium wave speed $C_{\rm e}$. The scattering at minima on average means that the excitation would be a quasi-mode meaning that the wave can coherently propagate over a certain distance. However, since the scattering may still be weak, we may approximate $\langle G\rangle$ by

$$\langle G\rangle \approx \frac{1}{(p-q-\Sigma)}, \tag{3}$$

where the self-energy $\Sigma \approx \langle T\rangle \approx nt$ to the first order in scattering strength, $q = \omega/C_{\rm e}$, and p is the Fourier-transform variable. According to the condition of elastic scattering ($p = q$), the minima of $\langle T\rangle$ can be identified by $S_{\rm D}(q,\omega) = -{\rm Im}\langle G\rangle/\pi$, where $S_{\rm D}$ is the density of state maxima and $\langle G\rangle \approx -1/nt$. The density of state maxima correspond directly to the minima in scattering, which give the best condition for the dispersion relation of the acoustic quasi-mode.

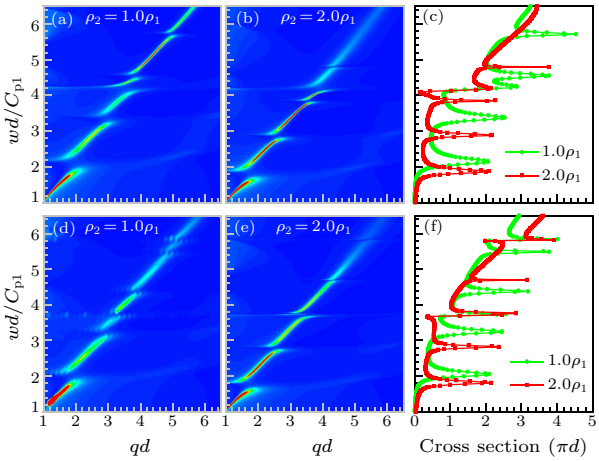

Fig. 2. The value of $S_{\rm D}$ plotted as a function of the normalized frequency $\omega d/C_{\rm p1}$ and wave vector qd, where $C_{\rm p1}$ is the longitudinal velocity for the water background. The magnitude is indicated by the colors. (a) The value of $S_{\rm D}$ for uncoated protein fibers immersed in water. (b) The value of $S_{\rm D}$ for coated protein fibers immersed in water with $\rho_2 = 2.0\rho_1$. (c) The renormalized scattering cross section plotted as a function of frequency $\omega d/C_{\rm p1}$ for uncoated protein fibers and coated protein fibers in water. (d) The value of $S_{\rm D}$ for uncoated PA fibers in water with $\rho_2 = 1.0\rho_1$. (e) The value of $S_{\rm D}$ for coated PA fibers in water. (f) The same as (c) but for uncoated and coated PA fibers. The filling fraction of the fiber is 0.33 and the filling fraction of the coating layer is 0.3.

We first consider the composite systems consisting of parallel coated protein fibers dispersed randomly in a water background and parallel coated polyamide (PA) fibers dispersed randomly in a water background. The filling fraction of the fiber φ_1 is 0.33 and the filling fraction of the coating layer φ_2 is 0.3. These two filling fractions determine the water-coating thickness used in the calculation. The material parameters are chosen as follows: $\rho_3 = 0.85\,{\rm g/cm^3}$, $C_{\rm p3} = 2.3\,{\rm km/s}$, and $C_{\rm s3} = 1.1\,{\rm km/s}$ for protein fibers; $\rho_3 = 1.15\,{\rm g/cm^3}$, $C_{\rm p3} = 2.3\,{\rm km/s}$, and $C_{\rm s3} = 0.95\,{\rm km/s}$ for PA fibers;

$\rho_2 = 2.0\,\mathrm{g/cm^3}$, $C_{\mathrm{p2}} = 1.49\,\mathrm{km/s}$ for the coating layer; $\rho_1 = 1.0\,\mathrm{g/cm^3}$ and $C_{\mathrm{p1}} = 1.49\,\mathrm{km/s}$ for the water background; where ρ, C_{p}, and C_{s} are, respectively, the density, and the p-wave and s-wave velocities, and subscripts 1, 2 and 3 correspond to the liquid background, coating layer and core, respectively. In Figs. 2(a)–2(e), we plot, in color, the calculated S_{D} as a function of the dimensionless frequency $\omega d/C_{\mathrm{p1}}$ and dimensionless wave vector qd, for uncoated protein fibers immersed in water, coated protein fibers immersed in water, uncoated PA fibers immersed in water and coated PA fibers immersed in water, respectively. Significant dispersions are observed. There is one acoustic mode indicated by the outstanding colors. The dispersion curve, defined by the peaks, is accurately determined because the widths of the peaks are substantially less than their central frequencies. It should be noted that the observed modes are quasi-modes with a limited lifetime. Remarkably, at several intermediate frequency regimes we find gaps in the excitation spectra. For the protein fibers, comparing S_{D} for coated protein fibers immersed in water ($\rho_2 = 2.0\rho_1$) (shown in Fig. 2(b)) with S_{D} for the uncoated protein fibers immersed in water ($\rho_2 = 1.0\rho_1$) (shown in Fig. 2(a)), we can see that in the high frequency regime, the magnitude of the excitation spectra weakens and the gaps disappear when the protein fibers are coated. To explain the mode and the gaps, we demonstrate the behavior of the scattering cross section in Fig. 2(c). It is the key to understanding the physical origin of the mode. Scattering cross sections are renormalized in units of $\omega d/C_{\mathrm{p1}}$. We can see that in the regime of renormalized frequency from 1 to 6.5, there are many peaks in the scattering cross section, which means strong resonance scattering of single fibers. The frequency positions of the peaks are found to correspond directly to the gaps in the dispersion relation. This correspondence suggests that the excited mode results from the anti-resonance of a single fiber, where the scattering is minimum. Comparing the scattering cross section of an uncoated protein fiber immersed in water with that of a coated protein fiber immersed in water, we can see the peak shift except for at the normalized frequency 4.12, which agrees with the change of band gap position in the dispersion relations. In addition, for the coated protein fibers, the scattering cross section peaks disappear in the high frequency regime, which corresponds to the gaps disappearing in the high frequency regime. For the PA fibers, comparing S_{D} for coated PA fibers immersed in water ($\rho_2 = 2.\rho_1$) (shown in Fig. 2(e)) with S_{D} for the uncoated PA fibers immersed in water ($\rho_2 = 1.0\rho_1$) (shown in Fig. 2(d)), we also observe the gap changes except the gaps at the normalized frequencies 3.75 and 5.8. The scattering cross section peaks shift exactly corresponding to the gap shifts (shown in Fig. 2(f)). Thus the coating layer affects the strong resonance scattering of single fibers, which leads to the frequency shift of the gaps in the medium frequency regime.

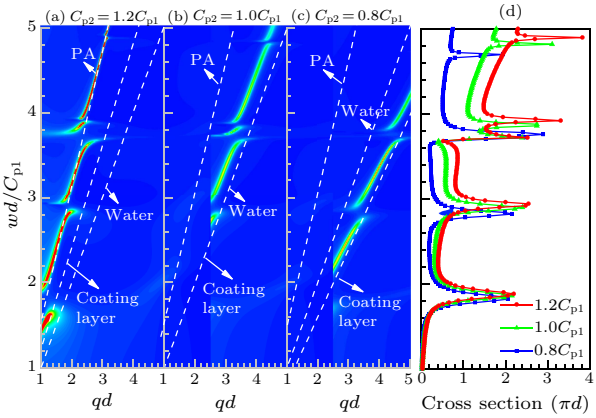

Fig. 3. The value of S_{D} for coated PA fibers immersed in water. The fibers are coated by liquid materials with different longitudinal wave velocities: (a) $C_{\mathrm{p2}} = 1.2C_{\mathrm{p1}}$, (b) $C_{\mathrm{p2}} = 1.2C_{\mathrm{p1}}$, and (c) $C_{\mathrm{p2}} = 1.2C_{\mathrm{p1}}$. The three white dashed lines indicate the dispersion curve for the longitudinal wave in the pure liquid phase (water and coating layer), and for the longitudinal wave in the solid phase (PA), respectively. (d) The renormalized scattering cross section for PA fibers coated by different liquid materials.

Since the coating layer plays an important role in the acoustic frequency gap in two-dimensional dispersed random media, we discuss how the elastic properties of the coating layer affect the acoustic dispersions in random systems. In the following, we employ a coated PA fiber-water system to demonstrate. In Figs. 3(a)–3(c), we plot S_{D} for parallel coated PA fibers dispersed randomly in a water background. The fibers are coated by liquid materials with different longitudinal sound velocities. Here the filling fraction of the PA fiber φ_1 is 0.33 and the filling fraction of the coating layer φ_2 is 0.3. The material parameters are chosen as $\rho_3 = 1.15\,\mathrm{g/cm^3}$, $C_{\mathrm{p3}} = 2.3\,\mathrm{km/s}$, and $C_{\mathrm{s3}} = 0.95\,\mathrm{km/s}$ for PA fibers; and $\rho_2 = 1.8\,\mathrm{g/cm^3}$ for the coating layer. One band of ridges is clearly seen in outstanding colors. We can see that when $C_{\mathrm{p2}} = 1.2C_{\mathrm{p1}}$, the dispersion relation is very close to the pure PA dispersion relation, which is marked by white dashed lines. This means that the excited mode has a similar velocity to that of the PA. When $C_{\mathrm{p2}} = C_{\mathrm{p1}}$, the dispersion relation exists between the pure PA dispersion relation and the pure water dispersion relation, where the water dispersion relation should coincide with the dispersion relation of the coating materials. This means that the excited mode has an intermediate velocity between those of the PA and water. These tendencies are in accord with general intuition. When $C_{\mathrm{p2}} = 0.8C_{\mathrm{p1}}$, the dispersion relation of the pure coating material falls below that of the water, and the dispersion relation of the random medium is seen to exist between those of the water and the coating materials, which means that the speed of sound in the random medium is less than the speed of sound in the water but larger than the

speed of sound in the coating materials. Moreover, small gaps are found in the dispersion relation relative to the coating layer. We focus on the two gaps at renormalized frequencies 3.65 and 3.85. When the longitudinal sound velocity in the pure coating materials C_{p2} decreases, the higher gap is close to the lower gap. When $C_{p2} = 0.8C_{p1}$, the higher gap merges into the lower gap, which leads to a wider gap. To understand this, we also employ scattering cross section, as shown in Fig. 3(d). When $C_{p2} = 1.2C_{p1}$, there exist two peaks at the renormalized frequencies 3.65 and 3.85, which exactly correspond to the two gaps in the dispersion relation. It should be pointed out that the peak with lower frequency 3.65 has the character of a Fano-resonance peak. Surprisingly, when $C_{p2} = 1.0C_{p1}$, the peak with higher frequency shifts to the low frequency, but the peak with lower frequency 3.65 does not move. When $C_{p2} = 0.8C_{p1}$, the peak with the higher frequency couples with the peak with the lower frequency. At this moment, the higher gap merges into the lower gap.

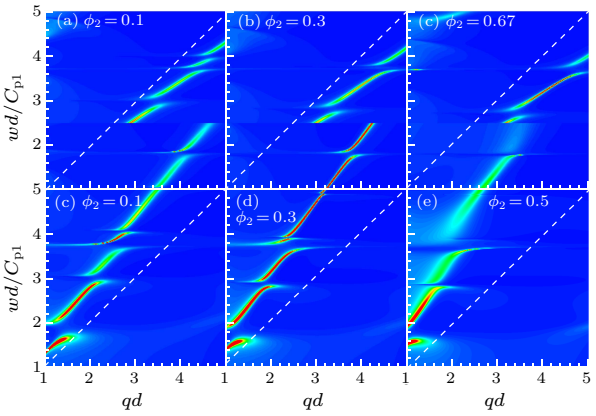

Fig. 4. The value of S_D for coated PA fibers in water. The filling fraction of the PA fiber is 0.33. (a)–(c) The longitudinal sound velocity in the coating materials is $C_{p2} = 0.8C_{p1}$. The filling fractions of the coating layer are 0.1, 0.2 and 0.67, respectively. (d)–(f) The longitudinal sound velocity in the coating materials is $C_{p2} = 1.2C_{p1}$. The volume fractions of the coating layer are 0.1, 0.2 and 0.5, respectively. The white dashed lines indicate the dispersion curve for the longitudinal wave in the water.

In Fig. 4, we plot S_D for coated PA fibers immersed in water with different filling fractions of the coating layer φ_2. Here we choose the density of the coating materials ρ_2 as $1.8\,\mathrm{g/cm^3}$. The upper panels show the evolution of the dispersion relation for $C_{p2} = 0.8C_{p1}$, implying that the sound speed in the coating materials is lower than that in water. The lower panels show the evolution of the dispersion relation for $C_{p2} = 1.2C_{p1}$, which illustrates that the sound speed in the coating materials is higher than that in water. Remarkably, when $C_{p2} = 0.8C_{p1}$, the dispersion relations for all filling fractions we considered fall below the dispersion relation of water. When $C_{p2} = 1.2C_{p1}$, the dispersion relations for all filling fractions we considered exist above the dispersion relation of water. In particular, in (a)–(c), we can see that as the filling fraction of the coating layer φ_2 increases, the velocity of the excited mode decreases, while in (d)–(f), as the volume fraction of the coating layer φ_2 increases, the velocity of the excited mode increases, which is exactly opposite to the trend of the upper excited mode. At low filling fraction, the two dispersion relations will converge to the dispersion relation of the random dispersed medium consisting of uncoated PA fibers immersed in water. Thus the velocity of the excited mode can be modulated by changing the longitudinal sound velocity and the filling fraction of the coating layer.

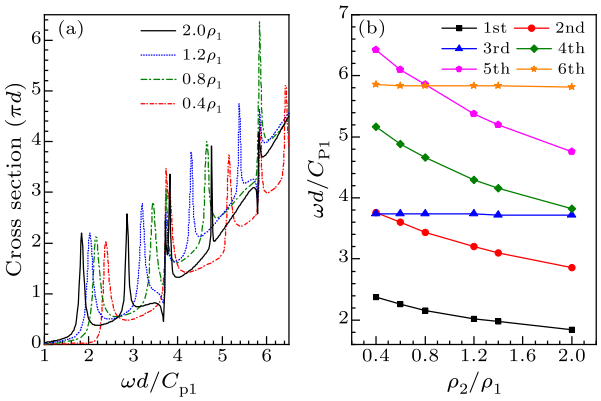

Fig. 5. (a) The renormalized scattering cross section for different densities of coating layer. (b) The frequency location of the gaps in dispersion relation plotted as a function of the density ratio ρ_2/ρ_1.

We have found that the frequency positions of the peaks in the scattering cross section correspond to the acoustic gaps in the dispersion relation, and the existence of excited modes depends on the single cylinder's strong resonance scattering. Next, we discuss the effect of the elastic properties of the coating materials on the acoustic gaps. In Fig. 5(a), we demonstrate the behavior of the scattering cross section for different densities of the liquid coating layer. In Fig. 5(b), the frequency locations of the gaps in the dispersion relation are plotted as a function of the density ratio ρ_2/ρ_1. Here we take the sound speed in the coating layer as $C_{p2} = 1.2C_{p1}$. It can be seen that there are 6 peaks in the regime of renormalized frequency from 1 to 6.5. The frequency of the 1st, 2nd, 4th and 5th peaks decrease with increasing density ratio ρ_2/ρ_1. Correspondingly, the 1st, 2nd, 4th and 5th gaps decrease with increasing density ratio ρ_2/ρ_1. However, the 3rd and 6th peaks are immovable and show the character of Fano-resonance. The 3rd and 6th gaps also keep the original frequency. This means that the coating layer is nothing to do with the gaps from the Fano-resonance scattering.

Furthermore, we demonstrate the behaviors of the scattering cross section and gaps for different wave speeds for the coating layer in Fig. 6. Here we take the density of the coating layer as $\rho_2 = 1.8\,\mathrm{g/cm^3}$. In

Fig. 6(a), there exist 6 scattering cross section peaks in the regime of renormalized frequency from 1 to 6.5 for every wave speed ratio C_{p2}/C_{p1}. When the wave speed ratio C_{p2}/C_{p1} increases, the 1st, 2nd, 4th and 5th peaks shift to the higher frequency. It can also be found that the 3rd and 6th peaks are Fano-resonance peaks. In the regime we discuss, they hardly move. In Fig. 6(b), the frequency locations of the gaps in dispersion relation are plotted as a function of the wave speed ratio C_{p2}/C_{p1}. For the 1st, 2nd, 4th and 5th gaps, at the beginning, when the wave speed ratio C_{p2}/C_{p1} increases, the gap frequency rises. When the wave speed ratio C_{p2}/C_{p1} increases to 1.2, the rising speed of the gap frequency slows down. For the 3rd and 6th gaps, the function curve is flat. It is confirmed again that the coating layer is nothing to do with the gaps originating from the Fano-resonance scattering. Above all, on the one hand, when the density ratio is in the range from 0.4 to 2.0, most band gaps can be modulated by the density of the coating layer materials, and when the wave speed ratio is in a range from 0.4 to 1.2, most band gaps can be modulated by the wave speed of the coating layer materials. On the other hand, in our considered regime, the function relationship between gap frequency and density of coating layer, as well as between gap frequency and wave speed of coating layer could be used as a datum reference for quantification of elastic parameters, and thus it is expected to be applied to precisely modulating the frequency of the sound insulation.

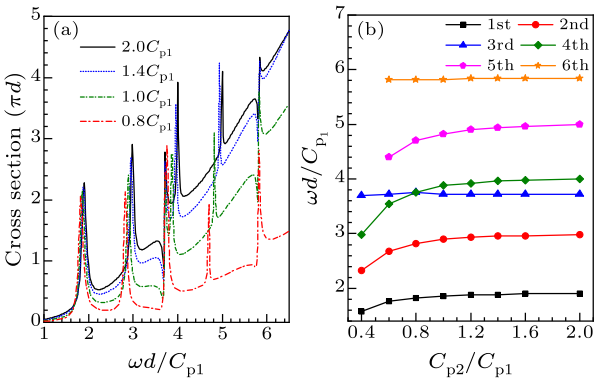

Fig. 6. (a) The renormalized scattering cross section for different wave speeds for the coating layer. (b) The frequency location of the gaps in the dispersion relation plotted as a function of the wave speed ratio C_{p2}/C_{p1}.

In conclusion, we have investigated acoustic wave propagation in two-dimensional disordered media consisting of parallel coated fibers dispersed in water. Using the GCPA approach based on the principle of locating the minima of T, which concerns a double-coating fiber, the dispersion relations of the system are investigated. Numerical results show that a number of frequency gaps exist in the excitation spectra when the wavelength is comparable to the scale of the inhomogeneities. This is due to the strong resonance scattering. We find that the frequency gaps change subtly with the density and sound velocity of the coating layer. To better understand this change, we calculate the cross section of scatterers. The frequency variations of the scattering resonance peaks just correspond to the frequency variations of the gaps. It is also found that when C_{p2} is smaller than C_{p1}, the velocity of the excited mode decreases with increasing filling fraction of the coating layer φ_2. In contrast, when C_{p2} is larger than C_{p1}, the velocity of the excited mode increases with the filling fraction of the coating layer φ_2. The properties of these excited modes can be precisely modulated by changing the sound velocity and the filling fraction of the coating layer. Our model can be extended to a more complicated random medium consisting of multi-coating scatterers, and more subtle characteristics will be observed in the acoustic resonance scattering process.

References

[1] Sheng P 1990 *Scattering and Localization of Classical Waves in Random Media* (Singapore: World Scientific)
Sheng P 1995 *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena* (San Diego: Academic Press)
[2] Economou E N and Sigalas M M 1993 *Phys. Rev. B* **48** 13434
[3] Martínez-Sala R, Sancho J, Sánchez J V et al 1995 *Nature* **378** 241
[4] Liu Z Y, Zhang X X, Mao Y W et al 2000 *Science* **289** 1734
[5] Cowan M L, Page J H and Sheng P 2011 *Phys. Rev. B* **84** 094305
[6] Leroy V, Strybulevych A, Page J H et al 2011 *Phys. Rev. E* **83** 046605
[7] Cobus L A, Skipetrov S E, Aubry A et al 2016 *Phys. Rev. Lett.* **116** 193901
[8] Zhang X, Dan H, Wu F G et al 2008 *J. Phys. D* **41** 155110
[9] Zhang S, Park Y S, Li J et al 2009 *Phys. Rev. Lett.* **102** 023901
[10] Liu F and Liu Z Y 2015 *Phys. Rev. Lett.* **115** 175502
[11] Chen H and Chan C T 2007 *Appl. Phys. Lett.* **91** 183518
[12] Mei J, Ma G C, Yang M et al 2012 *Nat. Commun.* **3** 756
[13] Ren S W, Belle L V, Claeys C et al 2019 *Mech. Syst. Signal Process.* **117** 138
[14] Zhao D G, Xiao M, Ling C W et al 2018 *Phys. Rev. B* **98** 014110
[15] Achaoui Y, Laude V, Benchabane S et al 2013 *J. Appl. Phys.* **114** 104503
[16] Ma G C, Yang M, Xiao S W et al 2014 *Nat. Mater.* **13** 873
[17] Lai Y, Wu Y, Sheng P et al 2011 *Nat. Mater.* **10** 620
[18] Wang P, Casadei F, Shan S C et al 2014 *Phys. Rev. Lett.* **113** 014301
[19] Jing X, Sheng P and Zhou M 1992 *Phys. Rev. A* **46** 6513
[20] Jing X, Sheng P and Zhou M 1991 *Phys. Rev. Lett.* **66** 1240
[21] Page J H, Sheng P, Schriemer H P et al 1996 *Science* **271** 634
[22] Zhang X, Liu Z Y, Wu F G et al 2006 *Phys. Rev. E* **73** 066604
[23] Qiu C Y, Ma Z F, Lu J Y et al 2012 *Phys. Lett. A* **376** 637
[24] Ding Y Q, Liu Z Y, Qiu C Y et al 2007 *Phys. Rev. Lett.* **99** 093904
[25] Wu S and Mei J 2016 *AIP Adv.* **6** 015204
[26] Gao H F, Zhang X, Wu F G et al 2016 *Acta Phys. Sin.* **65** 044301 (in Chinese)