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A Quasi-1D Potential for Bose Gas Phase Fluctuations *

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An elongated trap potential for cold atoms is designed based on a quadrupole-Ioffe configuration. Phase fluctuations in a Bose–Einstein condensate (BEC), which is confined by the trap, are studied. We simulate the atom density distribution induced by fluctuation after time of flight from this elongated trap potential and study the temperature measurement method related to the distribution. Furthermore, taking advantage of the tight confinement and radio frequency dressing technique, we propose a double well potential for splitting BECs. Our results are helpful for improving understanding of low-dimensional quantum gases and provide important guidance for atomic interferometry.

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One-dimensional (1D) systems[1−6] have rich physical properties that are dramatically different from those of their higher dimensional counterparts. For instance, a Luttinger liquid[7] cannot be described by the Fermi liquid theory, which is one of the rules for three-dimensional (3D) Fermi systems. In 1D (including quasi-1D) systems,[8,9] the phase fluctuations[10−13] are related to thermal excitations, and the density fluctuations are released along the axial direction in an ultracold Bose gas. These characters raise a series of non-trivial phenomena, for example, a Tonks gas[14] of impenetrable bosons. When $\omega_z \ll \omega_\mathrm{r}$, a distinguishing standard is provided,[15] demarcating a deeply 1D regime with $k_\mathrm{B}T \ll \hbar\omega_\mathrm{r}$ and an almost 3D regime with $k_\mathrm{B}T \simeq 3\hbar\omega_\mathrm{r}$, where k_B is the Boltzmann constant, T is the temperature as per Bose gas statistics, $\hbar$ is the reduced Planck constant, and ω_r and ω_z are the radial frequency and axial trapping frequency, respectively.

The realization of the Bose–Einstein condensate (BEC) provides a powerful platform for a series of low-dimensional experiments, and it has attracted much interest in the cold atom community. The ultracold Bose gas has a long coherent length and can exist without impurity doping. These kinds of properties make the Bose gas a perfect candidate for studying a 1D Bose system as well as atom interferometry,[16−20] atomic clocks[21,22] and quantum simulation.[23−27] Low-dimensional Bose gases are typically produced by trapping the pre-cooled atoms in an elongated magnetic or light potential.[28] After applying an additional cooling mechanism,[29,30] such as evaporation cooling, all atoms occupy the ground state of the system and have the same physical shape as well as confinement potential. The representative method is the atom chip method,[31−34] an integrated device in which both electric and magnetic fields are used to confine and manipulate cold atoms. The typical aspect ratio of the axial and radial directions in an atom chip is 1000:1, and the ultracold gas transforms into a nearly 1D system. However, in an atom chip system, the number of atoms can only be as high as 10^4, which is at least one order of magnitude lower than that in the other BEC setups, and this aspect reduces signal amplitude. An optical dipole trap[35−37] with large gradient light intensity is an alternative method of achieving a low-dimensional system and has been widely used for arbitrary confinement of neutral atoms. To trap enough atoms, a high-power dipole laser is required and, therefore, additional noise is introduced in the whole system.

In this Letter, we propose a simple and convenient way of magnetic field shaping of the ultracold atoms across the 3D regime to the quasi-1D regime. We design a quadrupole-Ioffe configuration (QUIC) trap,[38] which consists of quadrupole coils (anti-Helmholtz coils) and an Ioffe coil. The QUIC trap has been widely used for making BECs and manipulating cold atoms. Surprisingly, the trapping configuration, especially the aspect ratio, has not been explicitly studied. Here the relationship between radial trapping frequency and magnetic bottom offset is explicated. Hence, a large BEC aspect ratio is achieved, allowing the transformation into a quasi-1D regime system. Then, a phase fluctuation which causes density modulation after the BEC time of flight (TOF) is obtained. Finally, taking advantage of the elongated trap and radio frequency (RF) dressing[39,40] technique, we propose a new method for splitting cold atoms, which is an alternative to atom chip interferometry experiments.

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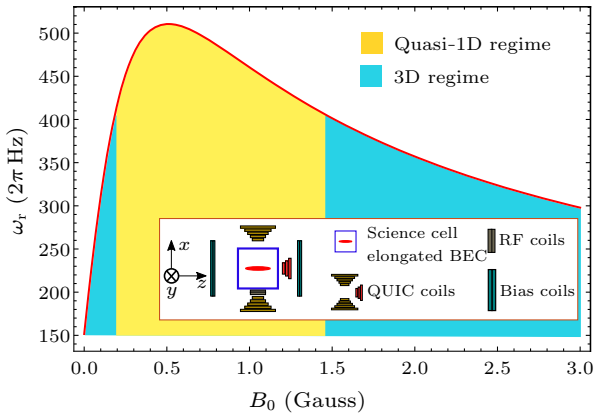

Fig. 1. Radial frequency behaviors controlled by trapping bias field. In a quadrupole-Ioffe configuration, the trap bottom value could significantly affect radial frequency. The radial frequency of a magnetic field within 3 G varies from $2\pi \times 150$ Hz to $2\pi \times 500$ Hz, while the axial trapping frequency remains the same. Then, a variable BEC aspect ratio can be achieved and the ultracold atom system changes from a 3D to a quasi-1D regime. The theoretical curve shows that the left side of the slope has a large trapping frequency gradient, which means that it is convenient to study the Bose gas quench behavior when such a potential value exists. The inset shows the configuration of the magnetic field coils in all directions. The levitation coils, which levitate atoms in the gravitational field, are not shown.

The inset in Fig. 1 shows a schematic of the QUIC design in all directions. A pair of quadrupole coils, each made of 126 turns of 1.5-mm enamel insulated wires, are placed along the x axis and symmetrized about the vacuum chamber. The numbers of initial and final turns of a single quadrupole coil are 5 and 16, respectively. The same wires of a tapered-shape Ioffe coil with 91 turns are placed along the z axis and 50 mm away from the BEC. The Ioffe coil starts with 1 turn and ends at 13. The science chamber is a square-shaped glass cell of length 30 mm. For this kind of potential, the universal magnetic field B can be written as

$$B(r,z) = B_0 + \frac{1}{2}\Big(\frac{B_{\rm r}'^2}{B_0} - \frac{B_z''}{2}\Big)r^2 + \frac{1}{2}B_z''z^2, \qquad (1)$$

where B is the total magnetic field, B_0 is the bias of the trap at minimum position, $B_{\rm r}'$ refers to the magnetic field gradient along the trap's radial (r) direction, and B_z'' is the curvature of the magnetic field along the trap's axial (z) direction. Then, we can obtain

$$\omega_r^2 = \frac{\mu_0}{m}\Big(\frac{B_r'^2}{B_0} - \frac{B_z''}{2}\Big), \qquad (2)$$

$$\omega_z^2 = \frac{\mu_0 B_z''^2}{m}, \qquad (3)$$

where m denotes the atom mass, and $\mu_0 = m_{\rm F} g_{\rm F} \mu_{\rm B}$ is the atom's magnetic moment with $m_{\rm F}$ the magnetic quantum number, $g_{\rm F}$ the Landé g factor, and $\mu_{\rm B}$ the Bohr magneton. From Eq. (3), we can see that the radial frequency $\omega_{\rm r}$ is inversely proportional to B_0. Then, a large range of $\omega_{\rm r}$ can be achieved by the scanning trap bias B_0, and the BEC confined in the trap becomes a quasi-1D system from a normal 3D situation. The magnetic field calculation is based on the Biot–Savart law and we use finite element analysis to simulate the design. According to the simulation when the currents in the quadrupole and Ioffe coils, $I_{\rm q}$ and $I_{\rm i}$, are both 20 A, i.e., $I_{\rm q} = I_{\rm i} = 20$ A, the maximal radial frequency ω_r of $2\pi \times 500$ Hz at $B_0 = 0.5$ Gauss is achieved. In Ref. [15], a crossover from a deeply 1D regime with $k_{\rm B}T \ll \hbar\omega_{\rm r}$ to an almost 3D regime with $k_{\rm B}T \simeq 3\hbar\omega_{\rm r}$ is studied. With our designed QUIC trap and $\omega_{\rm r} = 2\pi \times 500$ Hz, the thermal energy $k_{\rm B}T$ is smaller than transverse excitation energy $\hbar\omega_{\rm r}$, and the system is close to a deeply 1D regime. In this case, the BEC with an aspect ratio of 50 appears with phase fluctuations due to thermal excitations, which shows strong characteristics of a quasi-1D system.[10] Note that ω_z does not change when B_0 is varied. Figure 1 shows a direct relationship between $\omega_{\rm r}$ and B_0. With the help of bias coils, B_0 at the trap's bottom is changed from zero to 3 G, and, simultaneously, $\omega_{\rm r}$ changes from $2\pi \times 150$ Hz to $2\pi \times 500$ Hz. The blue areas denote the 3D regime while the yellow area denotes the quasi-1D system with a maximum BEC aspect ratio of ∼50. The figure shows interesting behavior in that the curve on the left has a large gradient for $\omega_{\rm r}$. In this case, one can obtain a rapidly changing $\omega_{\rm r}$, which is proportional to the BEC aspect ratio, by controlling B_0. This behavior can be used to study 1D phase fluctuation quenching[41,42] and spontaneous symmetry breaking.[43]

We then implement the above-mentioned idea, which involves scanning B_0 firstly, followed by the aspect ratio λ, to study phase fluctuations under different aspect ratios. In the QUIC trap, when B_0 is 0.25 G and 8 G (not shown in the figure), the BEC aspect ratios are $\lambda = 45$ and 20. According to the phase fluctuation theory, the more elongated the BEC, the higher the density modulation for the Bose gas after the TOF. As an elongated condensate in the Thomas–Fermi regime has different phases across the whole section, the normalized density fluctuations are given by[10]

$$(\sigma_{\rm BEC})^2 = \sqrt{\frac{\ln\rho}{\pi}}\frac{T}{\lambda T_\phi}\Big(\sqrt{1+\sqrt{1+\Big(\frac{\hbar\omega_r\rho}{\mu\ln\rho}\Big)^2}} - \sqrt{2}\Big), \qquad (4)$$

where $\sigma_{\rm BEC}$ is the standard deviation of the experimental data from the fit in the central section of the condensate fraction (half width at full maximum), $\rho = \omega_r t_{\rm tof}$ with $t_{\rm tof}$ referring to the evolution time for BEC released from the magnetic trap, T is the system temperature, and T_ϕ is the characteristic temperature with $T_\phi \sim \omega_{\rm r}\hbar^2 N_0/\mu/k_{\rm B}$ (N_0 is the number of atoms and μ is the chemical potential). In the quasi-1D regime, the coherence length is smaller than the condensate size. For normal situations, the BEC sys-

tem has $N_0 = 1 \times 10^5$ atoms. A magnetic field of $\sim$10 G/cm, which is generated by levitation coils, provides an anti-gravity force for ^{87}Rb, and a long free fall time of 100 ms is achieved. Such a long evolution time enables us to adequately study density distribution resulting from the initial quantum phase fluctuations. A scientific charge-coupled device (CCD) camera with a pixel size of 10 μm is hired to record the density information. Taking all these conditions into account, we simulate the evolution of an elongated BEC released from a harmonic potential. Figure 2(a) shows the increase in the normalized density fluctuations with growing λ at system temperature $T = 70$ nK. Here $\lambda = 45$ is for the gray area and $\lambda = 20$ is for the purple area. Each curve is assigned a temperature uncertainty of $\pm$10 nK. Conversely, we can obtain the BEC temperature by fitting the above curves. A previous work[10] developed a powerful way to control temperature trends by controlling atom numbers. Taking advantage of that method, we lock the evolution time to 110 ms and study the phase fluctuations against λ at different temperatures (Fig. 2(b)). A common way to determine an ultracold Bose gas is to fit thermal wings from a bi-mode BEC. However, the method is invalid below a hundred nK since the thermal wings are very dilute. Our designed QUIC trap setup provides an effective tool for fitting the BEC temperature precisely from the phase fluctuations. Equation (4) should be valid so that the evolution time is larger than 17 ms since it contains a square root. This 17 ms dropping time is not a difficult condition in other experimental setups. We notice that a sharp transition slope does not exist between the 3D and quasi-1D Bose gases.

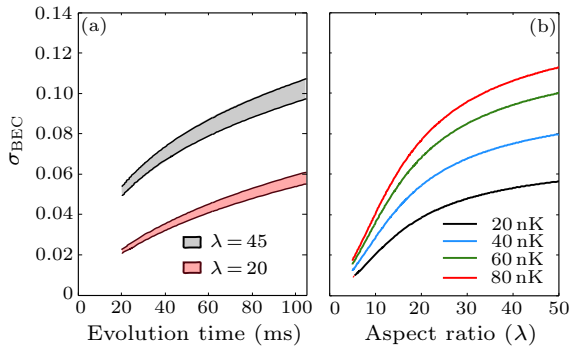

Fig. 2. Phase fluctuation-induced density modulation after atoms have been released from the elongated trap. (a) Fluctuation behavior in $\lambda = 45$ (gray) and $\lambda = 20$ BECs for different evolution times. In both the cases, the center temperature is 70 nK, and $\pm$10 nk is regarded as the temperature uncertainty (the colored area). (b) Phase fluctuations as a function of the trapping potential aspect ratio after 110 ms of expansion. Four different temperature curves are evaluated below 100 nK.

So far, we have presented a magnetic-based quasi-1D cold atom system and studied the behavior of elongated BEC phase fluctuations. Now, we analyze the phase-induced density modulation theory. Our aim is to numerically simulate momentum/density distribution on the CCD after TOF. Firstly, ^{87}Rb cold atoms are loaded into a trap with $\lambda = 50$ and evaporation cooling is conducted to cool the atoms to a temperature of 100 nK. The atoms now become a BEC and have the same aspect ratio as that of the trap. The size of the BEC in the axial direction in the trap is several tens of microns. Secondly, the random phase is added to the system. Consider that in the quasi-1D system, the healing length is much smaller than the BEC size. Thirdly, the BEC is released from the elongated trap and allowed to expand freely. All random phases are involved in this case, and condensate density is modulated during the time. Finally, a resonance light illuminates the BEC and all the density information is recorded on the CCD. Figure 3(a) shows that the random phases occur across the whole BEC and Fig. 3(b) shows the density modulation after 25 ms TOF.

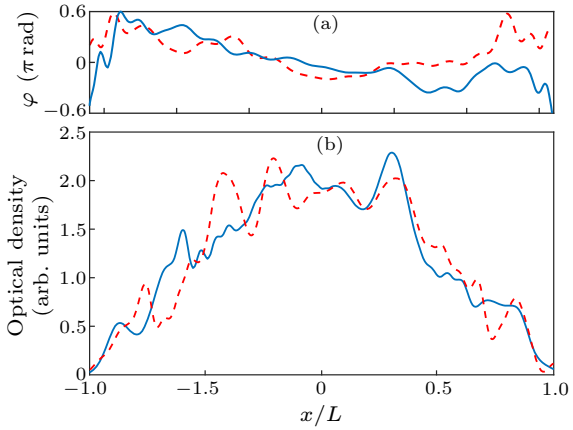

Fig. 3. Typical axial spatial distributions after TOF. Two random phase fluctuations (blue and red curves) are added to the BEC system along the x direction (a). Density modulation distributions for these two cases are simulated on the charge-coupled device (CCD) camera (b). Here the TOF is 17 ms.

The QUIC trap together with shim coils not only produces elongated confinement for cold atoms but also reveals great potential for creating double-slit BECs, which is the core idea of atomic chip interferometry. For atomic chip experiments, the radial trapping frequency usually exceeds $2\pi \times 1$ kHz and $\lambda \sim 500$. Such a high trapping frequency provides a larger gradient and helps split the BEC into two adjacent parts (also called the double well) using the RF dressing technique. We find that both the QUIC and atomic chip traps could share the same expression with regard to RF dressing potential

$$\boldsymbol{V}_{\rm RF}(\boldsymbol{r}) = \mu_0 \sqrt{\left(\frac{\hbar\omega_{\rm r}}{|\mu_{\rm B} g_F|} - |\boldsymbol{B}_0(\boldsymbol{r})|\right)^2 + \boldsymbol{\Omega}_{\rm RF}(\boldsymbol{r})^2} + m g \boldsymbol{r}, \tag{5}$$

where the first term under the square root refers to detuning, and the second term $\boldsymbol{\Omega}_{\rm RF}(\boldsymbol{r})$ is the Rabi frequency, which is generated by the presence of RF coils close to the science chamber. In the quasi-1D

gas, the long axis of the cold atoms lies in the z direction in our scheme, and we can obtain double well potential high enough to split the atoms. Figure 4 shows the potential configuration for three different Rabi frequency amplitudes. According to our calculations, when $B_{\rm RF}$ is 0.2 G, the trap could produce a barrier of 2 μK, which is much higher than the BEC transition temperature. Introduction of the gravitational field tilts all the potential curves. The final double trap bottoms leave a distance of ∼10 μm. This interval is sufficiently small for the two BEC interference patterns to always have one Gaussian envelope and a sine wave modulation on top of it. In contrast, this was not the case for the whole envelope in previous classic experiments.[44] Setting the Gaussian center as the reference, one can trace phase information from the modulation and calculate relevant parameter variations, which is the principle of atomic interferometry.

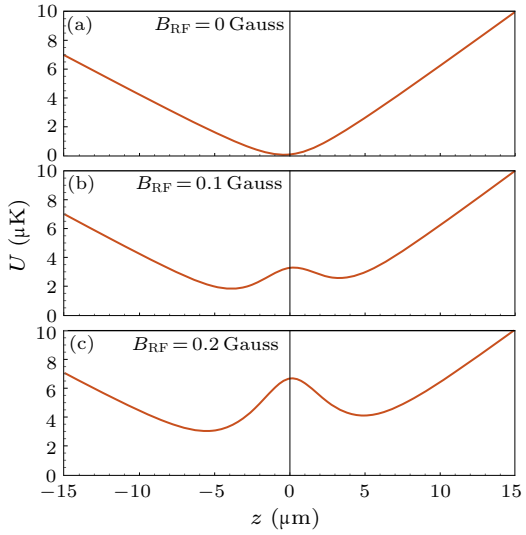

Fig. 4. Double well potential constituted by the QUIC trap and strong radio frequency coupling. Three coupling strengths are shown. At 0 or 0.1 G of radio-induced magnetic field strength, a double bottom or weak double trap does not exist. A large potential barrier of 2 μK is achieved with 0.2 G coupling.

In summary, we have designed an ultracold atom experimental platform to study coherence properties. A QUIC trap combined with an RF dressing technique is proposed. The designed trap is used to study elongated Bose gas behavior, especially in phase fluctuations. The standard deviation of fluctuation fitting provides a powerful way to measure atom temperatures below 100 nK, which is impossible with thermal wing fitting. We find that fluctuations grow as evolution time and aspect ratio increase, leading to a precise measurement of temperature. Then, the fluctuation-induced interference pattern is studied. We simulate the random phase across the whole BEC and obtain the typical density distribution on the CCD camera after a certain evolution time. Finally, taking advantage of the elongated trap, we develop a QUIC and RF dressing technique for atom interferometry in a normal quadrupole configuration. Compared to other atom chip experiments, this proposal is simpler to implement and introduces less phase noise due to the relatively small aspect ratio. Our magnetic field design can be used in most cold atom experiments in principle and has great practical potential in the atom interferometry domain.

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