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Distillability of Sudden Death in Qutrit-Qutrit Systems under Global Mixed Noise *

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Considering the influences of collective dephasing, multilocal qutrit-flip, local qutrit-flip, and the combination of global mixed noise, we study the dynamics of entanglement and the phenomenon of distillability of sudden death (DSD) in a qutrit-qutrit system under various decoherent noises. It is shown that the system always undergoes DSD when it interacts with multilocal and local qutrit-flip noise, and the time-determined bound entangled state is more dependent on different noises. Comparing with the cases of global mixed and collective dephasing noise, we conclude that the qutrit-flip noise is responsible for the DSD.

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Quantum entanglement, as a remarkable feature distinguishing classical physics from quantum physics,[1,2] has been applied in many fields such as quantum teleportation, superdense coding, and quantum cryptographic key distribution.[3] All of them show that it is superior to classical methods in information processing. However, any real quantum system should be regarded as an open system, since it inevitably interacts with the noise environment, which leads to quantum decoherence and degrades the entanglement of the system.[4,5] Thus, it is important and necessary to study the entanglement evolution under the influence of the decoherence. In recent years, great efforts have been taken to research the dynamics of entanglement since the work of Yu and Eberly.[6] They demonstrated that the entanglement between two qubits coupled with two independent noise environments turns out to be vanished in a finite time, resulting in entanglement sudden death (ESD). This surprising phenomenon, contrary to intuition on decoherence, also appears in many other scenarios,[7–12] including in high-dimensional quantum systems,[13–15] and has been confirmed experimentally in atomic ensembles and linear optical ensembles.[16,17]

By contrast, distillability of sudden death (DSD)[18] is another extremely interesting phenomenon in the evolution of entanglement in high-dimensional quantum systems. It originates from the important issue that bipartite entangled states can be divided into two categories. One is free entangled states, which can be distilled under local operations and classical communication (LOCC), and the other is bound entangled states, which means that no LOCC strategy is able to extract pure-state entanglement from the state no matter how many copies are available.[19] Unlike free entangled states, bound entangled states cannot be used independently for quantum information processing. The DSD exhibits such behavior that an initial free entangled state becomes bound entangled in a finite time. This discovery has attracted much interest.[20–32] Ali found that ESD and DSD may occur in the evolution of a two-qutrit system under global dephasing as well as under multilocal dephasing noise,[23] and the DSD induced by the amplitude damping noise can be completely avoided by a simple local unitary transformation.[24] Similar work has been reported on the two-qutrit system interacting with global, local, and multilocal depolarizing noise.[25] Further study was performed in the case of being under thermal reservoirs.[26,27] In addition, the DSD for two-qutrit systems in the presence of an external magnetic field and Dzyaloshinskii-Moriya interaction under decoherence was also investigated.[28,29] The phenomenon of DSD was reported in our previous works for a two-qutrit system coupled to an XY spin chain, and effective methods for controlling the DSD were proposed.[32] In this regard, it is interesting to discuss the new effects resulting from the evolution of the system under a global mixed noise composed of a collective noise different from the multilocal noise. Furthermore, we notice that the DSD in the two-qutrit system under qutrit-flip noise has not been explored. Therefore, in this work we investigate the dynamics of entanglement and the phenomenon of DSD in a qutrit-qutrit system under collective dephasing, multilocal qutrit-flip, local qutrit-flip, and the combination of global mixed noise.

In the study of entanglement dynamics, a very important issue is how to measure the degree of entanglement. As we know, the concurrence is regarded as a good measure of entanglement for spin-1/2 systems.[33] To quantify the entanglement of composed systems in higher-dimensional Herbert space, for example, qutrit-qutrit systems, negativity has been proven as an effective measure.[34] It is based on the

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trace norm of partial transpose ρ^T of the state ρ, and the definition of negativity is given as

$$N(\rho) = \frac{||\rho^T|| - 1}{2}, \tag{1}$$

where $\rho^T_{ij,kl} = \rho_{kj,il}$. According to the Peres separability criterion,[35] the negativity can only quantify partial entangled states. Specifically, we can only judge that a quantum state will be an entangled state if its negativity is positive, which means that it is negative under partial transposition (NPT). However, we cannot judge that the state must be a free entangled state because whether or not all NPT quantum states can be distilled is still an open problem. Meanwhile, we also cannot conclude whether a positive partial transpose (PPT) state with zero negativity is entangled or separable until some other measures or steps reveal its status. Hence, the bound entangled states cannot be quantified and detected by the negativity. In general, no unique criterion can be used to detect all bound entangled states. In this case, usually one takes the realignment criterion[36] to determine certain bound entangled states. The realignment criterion for a given density matrix ρ is defined as

$$R(\rho) = ||\rho^R|| - 1, \tag{2}$$

where $\rho^R_{ij,kl} = \rho_{ik,jl}$. For a separable state ρ, the realignment implies $R(\rho) \leq 0$. For a PPT state, the positive value of the quantity $R(\rho)$ can prove the bound entangled states. That is, the state can be determined as a bound entangled state if $N(\rho) = 0$ and $R(\rho) \geq 0$. However, we should note that the realignment criterion cannot detect all the bound entangled states. For example, if $N(\rho) = 0$ and $R(\rho) = 0$, we cannot determine whether or not the state is bound entangled.

In this work, we study the dynamics of a qutrit-qutrit system under a noisy environment. We suppose that the qutrit-qutrit system and the noisy environment are initially in the product density matrix form $\rho(0) = \rho_{\rm AB}(0) \otimes \rho_E(0)$. After time evolution, the reduced density matrix of the qutrit-qutrit system can be obtained as $\rho_{\rm AB}(t) = {\rm Tr}_{\rm E}[U(t)\rho(0)U^\dagger(t)]$, where ${\rm Tr}_{\rm E}$ means tracing out the environment variables and $U(t)$ is the unitary evolution operator generated by the total Hamiltonian. Assuming $\{|e\rangle_k\}$ is an orthonormal basis for the environment and $\rho_E(0) = |e_0\rangle\langle e_0|$ is the initial environmental state, then we can rewrite the expression of the evolution in virtue of the so-called Kraus operator,

$$\begin{aligned}\rho_{\rm AB}(t) &= \sum_k \langle e_k|U(t)\left(\rho_{\rm AB}(0)\otimes|e_0\rangle\langle e_0|\right)U^\dagger(t)|e_k\rangle \\ &= \sum_k M_k \rho_{\rm AB}(0) M_k^\dagger,\end{aligned} \tag{3}$$

with the Kraus operators $M_k(t) = \langle e_k|U(t)|e_0\rangle$ acting only on the state space of the system. The Kraus operators satisfy the completeness relation $\sum_k M_k^\dagger M_k = 1$.

Let us consider the qutrit-qutrit system interacting with a noisy environment both collectively and individually. The qutrits are spatially separated and have no direct interaction with each other. Collective coupling refers to the situation when both the qutrits are influenced by the same dephasing environment, and multilocal coupling means that each qutrit is coupled to its own independent qutrit-flip environment. When the system is influenced by both collective dephasing and multilocal qutrit-flip noise at the same time, it can be regarded as a system interacting with a global mixed noisy environment. The qutrit-flip noise describes the effect of destroying the information contained in the phase relations without an exchange of energy.[3] The action of the qutrit-flip noise on single qutrit A or qutrit B can be described by the following Kraus operators[37]

$$\begin{aligned}
&F_1^A = \sqrt{1-\frac{2\gamma_A}{3}}I_3\otimes I_3, \quad F_2^A = \sqrt{\frac{\gamma_A}{3}}\begin{pmatrix}00 & 1\\ 10 & 0\\ 01 & 0\end{pmatrix}\otimes I_3,\\
&F_3^A = \sqrt{\frac{\gamma_A}{3}}\begin{pmatrix}01 & 0\\ 00 & 1\\ 10 & 0\end{pmatrix}\otimes I_3,\\
&F_1^B = I_3\otimes\sqrt{1-\frac{2\gamma_B}{3}}I_3, \quad F_2^B = I_3\otimes\sqrt{\frac{\gamma_B}{3}}\begin{pmatrix}00 & 1\\ 10 & 0\\ 01 & 0\end{pmatrix},\\
&F_3^B = I_3\otimes\sqrt{\frac{\gamma_B}{3}}\begin{pmatrix}01 & 0\\ 00 & 1\\ 10 & 0\end{pmatrix},
\end{aligned} \tag{4}$$

with $\gamma_A = \gamma_B = 1 - \exp(-t\Gamma_1)$. Therefore, the evolved state of the qutrit-qutrit system under global mixed noise is given by

$$\rho_{\rm AB}(t) = \sum_k \sum_{i,j} \left(D_k^{\rm AB} F_j^B F_i^A\right)\rho_{\rm AB}(0)\left(F_i^{A\dagger} F_j^{B\dagger} D_k^{AB\dagger}\right), \tag{5}$$

where $D_k^{\rm AB}$ are the Kraus operators describing the collective dephasing noise,[23] which can be written as $D_1^{\rm AB} = {\rm diag}[\gamma,1,1,1,\gamma,1,1,1,\gamma]$, $D_2^{\rm AB} = {\rm diag}[\omega_1,0,0,0,\omega_2,0,0,0,\omega_2]$ and $D_3^{\rm AB} = {\rm diag}[0,0,0,0,\omega_3,0,0,0,\omega_3]$, where $\gamma = \exp(-t\Gamma_2)$, $\omega_1 = \sqrt{1-\gamma^2}$, $\omega_2 = -\gamma^2\sqrt{1-\gamma^2}$ and $\omega_3 = (1-\gamma^2)\sqrt{1+\gamma^2}$. It is easy to check that $D_k^{\rm AB}$ satisfies the relation $\sum_k D_k^{AB\dagger} D_k^{\rm AB} = I$.

In this work, we only consider a particular initial state for the two-qutrit system given as

$$\rho_{\rm AB}(0) = \frac{2}{7}|\psi_+\rangle\langle\psi_+| + \frac{\alpha}{7}\sigma_+ + \frac{5-\alpha}{7}\sigma_-, \tag{6}$$

where $2 \leq \alpha \leq 5$. In Eq. (6), the maximally entangled state $|\psi_+\rangle = \frac{1}{\sqrt{3}}(|01\rangle + |10\rangle + |22\rangle)$ is mixed with the separable states $\sigma_+ = \frac{1}{3}(|00\rangle\langle 00| + |12\rangle\langle 12| + |21\rangle\langle 21|)$ and $\sigma_- = \frac{1}{3}(|11\rangle\langle 11| + |20\rangle\langle 20| + |02\rangle\langle 02|)$. Here $\rho(0)$ is separable for $2 \leq \alpha \leq 3$, bound entangled for $3 < \alpha \leq 4$, and free entangled for $4 < \alpha \leq 5$.[38] After a straightforward calculation from Eq. (5), the evolution

of the initial state (6) can be expressed under the bases $\{|2,2\rangle, |2,1\rangle, |2,0\rangle, |1,2\rangle, |1,1\rangle, |1,0\rangle, |0,2\rangle, |0,1\rangle, |0,0\rangle\}$,

$$\begin{aligned}
&\rho_{11} = \rho_{66} = \rho_{88} = \frac{1}{9} + \frac{3a-7}{63}e^{-2\Gamma_1 t},\\
&\rho_{22} = \rho_{44} = \rho_{99} = \frac{1}{9} - \frac{1}{63}e^{-2\Gamma_1 t},\\
&\rho_{33} = \rho_{55} = \rho_{77} = \frac{1}{9} + \frac{8-3a}{63}e^{-2\Gamma_1 t},\\
&\rho_{24} = \rho_{42} = \frac{2}{63}(1+2e^{-2\Gamma_1 t}),\\
&\rho_{29} = \rho_{92} = \rho_{49} = \rho_{94} = \frac{2}{63}(1+2e^{-2\Gamma_1 t})e^{-\Gamma_2 t},\\
&\rho_{37} = \rho_{73} = \rho_{68} = \rho_{86} = \frac{2}{63}(1-e^{-2\Gamma_1 t}),\\
&\rho_{16} = \rho_{61} = \rho_{18} = \rho_{81} = \rho_{35} = \rho_{53} = \rho_{57} = \rho_{75}\\
&= \frac{2}{63}(1-e^{-2\Gamma_1 t})e^{-\Gamma_2 t}. \qquad (7)
\end{aligned}$$

In the following, we adopt the negativity and the realignment criterion to investigate the evolution of entanglement and the possibility of DSD in the qutrit-qutrit system under a global mixed noisy environment.

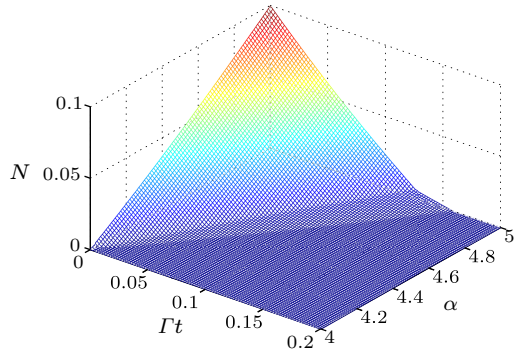

Fig. 1. The negativity $N(\rho_{\rm AB})$ as functions of α and Γt when the system is coupled with the global mixed noise. The parameters $\Gamma_1 = \Gamma_2 = \Gamma$.

In Fig. 1, we give the plot of the negativity as a function of the initial state parameter α and decay parameter Γt when the system undergoes the global mixed noise with $\Gamma_1 = \Gamma_2 = \Gamma$. One can observe that the negativity always decays to zero at a finite time in the range of $4 < \alpha \leq 5$, which indicates that all free entangled states will become PPT states at a finite time. This is also different from the result for the case of global dephasing noise,[23] where the state with $\alpha = 5$ becomes a PPT state only at infinity. Thus we can investigate the possibility of DSD in the two-qutrit system for all initial free entangled states.

Firstly the evolutions of negativity and realignment criterion for different values of parameter α, when only one qutrit undergoes qutrit-flip noise and there is no collective dephasing noise, are plotted in Figs. 2(a), 2(b), and 2(c). We find that when $\alpha = 4.2$, the negativity of the two-qutrit system becomes zero at finite time $\Gamma t \approx 0.065$, while the realignment criterion decays to zero at the time $\Gamma t \approx 0.339$. Note that the positive value of $R(\rho_{\rm AB})$ is in the region $0.065 \leq \Gamma t \leq 0.339$ corresponding to the entanglement of the PPT state. Therefore, an initial free entangled state becomes a bound entangled state at a finite time $\Gamma t \approx 0.065$, which is the phenomenon of DSD. However, it is worth noting that this realignment criterion fails to detect possible entanglement after the time $\Gamma t \approx 0.339$. Therefore, it is indicated that there is the possibility that the free entangled states experience DSD in the system. In addition, the nonzero values of the negativity and rearrangement rule are found to become longer with the increase of α. However, the DSD occurs in all the cases and the time determining the bound entangled state is only slightly shortened. This is different from the results in Refs. [24,28], where $R(\rho_{\rm AB})$ for a large α decays to zero before $N(\rho)$ disappears, which indicates that the rearrangement rule loses the ability to detect bound entangled states. Therefore, we can conclude that the possibility of precipitable DSD in the two-qutrit system is immune to the initial state parameter in this case.

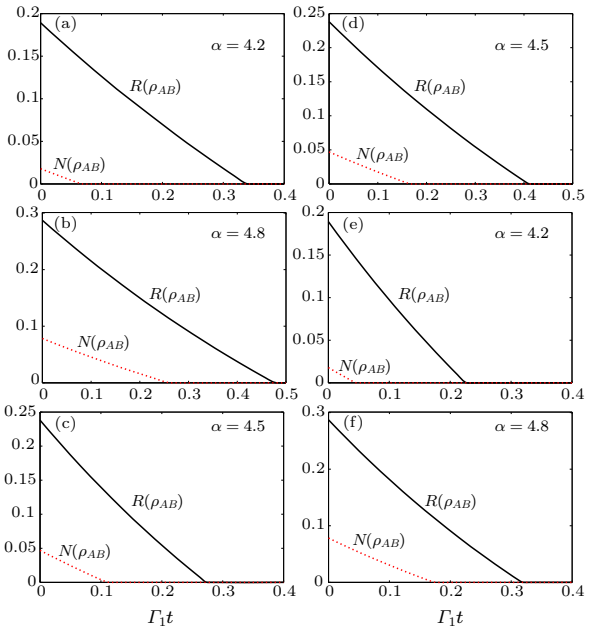

Fig. 2. (a, b, c) The negativity $N(\rho_{\rm AB})$ and the realignment criterion $R(\rho_{\rm AB})$ versus $\Gamma_1 t$ for different initial free entangled states when only one qutrit undergoes qutrit-flip noise. (d, e, f) The negativity $N(\rho_{\rm AB})$ and the realignment criterion $R(\rho_{\rm AB})$ versus $\Gamma_1 t$ for different initial free entangled states when both the qutrits are coupled to their local qutrit-flip environment. The parameter $\Gamma_2 = 0$.

Next we discuss the situation that both the qutrits are coupled to their local qutrit-flip noise in Figs. 2(d), 2(e), and 2(f); namely, we only consider the influence of multilocal qutrit-flip noise. As is expected, the variation trends of $N(\rho_{\rm AB})$ and $R(\rho_{\rm AB})$ are similar to the case when only one qutrit undergoes qutrit-flip noise. The phenomenon of DSD always appears, whereas the decay of $N(\rho_{\rm AB})$ and $R(\rho_{\rm AB})$ is strengthened and the time determining the bound entangled state is shortened. In Fig. 3, we show the evolution of the negativ-

ity and the realignment criterion for the case that only collective dephasing noise is considered when $\alpha = 4.5$. It can be seen that the negativity decays against $\Gamma_2 t$ first, and then it reaches a steady value. This implies that the initial free entangled state remains an NPT state until infinity. Therefore, the phenomena of ESD and DSD never occur in this case.

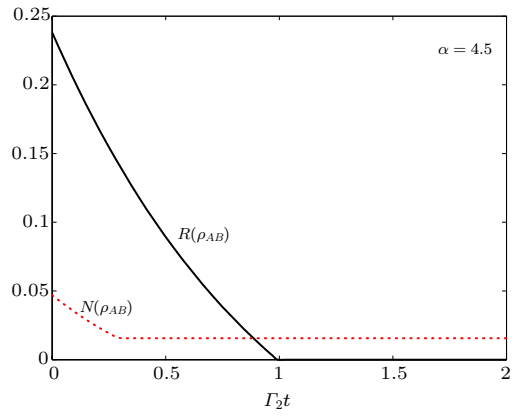

Fig. 3. The negativity $N(\rho_{\rm AB})$ and the realignment criterion $R(\rho_{\rm AB})$ versus $\Gamma_2 t$ when the two qutrits evolve under collective dephasing noise. The parameter $\Gamma_1 = 0$.

In Figs. 4(a), 4(b), and 4(c), we illustrate the dynamics of the negativity and the realignment criterion under the global mixed noise with $\Gamma_A = \Gamma_B = \Gamma$ for different initial free entangled states. Comparing with previous cases discussed above, one can see that the decay of $N(\rho_{\rm AB})$ and $R(\rho_{\rm AB})$ in this case is faster than that in other cases. In addition, the phenomenon of DSD always occurs though the time to determine the bound entangled state is notably shortened. We recall that only the collective dephasing noise cannot induce the DSD, so we can conclude that decoherence effects induced by the multilocal qutrit-flip noise are much stronger than those of collective dephasing noise, and the multilocal qutrit-flip noise is responsible for the DSD. If we reduce the influence of the multilocal qutrit-flip noise on the evolution of the two-qutrit system, for example, we let $\Gamma_2 = 5\Gamma_1$, we can find from Fig. 4(d), 4(e), and 4(f) that the phenomenon of DSD does not always occur for any initial free entangled state, and it only appears for some states with lower values of α in the range $4 < \alpha \leq 5$.

Finally, motivated by the results in Ref. [24], where a simple local unitary operation on the initial states can completely avoid DSD under amplitude damping, we investigate whether the local unitary operation can eliminate DSD in the evolution of the two-qutrit system coupled with global mixed noise. We consider the local unitary operator $U = I_3 \otimes \theta$, with $\theta = |0\rangle\langle 1| + |1\rangle\langle 0| + |2\rangle\langle 2|$. When this operator is applied locally to the state of Eq. (6), $\rho(0)$ is converted into the state σ_α, where

$$\sigma_\alpha = U\rho(0)U^\dagger = \frac{2}{7}|\widetilde{\psi}_+\rangle\langle\widetilde{\psi}_+| + \frac{\alpha}{7}\widetilde{\sigma}_+ + \frac{5-\alpha}{7}\widetilde{\sigma}_-, \quad (8)$$

with $|\widetilde{\psi}_+\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$, $\widetilde{\sigma}_+ = \frac{1}{3}(|01\rangle\langle 01| + |12\rangle\langle 12| + |20\rangle\langle 20|)$ and $\widetilde{\sigma}_- = \frac{1}{3}(|10\rangle\langle 10| + |21\rangle\langle 21| + |02\rangle\langle 02|)$. The evolution of σ_α in the noisy environment can be obtained straightforwardly in the same manner as carried out for the state $\rho_{\rm AB}(t)$. Our results show that σ_α exhibits more obvious decoherence behavior than $\rho_{\rm AB}(t)$, so the simple local unitary transformation cannot completely avoid DSD under global mixed noise.

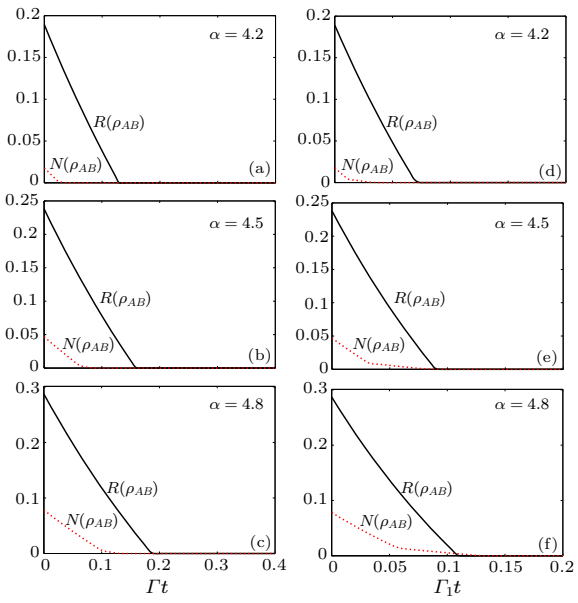

Fig. 4. (a, b, c) The negativity $N(\rho_{\rm AB})$ and the realignment criterion $R(\rho_{\rm AB})$ versus Γt for different initial free entangled states when the system evolves under global noise with the parameters $\Gamma_A = \Gamma_B = \Gamma$. (d, e, f) The negativity $N(\rho_{\rm AB})$ and the realignment criterion $R(\rho_{\rm AB})$ versus $\Gamma_1 t$ for different initial free entangled states when the system evolves under global mixed noise with the parameters $\Gamma_2 = 5\Gamma_1$.

In conclusion, the dynamics of entanglement and the DSD in a qutrit-qutrit system under collective dephasing, multilocal qutrit-flip, local qutrit-flip, and the combination of global mixed noise are discussed by using negativity and realignment criterion. It is shown that all initial free entangled states always become PPT states at a finite time when the system is under global, multilocal, and local qutrit-flip noise, but the negativity always keeps a nonzero value in the case of collective dephasing noise. Combining the negativity and realignment criterion, we find that the phenomenon of DSD always occurs when the system couples with global, multilocal, and local qutrit-flip noise, though the time-determined bound entangled state depends on different noises. The DSD cannot be determined for all initial free entangled states with the strengthening of the influence of collective dephasing noise or reduce the influence of qutrit-flip noise, so we conclude that qutrit-flip noise is a major cause of DSD. Furthermore, our results indicate that the phenomenon of DSD cannot be avoided by a simple local unitary transformation.

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