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Pairing Effects on Bubble Nuclei *

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In the framework of the Skyrme–Hartree–Fock–Bogoliubov approach with the SkT interaction, the pairing effects on the proton bubble structures of ^{46}Ar and ^{206}Hg are discussed. In calculations, three kinds of pairing forces (volume, surface and mixed pairing interactions) are used. For ^{46}Ar, it is shown that the bubble structure with the volume pairing is almost the same as that with the mixed pairing. The bubble with the surface pairing is less pronounced than those with the volume and mixed pairings. Analyzing the density distributions and occupation probabilities of the proton s states and the quasi-degeneracy between the proton $2s_{1/2}$ and $1d_{3/2}$ orbitals, we explain the difference between the bubble structure with the surface pairing and those with the volume and mixed pairings. For ^{206}Hg, it is seen that the proton density distribution with the surface pairing is different from those with the volume and mixed pairings in the whole region of the radial distance. In addition, it is found that the bubbles with the three pairing forces are different from each other and the least pronounced bubble is obtained with the surface pairing. Thus the selection of the pairing force is important for the study of the nuclear bubble structure.

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In nuclear physics, the Skyrme energy density functional theory is one of the most important microscopic methods to describe the nuclear structure.[1–4] It is well known that the Skyrme interaction is an effective zero-range nonlocal one, which was proposed in the 1950s.[5] In 1972, it was firstly applied to study the magic nuclear structures in the framework of the self-consistent mean-field method.[6] Since then, the Skyrme energy density functional theory (the Skyrme–Hartree–Fock (SHF) and Skyrme–Hartree–Fock–Bogoliubov (SHFB) approaches) has been widely used to investigate the bulk properties and microscopic structures of stable and exotic nuclei.[7–10]

Nowadays the study on exotic nuclear structure has attracted attention of many researchers.[11–13] Usually the exotic nuclei refer to the ones that are far from the β-stability line, whose structures are different from those of stable nuclei, for example, halo nuclei.[14] Studies indicate that the bubble nucleus is one kind of exotic nuclei, whose central density vanishes or is much less than the saturation density ($\rho_0 \sim 0.16\,\mathrm{fm}^{-3}$). Research of the bubble structure started from the work of Wilson.[15] In the pioneering work of Wilson, the low-lying excitations in spherical nuclei were discussed by employing the classical oscillations of bubbles. The first microscopic calculation was performed by the HF+BCS method with an interaction derived from a G-matrix.[16] Since then, bubble structures have been predicted in various mass regions from light to super-heavy nuclei.[17–44] Researchers suggested that the bubble formation mechanism of light nuclei is different from that of heavy nuclei. Usually, one thinks that the bubble structure formation of heavy and superheavy nuclei comes from the effect of the electrostatic repulsion by moving protons towards the nuclear surface.[36] For light nuclei, whose bubble structures originate from the low occupation probability of the s states. The absence of the $l = 0$ orbital contribution leads to a strong drop of the nuclear central density.[36] Very recently, Mutschler *et al.* reported the first experimental evidence for a depletion of the central proton density in ^{34}Si by the one-proton removal reaction technique.[45] The measured occupation probability of the proton $2s_{1/2}$ orbital is 0.17 ± 0.03,[45] which indicates that the pairing correlation effect in ^{34}Si is weak. It seems that the small deformation and weak pairing effect become the key factors for the bubble formation.

Many relevant studies showed that the bubble structure is weakened by the pairing correlation effect.[23,24] Thus in practical calculations the bubble structure should be dependent strongly on the pairing form. In the framework of the SHFB approach, a zero

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range density dependent pairing interaction is usually used, which includes the volume, surface and mixed pairing forces. Previous studies suggested that there is a noticeable difference between the neutron-rich nuclear structures calculated with the surface pairing force and those with the mixed and volume pairing forces.[46–48] These studies drive us to think whether the bubble structures are influenced by different types of pairing forces. This constitutes the motivation of this study. In our previous work, we studied the tensor force effect on the bubble structures of ^{34}Si, ^{46}Ar and ^{206}Hg using the Skyrme energy density functional theory.[21–25] In addition, to understand the bubble formation, the competition between the tensor force and pairing interaction was discussed.[23,24] Because the pairing in ^{46}Ar and ^{206}Hg is stronger than that in ^{34}Si, based on our previous work, in this Letter we make a discussion on the bubble structures of ^{46}Ar and ^{206}Hg using different pairing force forms in the framework of the SHFB approach.

Previous studies show that the bubble existence of ^{46}Ar and ^{206}Hg is rather model dependent.[18,21–23,25,26,35,40–42] In the present calculation, the SkT[49] interaction, i.e., a frequently used Skyrme interaction, is applied with the HFBRAD code.[4] The box and mesh sizes are selected to be 30 fm and 0.1 fm, respectively. In the pairing channel, the zero range density dependent pairing interaction is written as[4]

$$V_{\rm pair} = \left(t'_0 + \frac{t'_3}{6}\rho^{\gamma'}\right)\delta, \tag{1}$$

where $\gamma' = 1$; $t'_3 = 0$, $-37.5t'_0$ and $-18.5t'_0$ correspond to the volume, surface and mixed pairing forces, respectively, and t'_0 is the pairing strength parameter, which is adjustable. Usually t'_0 is determined by the empirical pairing energy gap, whose value can be extracted by the three-point or five-point formula.[50–52] The empirical $\Delta_{\rm p}$ values of ^{46}Ar and ^{206}Hg by the five-point formula are 2.012 MeV and 0.805 MeV, respectively.

In fact, $\Delta_{\rm p}$ is influenced by the deformation, shell effect, isospin effect, and other nuclear structure effects. Thus it is not easy to discuss the bubble structure of ^{46}Ar influenced by different pairing force forms when the accurate pairing force is unclear. For the sake of discussion, the proton pairing gap of ^{46}Ar is assumed to be $\frac{1}{3}\Delta_{\rm p}$ (0.671 MeV), $\frac{2}{3}\Delta_{\rm p}$ (1.341 MeV), and $\Delta_{\rm p}$ (2.012 MeV). Here t'_0 values are then determined by the three pairing gap values for the three pairing force forms. Lastly, the proton density distribution of ^{46}Ar is calculated within the SHFB approach by inputting the established t'_0 values.

The proton density distributions of ^{46}Ar with the SkT interaction using the t'_0 values determined by the three values of the pairing energy gap ($\frac{1}{3}\Delta_{\rm p}$, $\frac{2}{3}\Delta_{\rm p}$, and $\Delta_{\rm p}$) are plotted in Fig. 1. For each pairing gap, the density distributions as a function of the radial distance r in the cases of the volume, surface and mixed pairing forces are shown. From Fig. 1, the following features can be seen: (i) For each pairing gap, the density distributions extracted by the volume and mixed pairings are almost the same and the proton bubble in the case of the surface pairing is less pronounced than those in the cases of the volume and mixed pairings. (ii) The bubbles are evident in all three pairing gap cases. (iii) The bubbles become less and less pronounced and the bubble by the surface pairing is more and more different from those by the volume and mixed pairings with the increase of the pairing gap.

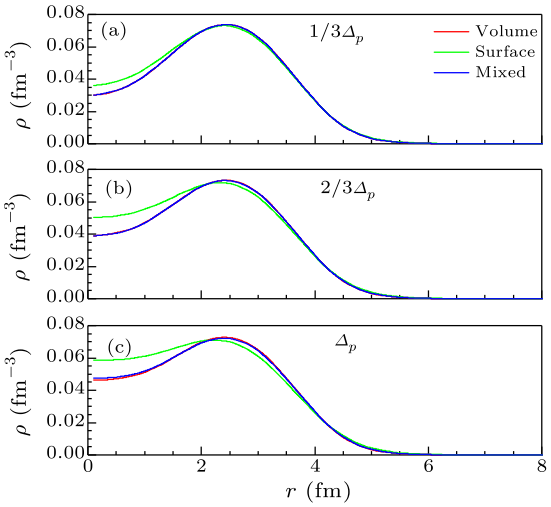

Fig. 1. The proton density distributions of ^{46}Ar with the volume, surface and mixed pairing forces. In the SHFB calculations, t'_0 values are determined by the pairing gaps ($\frac{1}{3}\Delta_{\rm p}$ (a), $\frac{2}{3}\Delta_{\rm p}$ (b), and $\Delta_{\rm p}$ (c)).

Table 1. The t'_0 values, DF values, $v^2_{\rm p}$ values of the proton $2s_{1/2}$ orbital and the energy gap values ΔE between the $2s_{1/2}$ and $1d_{3/2}$ states in ^{46}Ar for the three pairing gaps with the SkT interaction.

	$\frac{1}{3}\Delta_{\rm p}$			$\frac{2}{3}\Delta_{\rm p}$			$\Delta_{\rm p}$		
	volume	surface	mixed	volume	surface	mixed	volume	surface	mixed
t'_0 (MeV·fm^{-3})	−247.5	−717.4	−393.8	−267.0	−752.2	−427.2	−290.5	−788.4	−458.2
DF	59.32%	50.73%	59.03%	46.79%	30.00%	46.86%	36.49%	17.73%	34.35%
$v^2_{\rm p}$	0.052	0.114	0.056	0.158	0.313	0.161	0.250	0.437	0.269
ΔE (MeV)	3.441	2.554	3.452	3.289	1.868	3.323	3.203	1.270	2.998

Nuclear interior density is mainly from the contribution of the s states. To analyze the reason why the bubbles with the volume and mixed pairings are stronger than those with the surface pairing, by taking the pairing gap to be $\frac{2}{3}\Delta_{\rm p}$ as an example, the density distributions of the $1s_{1/2}$ and $2s_{1/2}$ states, $\rho_{1s_{1/2}}$ and $\rho_{2s_{1/2}}$, and their sum $\rho' = \rho_{1s_{1/2}} + \rho_{2s_{1/2}}$ for the volume, surface and mixed pairings are plotted in Fig. 2. From Fig. 2, one can see that for each case the central densities with the volume and mixed pairings are lower than that with the surface pairing. Thus the bubble with the surface pairing is the weakest.

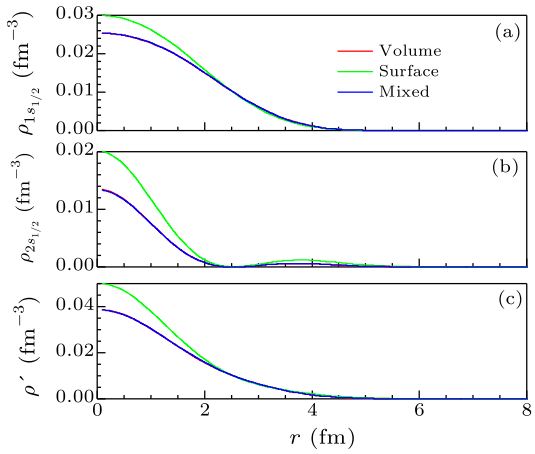

Fig. 2. The density distributions with the volume, surface and mixed pairing forces of the single proton $1s_{1/2}$ state (a) and $2s_{1/2}$ state (b) of ^{46}Ar, and their sum (c).

The experimental energy difference between the $2s_{1/2}$ and $1d_{3/2}$ states for ^{48}Ca is just 0.40 MeV.[6] Thus the two states are quasi-degenerate. As to ^{46}Ar, it can be seen as a nucleus resulted from removing two protons from ^{48}Ca. Thus the bubble structure of ^{46}Ar is not only associated with the occupation probability of the $2s_{1/2}$ orbital but also related to the energy splitting of the pseudo-spin partners. To quantify the bubbles of ^{46}Ar and further understand the above mentioned features, the established t'_0 values, the depletion factor (DF) values,[34–38] the occupation probability $v^2_{\rm p}$ values of the $2s_{1/2}$ orbital, and the energy gap values ΔE between the proton $2s_{1/2}$ and $1d_{3/2}$ states for the three pairing gaps are listed in Table 1. Note that DF is defined as DF$=\frac{\rho_{\rm max}-\rho_{\rm c}}{\rho_{\rm max}}\times 100\%$,[34–38] which represents the reduction of the density at the nuclear center relatively to its maximum value. Here $\rho_{\rm max}$ and $\rho_{\rm c}$ are the values of the nucleon density at its maximum and at the nuclear center, respectively. The first feature can also be seen from the DF values in the fourth line of Table 1. The third line of Table 1 shows that the differences between the t'_0 values with the volume pairing and those with the mixed pairing are not so large so that the $v^2_{\rm p}$ values of the $2s_{1/2}$ state in the two cases are close to each other. As a result, the bubbles with the volume pairing and the mixed pairing are very similar. However, for the surface pairing the absolute t'_0 value is much larger than those of the volume and mixed pairings for each kind of pairing gap, which leads to larger $v^2_{\rm p}$ values in the surface pairing case. Thus the bubbles with the volume and mixed pairings are more pronounced than that with the surface pairing. As to the pseudospin symmetry, for each pairing gap the ΔE value with the surface pairing is much smaller than those with the volume and mixed pairings, which can be seen from the last line of Table 1. It indicates that the pseudospin symmetry with the surface pairing is preserved better than those with the two other pairings. The smaller the ΔE value is, the larger the $v^2_{\rm p}$ value is. Thus the bubble with the surface pairing becomes weaker. In addition, as can be seen from the last line of Table 1, all the $v^2_{\rm p}$ values are small. The maximum v^2 value is just 0.437. These small $v^2_{\rm p}$ values lead to the bubble appearance. As to the third feature, it can also be understood by comparing the DF values for the three kinds of pairing gaps. Taking the volume pairing force as an example, one can see that the absolute t'_0 values and $v^2_{\rm p}$ values become larger and larger with the increase of the pairing gap, thus the bubble becomes less and less pronounced. Meanwhile, from the fifth line of Table 1 one can see that the $v^2_{\rm p}$ differences in the cases of the volume (mixed) pairing and the surface pairing become larger and larger with the increase of the pairing gap. Consequently, the bubble differences become more and more evident.

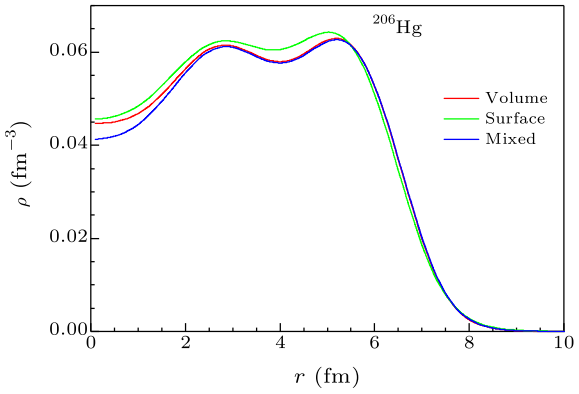

Fig. 3. The proton density distributions of ^{206}Hg with the volume, surface and mixed pairing forces.

As to ^{206}Hg, the proton density distributions with the three pairing forces for the $\Delta_{\rm p}$ case are plotted in Fig. 3. From Fig. 3, one can see that the density distribution with the surface pairing is different from those with the volume and mixed pairings in the whole r range. The density distributions in the small r region (the bubble structures) with the three pairing forces are different from each other. The extracted DF values are 27.35%, 26.83% and 32.48% for the

volume, surface and mixed pairings, indicating that the bubbles with the volume and mixed pairings are stronger than that with the surface pairing, which is the same as the case of ^{46}Ar. However, it is found that the bubble with the mixed pairing force is the most evident. Meanwhile, the bubbles with the volume pairing and the surface pairing are close to each other. As the discussion of ^{46}Ar, we extract the ΔE values between the $3s_{1/2}$ and $2d_{3/2}$ states (the two states are pseudo-spin partners). The ΔE values with the volume, surface and mixed pairings are 1.695 MeV, 1.668 MeV and 1.925 MeV, respectively. It is easy to know that the smaller the ΔE is, the smaller the DF value is. Thus the bubble structure of ^{206}Hg is indeed dependent on the quasi-degeneracy degree of the $3s_{1/2}$ and $2d_{3/2}$ states. According to the conclusions of Refs. [46-48] and the present study, one can see that the choice of the pairing forces is important for the study of nuclear structure within the SHFB approach. In fact, in the particle-particle channel the volume and surface pairings are two kinds of extreme cases. The volume pairing is not dependent on the nuclear density. The surface pairing is a strong density dependent one. According to the work on the odd-even mass difference,[46–48] the mixed pairing (the average of the volume and surface pairings) could be better than the other two pairings.

In summary, the bubble structures of ^{46}Ar and ^{206}Hg influenced by the volume, surface and mixed pairing forces have been studied by the SHFB approach with the SkT interaction. Because the pairing gap in ^{46}Ar can not be extracted accurately by the so-called five point formula, t_0' values were determined by the pairing gap values of $\frac{1}{3}\Delta_{\rm p}$, $\frac{2}{3}\Delta_{\rm p}$ and $\Delta_{\rm p}$. It is shown that for each pairing gap the bubble structures of ^{46}Ar for the three kinds of pairing forces are evident. However, for each pairing gap the bubble structures extracted by the volume and mixed pairings are almost the same and the bubble structure by the surface pairing is less pronounced than those by the volume and mixed pairings. By analyzing the density distributions and occupation probabilities of the s states and the quasi-degeneracy between the $2s_{1/2}$ and $1d_{3/2}$ states, the similarities and differences of the bubble structure with the three pairing forces have been explained. For ^{206}Hg, it is seen that in the whole r region the density distributions with the volume and mixed pairings are different from that with the surface pairing. Meanwhile, it is found that the bubbles with the three pairings are different from each other and the bubble with the surface pairing is the least pronounced. Therefore, the pairing force form is important for the nuclear structure study with the SHFB approach and a precise pairing force is called for. The pairing force given by the Richardson model[53] may be helpful for developing the SHFB theory.

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