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Quantization of Horizon Area of Kerr–Newman–de Sitter Black Hole

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Using the adiabatic invariant action and applying the Bohr–Sommerfeld quantization rule and first law of black hole thermodynamics, a study of the quantization of the entropy and horizon area of a Kerr–Newman–de Sitter black hole is carried out. The same entropy spectrum is obtained in two different coordinate systems. It is also observed that the spacing of the entropy spectrum is independent of the black hole parameters. Also, the corresponding quantum of horizon area is in agreement with the results of Bekenstein.

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A complete and consistent quantum theory of gravity is still not fully developed though its necessity was recognized as early as the 1930s. It is believed that black holes should play an important role in the quantum theory of gravity. In the early attempts to quantize black holes, Bekenstein[1–4] suggested that in quantum theory the black hole mass spectrum must be discrete and highly degenerate unlike the continuous black hole mass spectrum in general relativity. In addition, the black hole surface area spectrum must be discrete and equispaced with a degeneracy corresponding to the black hole entropy associated with the area eigenvalue. It was also observed that the horizon area of a non-extremal black hole behaves like a classical adiabatic invariant which corresponds to a quantum entity with discrete spectrum. Bekenstein[3] found a lower bound on the increase in the black hole surface area in quantum theory, which is given by

$$(\Delta A)_{\min} = 8\pi\ell_{\rm p}^2,$$

where $\ell_{\rm p} = (\sqrt{G/c^3})^{1/2}\hbar^{1/2}$ is the Plank length. Hod[5] also found a similar lower bound on the increase of the black hole area caused by the assimilation of charged particles, which is given by

$$(\Delta A)_{\min} = 4\ell_{\rm p}^2.$$

Following Bohr's correspondence principle, Hod[5] used quasinormal mode frequencies of the Schwarzschild black hole as the transition frequency in quantum theory and determined the surface area spectrum, which is given by

$$A_n = (4\ell_{\rm p}^2\ell n 3)n, \quad n = 1, 2, 3, \ldots$$

It is characterized by an equidistant area spacing of $4\ell_{\rm p}^2\ell n 3$. Drayer[6] also found a similar area spacing based on loop quantum gravity.

In a semiclassical study, Padmanabhan and Patel[7] observed that in all static spacetimes with horizons, the area of the horizon as measured by any observer blocked by the same horizon must be quantized. Also, the change in area of the horizon must also be quantized with minimum detectable change of the order of $\ell_{\rm p}^2$. Based on Hod's ideas, Kunstatter[8] considered the quasinormal frequency as a fundamental frequency for a black hole and the quantity $I = \int dE/\omega(E)dE$ as an adiabatic invariant, where $\omega(E)$ is the vibrational frequency of a system with energy E. Maggiore[9] introduced a new interpretation of the black hole quasinormal modes, namely that the relaxation behavior of a perturbed black hole governed by the complex quasinormal frequencies $\omega = \omega_R + i\omega_I$ is the same as that of the damped harmonic oscillator whose real frequencies are $[\omega_{\rm R}^2 + \omega_{\rm I}^2]^{1/2}$ rather than $\omega_{\rm R}$ used by Hod. Vagenas[10] extended Kunstatter's approach to the case of a Kerr black hole using an adiabatic invariant of the form $I = \int(dM - \Omega dJ)/\omega_{\rm o}$, where Ω and J are the angular velocity and angular momentum of the black hole, respectively, and $\omega_{\rm o}$ is the quasinormal mode frequency of the black hole. Using this adiabatic invariant and considering that in the semiclassical regime, the Kerr black hole must be far away from extremality so that $J \ll M^2$, Medved[11] showed that the Kerr area spectrum is asymptotically identical for the two methods (Hod's and Kunstatter's methods along with modifications suggested by Maggiore and Vagenas) and obtained a universal form for the Schwarzschild and Kerr black hole spectra. Applying quasinormal mode frequencies, the entropy and the area spectra of black holes have been studied in different spacetimes.[12,13] Based on the standard commutation rules and quantization of angular momentum related to the Euclidean Rindler space of black holes, Ropotenko[14] derived the quantization of the black hole horizon area and the area spectrum is applicable to all kinds of black holes. In 2011, Majhi and Vagenas[15] proposed a new approach using an adiabatic invariant quantity of the form $\int p_i dq_i = \int\int_0^H \frac{dH'}{\dot{q}_i} dq_i$ and the Bohr–

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Sommerfeld quantization condition where p_i is the momentum conjugate to q_i. Jiang and Han[16] used the covariant action $I = \oint p_i dq_i = \oint \frac{dH}{\dot{q}_i} dq_i$ as an adiabatic invariant quantity and studied the horizon area of the Schwarzschild black hole in both Schwarzschild and Painlevé coordinates. Showing that the time period of the outgoing wave is related to the vibrational frequency of the perturbed black hole, Zeng *et al.*[17] quantized the horizon area of Schwarzschild and Kerr black holes. More works have been carried out to quantize black hole entropy and horizon area.[18–23]

The Kerr–Newman–de Sitter (KNdS) metric in Boyer–Lindquist coordinates can be written as[24]

$$\begin{aligned} ds^2 =& -\frac{\Delta - \Delta_\theta a^2 \sin^2\theta}{\rho^2 \Xi^2} dt^2 + \frac{\rho^2}{\Delta} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \\ & - \frac{2a[\Delta_\theta (r^2 + a^2) - \Delta] \sin^2\theta}{\rho^2 \Xi^2} dt d\phi \\ & + \frac{\Delta_\theta (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\rho^2 \Xi^2} \sin^2\theta d\phi^2, \end{aligned} \quad (1)$$

where

$$\begin{aligned} \rho^2 =& r^2 + a^2 \cos^2\theta, \quad \Xi = 1 + \frac{1}{3}\Lambda a^2, \\ \Delta_\theta =& 1 + \frac{1}{3}\Lambda a^2 \cos^2\theta, \\ \Delta =& (r^2 + a^2)\Big(1 - \frac{1}{3}\Lambda r^2\Big) - 2Mr + Q^2. \end{aligned} \quad (2)$$

This line element represents the KNdS solution for $\Lambda > 0$, and the anti-KNdS solution for $\Lambda < 0$. Here M and Q are the mass and the charge of the black hole, respectively. The KNdS solution has singularity when

$$\Delta = r^2 + a^2 - 2Mr + Q^2 - \frac{1}{3}\Lambda r^2 (r^2 + a^2) = 0. \quad (3)$$

We consider only the case $\Lambda > 0$, in which Eq. (3) has four real roots. The case $\Lambda < 0$ is ignored, since Eq. (3) gives imaginary roots. The four real roots of Eq. (3), that is, r_c, r_h, r_1 and r_- ($r_c > r_h > r_1 > 0 > r_-$) satisfy the relation as

$$\begin{aligned} & (r - r_c)(r - r_h)(r - r_1)(r - r_-) \\ =& -\frac{3}{\Lambda}\Big[r^2 + a^2 - 2Mr + Q^2 - \frac{1}{3}\Lambda r^2 (r^2 + a^2)\Big]. \end{aligned} \quad (4)$$

The largest root r_c represents the location of the cosmological horizon, r_h is the location of the black hole event horizon, and r_1 is the location of the Cauchy horizon. The smallest negative root, r_- indicates another cosmological horizon on the other side of the ring singularity at $r = 0$ and another infinity.[25] We factorize Δ into the following form[26]

$$\Delta = (r - r_h)\Delta'(r_h), \quad (5)$$

where $\frac{d\Delta}{dr} = \Delta'(r_h)$. The metric (1) has a singularity at the radius of the event horizon. A coordinate system analogous to the Painlevé coordinate system will be established. We see that the new coordinate system is well behaved at the event horizon and we will discuss the dragging coordinate system. Let $\frac{d\phi}{dt} = -\frac{g_{14}}{g_{44}} = \Omega$. The new line element of KNdS is

$$ds^2 = \hat{g}_{11} dt_k^2 + \frac{\rho^2}{\Delta} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2, \quad (6)$$

where

$$\hat{g}_{11} = -\frac{\Delta \Delta_\theta \rho^2}{\Xi^2 [\Delta_\theta (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta]}. \quad (7)$$

The angular velocity at the event of horizon is

$$\Omega_h = \frac{a}{r_h^2 + a^2}. \quad (8)$$

The surface gravity of the black hole is given by

$$\kappa = \lim_{\hat{g}_{11} \to 0} \Big(-\frac{1}{2}\sqrt{\frac{-g^{22}}{\hat{g}_{11}}} \frac{d\hat{g}_{11}}{dr} \Big) = \frac{\Delta'(r_h)}{2\Xi(r_h^2 + a^2)}. \quad (9)$$

To study the black hole entropy, the line element of the Euclideanized KNdS black hole is obtained from Eq. (6) by transforming $t_k \to -i\tau$ as

$$ds^2 = \frac{\Delta \Delta_\theta \rho^2 \, d\tau^2}{\Xi^2 [\Delta_\theta (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta]} + \frac{\rho^2}{\Delta} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2. \quad (10)$$

The quantum phenomena, such as tunneling radiation,[27–32] quantization of black hole entropy and area[33–37] are observed at the black hole event horizon. The close contour integral can be used by taking a close path from $q_i^{\rm out}$ (outside the horizon) to $q_i^{\rm in}$ (inside the horizon), near the event horizon. By adopting this closed integral for the charged black hole, the adiabatic invariant quantity can be expressed as

$$I_{\rm adia} = \oint p_i dq_i = \int_{q_i^{\rm in}}^{q_i^{\rm out}} p_i^{\rm out} dq_i + \int_{q_i^{\rm out}}^{q_i^{\rm in}} p_i^{\rm in} dq_i, \quad (11)$$

where $q_i^{\rm out}$ and $q_i^{\rm in}$ indicate the Euclideanized KNdS spacetime coordinates, and $p_i^{\rm out}$ and $p_i^{\rm in}$ represent the canonical momenta of the coordinates $q_i^{\rm out}$ and $q_i^{\rm in}$, respectively. For quantization of the entropy of the charged black hole, the expression of adiabatic covariant action in the charged black hole will be used. The integral $\int_{q_i^{\rm in}}^{q_i^{\rm out}} p_i^{\rm out} dq_i$ can be written as

$$\begin{aligned} \int_{q_i^{\rm in}}^{q_i^{\rm out}} p_i^{\rm out} dq_i =& \int_{q_i^{\rm in}}^{q_i^{\rm out}} \int_0^{p_i^{\rm out}} dp_i^{\rm out\prime} dq_i \\ =& \int_{\tau_{\rm in}}^{\tau_{\rm out}} \int_0^H dH' d\tau + \int_{q_j^{\rm in}}^{q_j^{\rm out}} \int_0^H \frac{dH'}{\dot{q}_j}, \end{aligned} \quad (12)$$

where H is the Hamiltonian of the system, q_j represent the space-like coordinates, $\int_{\tau_{\rm in}}^{\tau_{\rm out}} \int_0^H \frac{dH'}{\dot{q}_j}$ signifies the classical action I_c, and satisfies $I_c = \int_{\tau_{\rm in}}^{\tau_{\rm out}} L d\tau$, and

L is the Lagrangian function. The effect of the electromagnetic field should be considered when a charge particle tunnels out of the horizon. Then the matter gravity system consists of the black hole and the electromagnetic field outside the black hole. We observe that there is a dragging effect of the coordinate system in the rotating black hole and also in the dragging coordinate system, and the matter field in the ergosphere near the horizon should be described. It is also observed that the coordinate ϕ does not appear in the dragging metric (6). Therefore, ϕ is an ignorable coordinate in the Lagrangian function. For eliminating the two degrees of freedom, the classical action can be written as

$$\begin{aligned}\int_{q_j^{\rm in}}^{q_j^{\rm out}}\int_0^H \frac{dH'}{\dot{q}_j} &= \int_{\tau_{\rm in}}^{\tau_{\rm out}}(L-P_\phi\dot{\phi}-P_{A_\tau}\dot{A}_\tau)d\tau,\\ &= \int_{r_{\rm in}}^{r_{\rm out}}\int_0^{p_r} dP_r' dr - \int_{\phi_{\rm in}}^{\phi_{\rm out}}\int_0^{p_\phi} dP_\phi' d\phi\\ &\quad - \int_{A_\tau^{\rm in}}^{A_\tau^{\rm out}}\int_0^{P_{A_\tau}} dP'_{P_{A_\tau}} dP_{A_\tau},\\ &= \int_{\tau_{\rm in}}^{\tau_{\rm out}}\Big[\int_0^H dH'\Big|_{(r;\phi,P_\phi;A_\tau,P_{A_\tau})}\\ &\quad - \int_0^H dH'\Big|_{(\phi;r,P_r;A_\tau,P_{A_\tau})}\\ &\quad - \int_0^H dH'\Big|_{(A_\tau;r,P_r;\phi,P_\phi)} d\tau\Big],\\ &= \int_{\tau_{\rm in}}^{\tau_{\rm out}}\int_0^H dH' d\tau, \end{aligned} \tag{13}$$

where P_r, P_ϕ and P_{A_τ} are canonical momenta with respect to r, ϕ and A_τ, and the expression of Hamilton's canonical equations can be written as

$$\begin{aligned}&\dot{r} = \frac{dH}{dP_r}\Big|_{(r;\phi,P_\phi;A_\tau,P_{A_\tau})},\\ &dH'|_{(r;\phi,P_\phi;A_\tau,P_{A_\tau})} = dM',\\ &\dot{\phi} = \frac{dH}{dP_\phi}\Big|_{(\phi;r,P_r;A_\tau,P_{A_\tau})},\\ &dH'\,|_{(\phi;r,P_r;A_\tau,P_{A_\tau})} = \Omega_h dJ'\\ &\dot{A}_\tau = \frac{dH}{dP_{A_\tau}}\,|_{(A_\tau;r,P_r;\phi,P_\phi)},\\ &dH'\,|_{(A_\tau;r,P_r;\phi,P_\phi)} = \Phi_h dQ'. \end{aligned} \tag{14}$$

Using Eqs. (13) and (14), Eq. (12) can be rewritten as

$$\int p_i dq_i = 2\int_{r_{\rm in}}^{r_{\rm out}}\int_{(0,0,0)}^{(M,J,Q)} \frac{(dM'-\Omega_h dJ'-\Phi_h dQ)}{\dot{r}_{\rm out}} dr, \tag{15}$$

where $\dot{r}_{\rm in}$ denotes the radial outgoing path when a particle tunnels out, and $r_{\rm in}$ and $r_{\rm out}$ are the locations of the outer horizon before and after tunneling, respectively. After performing the same kind of analysis as above, we obtain

$$\int_{q_i^{\rm out}}^{q_i^{\rm in}} p_i^{\rm in} = 2\int_{r_{\rm out}}^{r_{\rm in}}\int_{(0,0,0)}^{(M,J,Q)} \frac{(dM'-\Omega_h dJ'-\Phi_h dQ)}{\dot{r}_{\rm in}} dr, \tag{16}$$

where $\dot{r}_{\rm in}$ corresponds to the radial ingoing path during the particle tunneling process. Therefore, the adiabatic covariant action can be written as

$$\begin{aligned} I_{\rm adia} &= \oint p_i dq_i\\ &= 2\int_{r_{\rm in}}^{r_{\rm out}}\int_{(0,0,0)}^{(M,J,Q)} \frac{(dM'-\Omega_h dJ'-\Phi_h dQ)}{\dot{r}_{\rm out}} dr\\ &\quad + 2\int_{r_{\rm out}}^{r_{\rm in}}\int_{(0,0,0)}^{(M,J,Q)} \frac{(dM'-\Omega_h dJ'-\Phi_h dQ)}{\dot{r}_{\rm in}} dr. \end{aligned} \tag{17}$$

Applying the adiabatic covariant action, the area spectrum for the KNdS black hole can be obtained in different coordinate systems. From Eq. (10), the outgoing (ingoing) radial null paths $ds^2 = d\theta^2 = 0$ can be written as

$$\dot{r} \equiv \frac{dr}{d\tau} = \pm i\frac{\Delta\sqrt{\Delta_\theta}}{\Xi\sqrt{[\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta]}}, \tag{18}$$

where the positive sign corresponds to the outgoing radial null path and the negative sign stands for the ingoing null path, that is,

$$\begin{aligned}\dot{r}_{\rm out} &= +i\frac{\Delta\sqrt{\Delta_\theta}}{\Xi\sqrt{[\Delta_\theta(r^2+a^2)-\Delta a^2\sin^2\theta]}},\\ \dot{r}_{\rm in} &= -i\frac{\Delta\sqrt{\Delta_\theta}}{\Xi\sqrt{[\Delta_\theta(r^2+a^2)-\Delta a^2\sin^2\theta]}}. \end{aligned} \tag{19}$$

From Eqs. (17) and (19), the adiabatic covariant action

$$\begin{aligned} I_{\rm adia} &= \oint p_i dq_i = -4i\int_{r_{\rm in}}^{r_{\rm out}}\int_{(0,0,0)}^{(M,J,Q)}\\ &\quad \cdot\frac{\Xi\sqrt{[\Delta_\theta(r^2+a^2)^2-\Delta a^2\sin^2\theta]}}{\Delta\sqrt{\Delta_\theta}}\\ &\quad \times(dM'-\Omega_h dJ'-\Phi_h dQ)dr\\ &= 4\pi\int_{(0,0,0)}^{(M,J,Q)}\frac{\Xi(r_{\rm h}^2+a^2)}{\Delta'(r_{\rm h})}\\ &\quad \cdot(dM'-\Omega_h dJ'-\Phi_h dQ)\\ &= \int_{(0,0,0)}^{(M,J,Q)}\frac{(dM'-\Omega_h dJ'-\Phi_h dQ)}{T}. \end{aligned} \tag{20}$$

From the first law of black hole thermodynamics, we obtain

$$dM = TdS + \Omega_h dJ + \Phi_h dQ, \tag{21}$$

the adiabatic covariant action in terms of entropy is given by

$$I_{\rm adia} = \oint p_i dq_i = \hbar S. \tag{22}$$

Implementing the Bohr–Sommerfeld quantization rule

$$\oint p_i dq_i = nh = 2\pi n\hbar, \quad n = 1, 2, 3, \ldots \tag{23}$$

the entropy spectrum of the KNdS black hole is given by

$$S = 2\pi n, \quad n = 1, 2, 3, \ldots \tag{24}$$

and the spacing of the entropy spectrum

$$\triangle S = S_{n+1} - S_n = 2\pi. \tag{25}$$

It implies that the entropy spectrum is discrete for the KNdS black hole. The relation between the entropy and horizon area is given by

$$S = \frac{A}{4\ell_p^2}, \tag{26}$$

and the spacing of the horizon area is given by

$$\Delta A = 8\pi\ell_p^2, \tag{27}$$

where $\ell_{\rm p} = (\sqrt{G/c^3})^{1/2}\hbar^{1/2}$ stands for the Plank length. Therefore, the entropy spectrum is equally spaced and independent of the parameters of the KNdS black hole and in agreement with Bekenstein's results.

Introducing the Painlevé-type coordinate transformation[38]

$$dt_k = dT + F(r,\theta)dr + G(r,\theta)d\theta, \tag{28}$$

where

$$F^2(r,\theta) = \frac{\Xi^2[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}{\Delta^2\Delta_\theta\rho^2}, \tag{29}$$

and

$$G(r,\theta) = \int \frac{\partial F(r,\theta)}{\partial\theta}dr + D(\theta), \tag{30}$$

we obtain the KNdS black hole metric in the dragged Painlevé coordinate

$$\begin{aligned} ds^2 =& -\frac{\Delta\Delta_\theta\rho^2}{\Xi^2[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}dT^2 \\ &+ \frac{2}{\Xi}\sqrt{\frac{\rho^2\Delta_\theta(\rho^2-\Delta)}{[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}}dTdr \\ &- \frac{2G(r,\theta)\Delta\Delta_\theta\rho^2}{\Xi^2[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}dTd\theta \\ &+ \frac{\rho^2}{\Delta_\theta}\Big[1 - \frac{\Delta\Delta_\theta^2 G^2(r,\theta)}{[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}\Big]d\theta^2 \\ &+ dr^2 + \frac{2G(r,\theta)}{\Xi} \\ &\cdot\sqrt{\frac{(\rho^2-\Delta)\rho^2\Delta_\theta}{[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}}drd\theta. \end{aligned} \tag{31}$$

The components of metric (31) are well behaved, not diverging at the horizon and are a flat Euclidean space in the radial to the constant time slice. In addition, ∂_T is a time-like Killing vector field, which keeps the spacetime stationary. The line element (31) should satisfy Landau's condition of coordinate clock synchronization. In the dragged Painlevé coordinate system, the Euclideanized metric is obtained by a transformation $T \to -i\tau'$ in the metric (31). Therefore, the corresponding radial null geodesic metric is given by

$$\begin{aligned} \dot{r} =& i\frac{\sqrt{\Delta_\theta}}{\Xi\sqrt{\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}} \\ &\cdot[\pm\rho^2 - \sqrt{\rho^2(\rho^2-\Delta)}], \end{aligned} \tag{32}$$

where the positive (negative) sign stands for the outgoing (ingoing) radial null paths, namely

$$\begin{aligned} \dot{r}_{\rm out} =& i\frac{\sqrt{\Delta_\theta}}{\Xi\sqrt{\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}} \\ &\cdot[\rho^2 - \sqrt{\rho^2(\rho^2-\Delta)}] \\ \dot{r}_{\rm in} =& -i\frac{\sqrt{\Delta_\theta}}{\Xi\sqrt{\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]}} \\ &\cdot[\rho^2 + \sqrt{\rho^2(\rho^2-\Delta)}]. \end{aligned} \tag{33}$$

Inserting Eq. (33) into Eq. (17), we obtain

$$\begin{aligned} I_{\rm adia} =& \oint p_i dq_i = -2i\int_{r_{\rm in}}^{r_{\rm out}}\int_{(0,0,0)}^{(M,J,Q)} \Big\{\Xi[\Delta_\theta(r^2+a^2)^2 \\ &- \Delta a^2\sin^2\theta]^{1/2}(dM' - \Omega_h dJ' - \Phi_h dQ)\Big\} \\ &\cdot\Big\{\sqrt{\Delta_\theta}[\rho^2 - \sqrt{\rho^2(\rho^2-\Delta)}]\Big\}^{-1}dr \\ &+ 2i\int_{r_{\rm in}}^{r_{\rm out}}\int_{(0,0,0)}^{(M,J,Q)} \Big\{\Xi\sqrt{[\Delta_\theta(r^2+a^2)^2 - \Delta a^2\sin^2\theta]} \\ &\cdot(dM' - \Omega_h dJ' - \Phi_h dQ)\Big\} \\ &\cdot\Big\{\sqrt{\Delta_\theta}[\rho^2 + \sqrt{\rho^2(\rho^2-\Delta)}]\Big\}^{-1}dr. \end{aligned} \tag{34}$$

Note that the integral of the second term in Eq. (34) has no contribution to the dragged Painlevé coordinate.[22] Therefore, the adiabatic covariant action can be written as

$$\begin{aligned} I_{\rm adia} =& \oint p_i dq_i = -2i\int_{r_{\rm in}}^{r_{\rm out}}\int_{(0,0,0)}^{(M,J,Q)} \Big\{\Xi[\Delta_\theta(r^2+a^2)^2 \\ &- \Delta a^2\sin^2\theta]^{1/2}(dM' - \Omega_h dJ' - \Phi_h dQ)\Big\} \\ &\cdot\Big\{\sqrt{\Delta_\theta}[\rho^2 - \sqrt{\rho^2(\rho^2-\Delta)}]\Big\}dr \\ =& 4\pi\int_{(0,0,0)}^{(M,J,Q)}\{\Xi(r_{\rm h}^2+a^2)(dM' - \Omega_h dJ' \\ &- \Phi_h dQ)\}\{\Delta'(r_{\rm h})\}. \end{aligned} \tag{35}$$

Using the first law of black hole thermodynamics, $dM = TdS + \Omega_{\rm h} dJ + \Phi_{\rm h} dQ$ and the Bohr–Sommerfeld quantization rule, $\oint p_i dq_i = nh$, the quantization of the entropy is obtained as

$$S = 2\pi n, \tag{36}$$

where $n = 1, 2, 3, \ldots$ and the spacing of the entropy spectrum is

$$\Delta S = 2\pi. \tag{37}$$

Utilizing the relation between the entropy and horizon area

$$\Delta S = \frac{\Delta A}{4\ell_p^2}, \tag{38}$$

the spacing of the area spectrum is given by

$$\Delta A = 8\pi\ell_p^2. \tag{39}$$

Hence, we have recovered Bekenstein's original result in the two different coordinates, namely, the dragged spherical coordinate and the dragged Painlevé coordinate. The results show that the black hole area spectrum is uniformly spaced and independent of the parameters of the black hole, e.g., mass M, angular momentum J and charge Q, in agreement with the earlier results based on different techniques.[9−11,17,39] The universal form for the spectra of Schwarzschild and Kerr black holes observed by Vagenas[11] has been extended to KNdS black holes.

In summary, we have studied the quantization of the horizon area and entropy of KNdS black holes in different coordinates using the modified adiabatic covariant action. To study the spectroscopy of black holes, the expression of the adiabatic invariant quantity in the dragged coordinate system is presented. Then, via modified adiabatic covariant action, implementing the Bohr–Sommerfeld quantization rule and the first law of black hole thermodynamics, the entropy spectra and the area of the KNdS black hole in different coordinates are obtained near the horizon. These results imply that the entropy spectrum and the area spectrum are equally spaced and do not depend on the black hole parameters. Our work is consistent with the obtained result in the literature by Maggiore with the quasinormal modes and the initial result obtained by Bekenstein is also confirmed. Using different coordinate systems we obtain the same value of area and entropy spectra of the black hole, which is a physically required result since the area spectrum should be invariant under the coordinate transformations. This technique gives a suitable method to quantize the horizon area for various black holes, especially for those with a complicated background spacetime like the KNdS black hole.

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