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Quantifying Process Nonclassicality in Bosonic Fields *

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Nonclassicality of optical states, as a key characteristic of bosonic fields, is a valuable resource for quantum information processing. We investigate the generation of nonclassicality in quantum processes from a quantitative perspective, introduce three information-theoretic measures of nonclassicality for quantum-optical processes based on the Wigner–Yanase skew information and coherent states, and illustrate their physical significance through several well-known single-mode quantum processes.

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The characterization and quantification of nonclassicality are attracting great interest in quantum optics.[1−8] For a quantum state ρ, nonclassicality is generally characterized in terms of the Glauber–Sudarshan representation, or the P function of the quantum state.[9−11] Specifically, for a quantum state ρ with the P function $P(\alpha)$,

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha| d^2\alpha,$$

where $|\alpha\rangle$ is the coherent state, and the integration is taken over the complex plane for $d^2\alpha = dxdy = rdrd\theta$ with $\alpha = x + iy = re^{i\theta}$. The quantum state ρ is said to be a classical state if $P(\alpha)$ is a non-negative regular function, or no more singular than the Dirac δ function. Otherwise, it is nonclassical.[12] Nonclassical states such as squeezed states, Fock states and Schrödinger cat states are crucial for implementation of quantum information and communication protocols.[13−17]

Various measures have been proposed to quantify nonclassicality of quantum states, including the distanced-based measure of nonclassicality, nonclassical depth, the entanglement potential, etc.[18−32] Inspired by the fact that the nonclassicality of quantum states in quantum optics is closely related to the particle nature of photon; that is, the creation and annihilation operators (denoted by $a^\dagger$ and a, respectively), Luo and Zhang recently proposed a measure of nonclassicality for quantum states in terms of the generalized Wigner–Yanase skew information.[33] The measure is shown to be conceptually simple and mathematically computable. We adopt this measure to quantify the nonclassicality of quantum states to address the nonclassicality-generating capability of various quantum-optical processes in this work.

Nonclassical quantum states are indispensable in quantum information processing tasks,[34] therefore the preparation of nonclassical states in the laboratory is crucial.[35,36] In quantum optics, a general quantum process can be described by the map[37]

$$\mathcal{E}(\rho) = \int P(\alpha)\mathcal{E}(|\alpha\rangle\langle\alpha|)d^2\alpha.$$

Since the map may not be trace-preserving, the output state $\rho_\mathcal{E} \propto \mathcal{E}(\rho)$ can be obtained by normalization. Therefore, quantum processes include both quantum channels (i.e., completely positive trace-preserving operations, or CPTP operations) and other quantum maps such as the photon substraction or photon addition processes.

Characterization and quantification of the nonclassicality-generating capability of quantum processes have attracted numerous attention.[37−40] According to the work of Rahimi–Keshari *et al.*,[37] a quantum process is nonclassical if it transforms an input coherent state to a nonclassical state. Otherwise, it is classical. Therefore, the displacement operators $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ are classical because they transform coherent states to coherent states, while the squeezing operators $S(\xi) = \exp(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2})$ are nonclassical since they transform the coherent states to squeezed nonclassical states. Other classical processes exist, such as all passive canonical transformations, photon substraction, and nonclassicality-breaking channels.[41] Notice that these are all qualitative characterizations, and the question arises naturally as how to quantify the nonclassicality of a general quantum process in quantum optics.

Recently, Sabapathy investigated the quantifica-

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tion of nonclassicality and proposed a notion called process output nonclassicality,[38] while as the name indicates, the concept is based on the output of a channel. A channel is classical if and only if it is nonclassicality-breaking. Thus the two classifications proposed by Keshari *et al.* and Sebapathy are different.

In this Letter, we propose measures of nonclassicality for quantum process in terms of an information-theoretic measure of nonclassicality of quantum states, reveal their basic properties and physical significance. We also validate the measures for several important quantum processes.

Nonclassicality of quantum optical processes. For a quantum state ρ of single-mode bosonic fields with annihilation operator a and creation operator $a^\dagger$ satisfying the canonical commutation relation $[a, a^\dagger] = \mathbf{1}$, motivated by the generalized Wigner–Yanase skew information,[42–45] Luo and Zhang defined[33]

$$N(\rho) = \frac{1}{2}\mathrm{tr}[\sqrt{\rho}, a][\sqrt{\rho}, a]^\dagger$$

as a measure of nonclassicality of ρ. Here $[A, B] = AB - BA$ is the commutator between the operators A and B. Simple manipulation shows that

$$N(\rho) = \frac{1}{2} + \mathrm{tr}\rho a^\dagger a - \mathrm{tr}\sqrt{\rho}a^\dagger\sqrt{\rho}a.$$

It is proved that $N(\rho)$ has some nice properties; for example, it is non-negative and convex under classical mixing. Also for pure quantum states, $N(\rho) \geq \frac{1}{2}$ where the minimum value $\frac{1}{2}$ is achieved if and only if ρ is a coherent state (i.e., eigenstate of the annihilation operator $a|\alpha\rangle = \alpha|\alpha\rangle$). For classical states in the sense that the P function is a classical probability, $N(\rho) \leq \frac{1}{2}$ due to the convexity of $N(\rho)$, thus if $N(\rho) > \frac{1}{2}$, ρ must be nonclassical. Nonclassicality of well-known nonclassical states such as Fock states, squeezed states, cat states, and Gaussian states can be easily evaluated.

For a general quantum process $\mathcal{E}$, by restricting the input state ρ to be coherent states which are classical states, the nonclassicality of the quantum process may be characterized by the nonclassicality of the output quantum state (not necessarily normalized) $\mathcal{E}(\rho)$.[37] Employing the nonclassicality measure of quantum state $N(\rho)$ mentioned above, we propose the following measures to quantify the nonclassicality-generating capability of the quantum process $\mathcal{E}$:

(1) Maximal nonclassicality

$$N_{\max}(\mathcal{E}) = \max_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - N(|\alpha\rangle\langle\alpha|).$$

(2) Minimal nonclassicality

$$N_{\min}(\mathcal{E}) = \min_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - N(|\alpha\rangle\langle\alpha|).$$

(3) Average nonclassicality

$$N_{\mathrm{ave}}(\mathcal{E}) = \frac{1}{\pi}\int (N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - N(|\alpha\rangle\langle\alpha|))e^{-|\alpha|^2}d^2\alpha.$$

Note that we take the average over a Gaussian distribution for simplicity.

The nonclassicality measure $N_{\max}(\mathcal{E})$ has the following nice properties.

(1) Additivity: $N_{\max}(\mathcal{E}_1 \otimes \mathcal{E}_2) = N_{\max}(\mathcal{E}_1) + N_{\max}(\mathcal{E}_2)$ for quantum processes $\mathcal{E}_i$ acting on different quantum optical mode i, respectively, $i = 1, 2$. The additivity actually also holds for the other two nonclassicality measures $N_{\min}$ and N_{ave}.

To prove it, considering the input coherent state on the two modes $|\alpha_1\alpha_2\rangle$ of the quantum process $\mathcal{E}_1 \otimes \mathcal{E}_2$, the nonclassicality of the normalized output state can be calculated as

$$\begin{aligned}&N_{\max}(\mathcal{E}_1 \otimes \mathcal{E}_2(|\alpha_1\alpha_2\rangle\langle\alpha_1\alpha_2|))\\ =\,&N_{\max}(\mathcal{E}_1(|\alpha_1\rangle\langle\alpha_1|) \otimes \mathcal{E}_2(|\alpha_2\rangle\langle\alpha_2|))\\ =\,&N_{\max}(\mathcal{E}_1(|\alpha_1\rangle\langle\alpha_1|)) + N_{\max}(\mathcal{E}_2(|\alpha_2\rangle\langle\alpha_2|)),\end{aligned}$$

the additivity of nonclassicality of quantum processes follows immediately.

(2) Convexity: $N_{\max}(\sum_i p_i\mathcal{E}_i) \leq \sum_i p_i N_{\max}(\mathcal{E}_i)$. This follows immediately from the convexity of nonclassicality of quantum states. The convexity also holds for the average nonclassicality N_{ave}. However, for the minimal nonclassicality measure $N_{\min}$, due to the minimization in the definition, the convexity fails in general.

(3) Invariance under composition with displacement and phase-space rotation processes: The composition of an arbitrary quantum process $\mathcal{E}$ with the displacement quantum operations $\mathcal{E}_{\mathrm{D}}(\rho) = D(\alpha)\rho D(\alpha)^\dagger$ or the phase-space rotation operations $\mathcal{E}_{\mathrm{R}}(\rho) = e^{i\theta a^\dagger a}\rho e^{-i\theta a^\dagger a}$ will not change the nonclassicality of the quantum process $\mathcal{E}$, that is,

$$N_{\max}(\mathcal{E} \circ \mathcal{U}) = N_{\max}(\mathcal{U} \circ \mathcal{E}) = N_{\max}(\mathcal{E}),$$

for $\mathcal{U} = \mathcal{E}_{\mathrm{D}}, \mathcal{E}_{\mathrm{R}}$. The above relations also hold for $N_{\min}$ and N_{ave}.

Nonclassicality of some single-mode quantum processes. We evaluate the nonclassicality of some well-known and frequently encountered single-mode quantum-optical processes. For a single-mode quantum optical process $\mathcal{E}$, since the nonclassicality of coherent states is $\frac{1}{2}$, the measures of nonclassicality of the quantum process $\mathcal{E}$ can be further expressed as

$$N_{\max}(\mathcal{E}) = \max_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2}.$$

$$N_{\min}(\mathcal{E}) = \min_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2}.$$

$$N_{\mathrm{ave}}(\mathcal{E}) = \frac{1}{\pi}\int N(\mathcal{E}(|\alpha\rangle\langle\alpha|))e^{-|\alpha|^2}d^2\alpha - \frac{1}{2}.$$

(1) For the displacement quantum operations $\mathcal{E}_{\rm D}(\rho) = D(\alpha)\rho D(\alpha)^\dagger$, all the three measures of nonclassicality vanish, that is,

$$N_{\max}(\mathcal{E}_{\rm D}) = N_{\min}(\mathcal{E}_{\rm D}) = N_{\rm ave}(\mathcal{E}_{\rm D}) = 0,$$

since coherent states are transformed to coherent states. Similarly for the phase-space rotation operations $\mathcal{E}_{\rm R}(\rho) = e^{i\theta a^\dagger a}\rho e^{-i\theta a^\dagger a}$, we have

$$N_{\max}(\mathcal{E}_{\rm R}) = N_{\min}(\mathcal{E}_{\rm R}) = N_{\rm ave}(\mathcal{E}_{\rm R}) = 0.$$

(2) For the photon-substraction process[37]

$$\mathcal{E}(|\alpha\rangle\langle\alpha|) = a|\alpha\rangle\langle\alpha|a^\dagger.$$

Since coherent states are eigenstates of the annihilation operator $a|\alpha\rangle = \alpha|\alpha\rangle$, we have $\mathcal{E}(|\alpha\rangle\langle\alpha|) = |\alpha|^2|\alpha\rangle\langle\alpha|$. Consequently,

$$\begin{aligned} N_{\max}(\mathcal{E}) &= \max_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2} \\ &= \max_{|\alpha\rangle} N(|\alpha\rangle\langle\alpha|) - \frac{1}{2} \\ &= 0. \end{aligned}$$

Similarly, $N_{\min}(\mathcal{E}) = N_{\rm ave}(\mathcal{E}) = 0$. Therefore, one can not generate nonclassical states from coherent states through photon substraction, as has been pointed out in Ref. [46].

(3) For the single-photon addition process[47]

$$\mathcal{E}(|\alpha\rangle\langle\alpha|) = a^\dagger|\alpha\rangle\langle\alpha|a,$$

the nonclassicality $N_{\max}$ of the process can be evaluated as

$$\begin{aligned} N_{\max}(\mathcal{E}) &= \max_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2} \\ &= \max_{\alpha} \frac{1}{(|\alpha|^2+1)^2} \\ &= 1. \end{aligned}$$

The maximum is achieved when the input state is a vacuum state; in which case, the output state is the Fock state $|1\rangle$ with one photon.

The minimal nonclassicality measure can be easily calculated as follows:

$$\begin{aligned} N_{\min}(\mathcal{E}) &= \min_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2} \\ &= \min_{\alpha} \frac{1}{(|\alpha|^2+1)^2} \\ &= 0, \end{aligned}$$

which means that if we input a coherent state with sufficient large amplitude α into the photon-addition process, the increase in the nonclassicality of the output state will be almost negligible.

The average nonclassicality which takes an average over all input states is

$$\begin{aligned} N_{\rm ave}(\mathcal{E}) &= \frac{1}{\pi}\int \Big(N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2}\Big)e^{-|\alpha|^2} d^2\alpha \\ &= \frac{1}{\pi}\int_0^{2\pi} d\theta \int_0^\infty \frac{re^{-r^2}}{(r^2+1)^2} dr \\ &= \int_0^\infty \frac{e^{-t}}{(t+1)^2} dt \\ &\approx 0.1485. \end{aligned}$$

Actually, for n-photon addition process $\mathcal{E}(|\alpha\rangle\langle\alpha|) = a^{\dagger n}|\alpha\rangle\langle\alpha|a^n$, after some tedious calculations, we have $N_{\max}(\mathcal{E}) = n$. Therefore, the nonclassicality of the photon addition process increases with n. This is consistent with the previous observation[39,48] where the authors employed the entanglement potential as the nonclassicality measure and computed them numerically for single, two and three photon added coherent states.

(4) The Yurke–Stoler states[49]

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + i|-\alpha\rangle)$$

can be generated by sending a coherent state into a Kerr-like medium, and the process is actually a unitary evolution with Hamiltonian $H = \chi(a^\dagger a)^2$. At time $t = \frac{\pi}{2\chi}$, the output state of the process is

$$\begin{aligned} \mathcal{E}(|\alpha\rangle\langle\alpha|) &= e^{-i(\pi/2)(a^\dagger a)^2}|\alpha\rangle\langle\alpha|e^{i(\pi/2)(a^\dagger a)^2} \\ &= \frac{1}{2}(|\alpha\rangle + i|-\alpha\rangle)(\langle\alpha| - i\langle-\alpha|), \end{aligned}$$

which is a Schrödinger cat state, thus the process is also termed as the cat-generation process.[37] The nonclassicality of the Yurke–Stoler state can be directly computed as

$$\begin{aligned} N(|\phi\rangle\langle\phi|) &= \frac{1}{2} + \langle\phi|a^\dagger a|\phi\rangle - \langle\phi|a^\dagger|\phi\rangle\langle\phi|a|\phi\rangle \\ &= \frac{1}{2} + |\alpha|^2 - |\alpha|^2 e^{-4|\alpha|^2}. \end{aligned}$$

Therefore, the nonclassicality of the cat-generating process turns out to be

$$\begin{aligned} N_{\max}(\mathcal{E}) &= \max_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2} \\ &= \max_{\alpha}(|\alpha|^2 - |\alpha|^2 e^{-4|\alpha|^2}) \\ &= \infty, \end{aligned}$$

which is obtained when α tends to ∞. In this limiting case, the cat state $|\phi\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + i|-\alpha\rangle)$ can be regarded as a superposition of two orthogonal states, thus the nonclassicality of the output state is large. Actually, in realistic situations, there are limitations on the input quantum states, for example, the average input photon numbers cannot be arbitrarily large and are required to satisfy some restriction $\mathrm{tr}\rho a^\dagger a \leq n$,

and if one redefines the maximal nonclassicality measure as

$$N_n(\mathcal{E}) = \max_{|\alpha\rangle,|\alpha|^2\leq n} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2},$$

then it is bounded and satisfies $N_n(\mathcal{E}) \leq n$.

Similarly, the minimal nonclassicality of the cat-generating process can be calculated as

$$\begin{aligned} N_{\min}(\mathcal{E}) &= \min_{|\alpha\rangle} N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2} \\ &= \min_{\alpha}(|\alpha|^2 - |\alpha|^2 e^{-4|\alpha|^2}) \\ &= 0, \end{aligned}$$

which is obtained when the input state is the vacuum state. The output state is also the vacuum state. Thus no nonclassicality can be generated in this situation.

The average nonclassicality of cat-generating process is

$$\begin{aligned} N_{\text{ave}}(\mathcal{E}) &= \frac{1}{\pi}\int (N(\mathcal{E}(|\alpha\rangle\langle\alpha|)) - \frac{1}{2})e^{-|\alpha|^2} d^2\alpha \\ &= \frac{1}{\pi}\int_0^{2\pi} d\theta \int_0^{\infty} r^2(1-e^{-4r^2})e^{-r^2} r dr \\ &= \frac{24}{25}. \end{aligned}$$

(5) For the squeezing processes with squeezing operators $S(\xi) = \exp(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi {a^\dagger}^2)$ with $\xi = |\xi|e^{i\theta}$ being the squeezing parameter, we consider the process

$$\mathcal{E}(|\alpha\rangle\langle\alpha|) = S(\xi)|\alpha\rangle\langle\alpha|S^\dagger(\xi) = |\alpha,\xi\rangle\langle\alpha,\xi|,$$

where $|\alpha,\xi\rangle = S(\xi)|\alpha\rangle$ are the squeezed coherent states, since

$$\begin{aligned} \langle\alpha,\xi|a|\alpha,\xi\rangle &= \alpha\cosh|\xi| - \alpha^* e^{i\theta}\sinh|\xi|, \\ \langle\alpha,\xi|a^\dagger a|\alpha,\xi\rangle &= |\alpha|^2(\cosh^2|\xi| + \sinh^2|\xi|) + \sinh^2|\xi| \\ &\quad - (\alpha^*)^2 e^{i\theta}\sinh|\xi|\cosh|\xi| \\ &\quad - \alpha^2 e^{-i\theta}\sinh|\xi|\cosh|\xi|, \end{aligned}$$

we can straightforwardly calculate the nonclassicality of the output state and obtain

$$N(|\alpha,\xi\rangle\langle\alpha,\xi|) = \frac{1}{2} + \sinh^2|\xi|,$$

which is independent of the input coherent states $|\alpha\rangle$, therefore we have

$$N_{\max}(\mathcal{E}) = N_{\min}(\mathcal{E}) = N_{\text{ave}}(\mathcal{E}) = \sinh^2|\xi|.$$

Thus the nonclassicality of the squeezing processes increases with the squeezing parameter $|\xi|$ of the channel, which coincides with our intuition.

(6) Considering the Gaussian noise mapping[50–52]

$$\Phi_m(\rho) = \frac{1}{\pi m}\int \exp\Big(-\frac{|\beta|^2}{m}\Big) D(\beta)\rho D^\dagger(\beta) d^2\beta,$$

where m is the average number of added thermal photons. The map transforms a Gaussian state into another Gaussian state. When the input state is a coherent state $|\alpha\rangle$, the output state can be calculated,

$$\begin{aligned} &\Phi_m(|\alpha\rangle\langle\alpha|) \\ &= \frac{1}{\pi m}\int \exp\Big(-\frac{|\beta|^2}{m}\Big) D(\beta)|\alpha\rangle\langle\alpha|D^\dagger(\beta) d^2\beta \\ &= \frac{1}{\pi m}\int \exp\Big(-\frac{|\tau-\alpha|^2}{m}\Big)|\tau\rangle\langle\tau| d^2\tau \\ &= D(\alpha)\rho_{\text{T}} D^\dagger(\alpha), \end{aligned}$$

where $\rho_{\text{T}} = \frac{1}{m+1}\sum_{n=0}^{\infty}(\frac{m}{m+1})^n|n\rangle\langle n|$ is a thermal state. The nonclassicality of a Gaussian state can be easily calculated. In this case

$$N(\Phi_m(|\alpha\rangle\langle\alpha|)) = \frac{1}{2(2m+1)}.$$

Therefore, the nonclassicality of this channel is

$$\begin{aligned} N_{\max}(\Phi) &= \frac{1}{2(2m+1)} - \frac{1}{2} \\ &= -\frac{m}{2m+1}. \end{aligned}$$

Here the nonclassicality is negative for $m \neq 0$, which indicates that the process is a classical process. It transforms the coherent states into classical mixture of coherent states which are more classical (i.e., less nonclassical), thus reduces nonclassicality. For general m, we have

$$-\frac{1}{2} \leq N_{\max}(\Phi) \leq 0.$$

Discussion. For single-mode bosonic quantum optical process, we have introduced several measures to quantify their nonclassicality-generating capability. We have also illustrated by explicit examples that these measures coincide with our intuition about nonclassicality and characterize nonclassicality of quantum processes from different perspectives. It will be desirable to explore the applications of these quantities in more physical processes.

Of course, there are many questions worth further investigation. For example, one may consider quantification of nonclassicality for multi-mode quantum processes, or consider the interplay of nonclassicality with other quantities such as entanglement and quantum correlations. In this work, we have quantified nonclassicality of quantum processes by limiting the input states to be coherent states. It is interesting to find what will happen if we remove this limitation and consider an arbitrary input state. This issue is related to the consideration of Sabapathy in Ref. [38], where a channel is considered to be classical if its output states are always classical for all input states. We hope that these problems will be addressed later.

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