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# Tripartite Entanglement Measures of Generalized GHZ State in Uniform Acceleration *

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*Using the single-mode approximation, we study entanglement measures including two independent quantities; i.e., negativity and von Neumann entropy for a tripartite generalized Greenberger–Horne–Zeilinger (GHZ) state in noninertial frames. Based on the calculated negativity, we study the whole entanglement measures named as the algebraic average $\pi_3$-tangle and geometric average $\Pi_3$-tangle. We find that the difference between them is very small or disappears with the increase of the number of accelerated qubits. The entanglement properties are discussed from one accelerated observer and others remaining stationary to all three accelerated observers. The results show that there will always exist entanglement, even if acceleration $r$ arrives to infinity. The degree of entanglement for all 1–1 tangles are always equal to zero, but 1–2 tangles always decrease with the acceleration parameter $r$. We notice that the von Neumann entropy increases with the number of the accelerated observers and $S_{\kappa_{\rm I}\zeta_{\rm I}}$ ($\kappa, \zeta \in (\rm A, B, C)$) first increases and then decreases with the acceleration parameter $r$. This implies that the subsystem $\rho_{\kappa_{\rm I}\zeta_{\rm I}}$ is first more disorder and then the disorder will be reduced as the acceleration parameter $r$ increases. Moreover, it is found that the von Neumann entropies $S_{\rm ABCI}$, $S_{\rm ABICI}$ and $S_{\rm AIBICI}$ always decrease with the controllable angle $\theta$, while the entropies of the bipartite subsystems $S_{2-2_{\rm non}}$ (two accelerated qubits), $S_{2-1_{\rm non}}$ (one accelerated qubit) and $S_{2-0_{\rm non}}$ (without accelerated qubit) first increase with the angle $\theta$ and then decrease with it.*

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The entanglement introduced in the pioneering work by EPR[1] and Schrödinger[2,3] in 1930s has become a key resource in quantum teleportation because of its quantum nonlocality feature.[4–7] We often use negativity[8,9] and relevant whole entanglement average measures $\pi_3$- and $\Pi_3$-tangles to describe the degree of the entanglement.[10] In addition, the von Neumann entropy[11–13] can also be used to describe the entanglement measures for the pure states. To date, many works have been published in bipartite and tripartite systems except for a few multipartite systems.[7,14–23] The shared entangled qubits allow us to perform some quantum teleportation.[24–26]

Recently, some interesting publications have been appeared in the noninertial frames.[18,22,23,27–43] The quantum information in noninertial frame, which is a combination of general relativity, quantum field theory and quantum information theory, has become an interesting research topic in recent years.[10,18] Among these contributions, we know that the entanglement is degraded when the observer is moving in an acceleration. Since the pioneering work on the tripartite entangled states,[18,32] most studies published focus on two states; i.e., Greenberger–Horne–Zeilinger (GHZ) state $|\rm GHZ\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$, W-state $|\rm W\rangle = (|100\rangle + |010\rangle + |001\rangle)/\sqrt{3}$ apart from other related states. Until now, most contributions made to this topic arise from the fact that their density matrices can be written as the form of an $X$ matrix and this allows us to calculate those quantities easily. Nevertheless, it should be pointed out that the calculation of the entanglement measures of the W-state described above is different from that of the GHZ state since the density matrix of the former cannot be written in the form of an $X$ matrix. We have found that the calculation of the entanglement measures of the tripartite W-state becomes complicated in comparison with that of the GHZ state.[40] Likewise, using the same calculation technique as that of Ref. [40], we have also studied the Werner mixed state[41] and a new type pseudo-pure state.[42] In particular, we have extended our precious study to the tetrapartite W-class state and GHZ state.[22,23] Stimulated by these studies, we will investigate the entanglement properties of a tripartite state initially entangled in a generalized GHZ state $|\psi_{\rm g}\rangle_{\rm ABC} = \cos\theta|000\rangle + \sin\theta|111\rangle, \theta \in [0, \pi/2]$ in the noninertial frames. There are two main purposes for this work. First, we study the negativity, the whole entanglement measures $\pi_3$-tangle and $\Pi_3$-tangle and the von Neumann entropy by considering all different cases from one accelerated observer to three accelerated observers. Second, we study how the angle parameter $\theta$ affects those entanglement measures. We notice that this generalized GHZ state is very different from the traditional GHZ state as mentioned above since a controllable angle $\theta$ is included. We find that the generalized GHZ state will reduce to the traditional one when the controllable angel $\theta = \pi/4$. Although some nice contributions related to this generalized GHZ state such as the three-tangle for mix-

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tures of generalized GHZ and generalized W state, the tripartite entanglement-dependence of tripartite nonlocality in noninertial frames and the influence of noise on tripartite quantum probe state have been made in Refs. [44–46], the present study has never been concerned to our best knowledge.

The initial generalized GHZ state has the form[10]

$$|\psi_{\rm g}\rangle_{\rm ABC} = \cos\theta|000\rangle + \sin\theta|111\rangle,\ \theta \in [0,\pi/2], \quad (1)$$

where $|000\rangle$ represents $|0\rangle_{\rm A}\otimes|0\rangle_{\rm B}\otimes|0\rangle_{\rm C}$ so does the state $|111\rangle$. Here we use subscripts A, B and C to denote those observers Alice, Bob and Charlie, respectively. Note that this initial entangled GHZ state will become maximally entangled when the controllable angle is given by $\theta = \pi/4$. The degree of entanglement of this state depends on the controllable angle $\theta$. However, this state would not be entangled anymore for the critical values $\theta = 0$ and $\theta = \pi/2$.

To study this entangled system in noninertial frames, we use the Rindler coordinates to describe a family of observers in uniform acceleration and divide Minkowski space-time into two inaccessible two regions I and II. The rightward accelerated observers are located in region I and causally disconnected from their analogous counterparts in region II.[47–50] The Minkowski vacuum state defined as the absence of any particle excitation in any of the modes can be expressed in terms of a product of two-mode squeezed states of the Rindler vacuum. Therefore, the respective transformations between two coordinated systems for the fermion field can be written as[29]

$$|0_{w_i}\rangle_{\rm M} = \cos r_i|0_{w_i}\rangle_{\rm I}|0_{w_i}\rangle_{\rm II} + \sin r_i|1_{w_i}\rangle_{\rm I}|1_{w_i}\rangle_{\rm II}, \quad (2)$$

$$|1_{w_i}\rangle_{\rm M} = |1_{w_i}\rangle_{\rm I}|0_{w_i}\rangle_{\rm II}, \quad (3)$$

where $\cos r_i = (e^{-2\pi\omega_i c/a_i}+1)^{-1/2}$ with $a_i$ being the acceleration of the $i$th accelerated observer, and $w_i$ its respective frequency. Due to the acceleration $a_i \in [0,\infty)$, one has $r_i \in [0,\pi/4)$. In this work, we are only interesting in the single-mode approximation case; i.e., $w_{\rm A,B,C,D} = w$ and also in uniform acceleration $a_{\rm A,B,C} = a$ for simplicity. Thus, we have $r_{\rm a} = r_{\rm b} = r_{\rm c} = r$. To illustrate how to expand $|\psi_{\rm g}\rangle_{\rm ABC}$ in the Rindler coordinates, as an example we give the explicit expression for the accelerated observer, say Charlie; i.e.,

$$|\psi_{\rm g}\rangle_{\rm ABCIC II} = \cos\theta\sin(r_c)|0_{\rm A}0_{\rm B}1_{\rm CI}1_{\rm CII}\rangle + \cos\theta\cos(r_c)\cdot|0_{\rm A}0_{\rm B}0_{\rm CI}0_{\rm CII}\rangle + \sin\theta|1_{\rm A}1_{\rm B}1_{\rm CI}0_{\rm CII}\rangle. \quad (4)$$

Since the moving observers are confined to region I, we have to trace out the part of the antiparticle state in region II. Let us apply Eqs. (2) and (3) to our $|\psi_{\rm g}\rangle_{\rm ABC}$ state (4). As mentioned earlier, we study this entangled system in three different cases. First, we study the case that only one observer Charlie is accelerated,

$$\rho_{\rm ABCI} = \begin{pmatrix} \beta^2\delta^2 & 0 & 0 & 0 & 0 & 0 & 0 & \beta\gamma\delta \\ 0 & \alpha^2\delta^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta\gamma\delta & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^2 \end{pmatrix}, \quad (5)$$

where $\alpha = \sin(r)$, $\beta = \cos(r)$, $\gamma = \sin(\theta)$, $\delta = \cos(\theta)$ and the density matrix $\rho_{\rm ABCI} = |\psi_{\rm g}\rangle_{\rm ABCI}\cdot{}_{\rm ABCI}\langle\psi_{\rm g}|$. Similarly, we can construct other density matrices.

Second, we consider the case when Bob and Charlie are accelerated, the density matrix is given by

$$\rho_{\rm ABICI} = \begin{pmatrix} \beta^4\delta^2 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_2 \\ 0 & \kappa_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \kappa_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha^4\delta^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \kappa_2 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^2 \end{pmatrix}, \quad (6)$$

with $\kappa_1 = \alpha^2\beta^2\delta^2$, $\kappa_2 = \beta^2\gamma\delta$.

Finally, we study the case that all observers Alice, Bob and Charlie are accelerated, the density matrix becomes

$$\rho_{\rm AIBICI} = \begin{pmatrix} \beta^6\delta^2 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_3 \\ 0 & \tau_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tau_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_2 & 0 \\ \tau_3 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_4 \end{pmatrix}, \quad (7)$$

with $\tau_1 = \alpha^2\beta^4\delta^2$, $\tau_2 = \alpha^4\beta^2\delta^2$, $\tau_3 = \beta^3\gamma\delta$, $\tau_4 = \delta^2\alpha^6 + \gamma^2$.

In what follows, we study the negativities and von Neumann entropies for three different cases.

The negativity has been calculated for many entangled systems since it quantifies the degree of the entanglement to see whether the entangled system is still entangled or not. An entangled system $\rho$ is entangled if there exists at least one negative eigenvalue for the partial transpose of the corresponding density matrix. The negativity for a tripartite state is defined as[10,40]

$$N_{\alpha(\beta\gamma)} = ||\rho^{T_\alpha}_{\alpha(\beta\gamma)}|| - 1 = 2\sum_{i=1}^{N}|\lambda_M^{(-)}|^i,\ N_{\alpha\beta} = ||\rho^{T_\alpha}_{\alpha\beta}|| - 1, \quad (8)$$

where $\lambda_M^{(-)}$ are the negative eigenvalues of the matrix $M$. The negativities defined in Eq. (8) describe the entanglements 1–2 tangle and 1–1 tangle, respectively. The notations $||\rho^{T_\alpha}_{\alpha(\beta\gamma)}||$ and $||\rho^{T_\alpha}_{\alpha\beta}||$ are the trace-norm of each partial transpose matrix. Here we have used the relation $||O|| = {\rm tr}\sqrt{O^\dagger O}$ for any Hermitian operator $O$.[51]

By tracing out the antiparticle in region Ⅱ, we try to find the negative eigenvalues of each density matrix and calculate the corresponding negativities in terms of Eq. (8). This will allow us to find all negativities including the 1–1 and 1–2 tangles by varying the quantities of accelerated qubits 1, 2 or 3. We first calculate the negativities for the 1–2 tangle. Unlike the complexity of the W-class case,[22,40] the simple and analytical expressions can be explicitly written as

$$
\begin{aligned}
&N_{\mathrm{A(BCI)}} = \sin(2\theta)\cos(r),\\
&N_{\mathrm{CI(AB)}} = \frac{1}{4}\{[\cos^2\theta(\cos(2\theta)(-20\cos(2r)+\cos(4r)\\
&\quad -13)+2(\cos(2r)+3)^2)]^{1/2}-4\cos^2\theta\sin^2 r\},\\
&N_{\mathrm{A(BICI)}} = \{\cos^4(\theta)\sin^8(r)+\sin^2(2\theta)\cos^4 r\}^{1/2}\\
&\quad -\cos^2\theta\sin^4 r,\\
&N_{\mathrm{BI(ACI)}} = \frac{1}{8}\{\cos^2\theta(\cos(4r)-1)\\
&\quad +2[\cos^2\theta\cos^4 r(-8\cos^2\theta\cos(2r)\\
&\quad +\cos(2\theta)(\cos(4r)-29)+\cos(4r)+35)]^{1/2}\},\\
&N_{\mathrm{AI(BICI)}} = \frac{1}{8}\{\cos^2\theta(\cos(4r)-1)+[\cos^2\theta\\
&\quad \cdot\cos^4 r[-57\cos(2\theta)+2\cos^2\theta(8\cos(4r)-4\cos(6r)\\
&\quad +\cos(8r))+(52-76\cos(2\theta))\cos(2r)+71]]^{1/2}\}, \qquad (9)
\end{aligned}
$$

where $N_{\mathrm{A(BCI)}} = N_{\mathrm{B(ACI)}}$, $N_{\mathrm{BI(ACI)}} = N_{\mathrm{CI(ABI)}}$ and $N_{\mathrm{AI(BICI)}} = N_{\mathrm{BI(AICI)}} = N_{\mathrm{CI(AIBI)}}$.

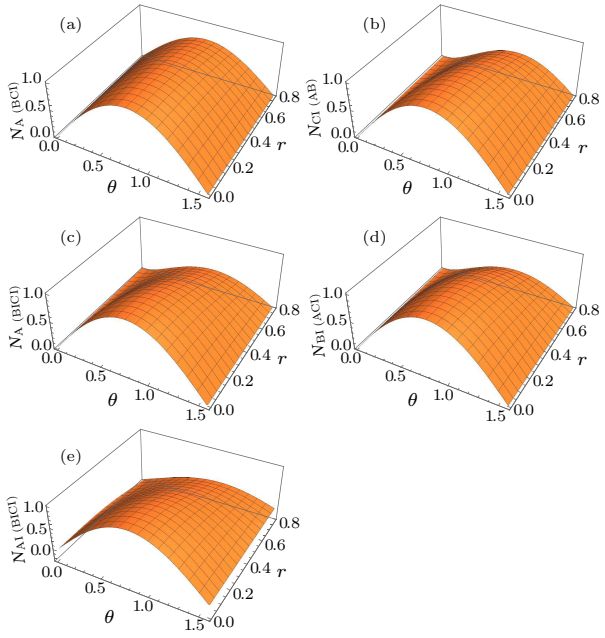


**Fig. 1.** The 1–2 tangles $N_{\mathrm{A(BCI)}}$ ($N_{\mathrm{B(ACI)}}$), $N_{\mathrm{CI(AB)}}$, $N_{\mathrm{A(BICI)}}$, $N_{\mathrm{BI(ACI)}}$ and $N_{\mathrm{AI(BICI)}}$ shown in (a), (b), (c), (d) and (e), respectively, versus both angle $\theta$ and parameter $r$.

We illustrate these 1–2 tangles in Fig. 1 and notice that the degree of the entanglement first increases with the controllable angle $\theta$ and then decreases with it, but always decreases with the acceleration parameter $r$. Note that the degree of entanglement for all of them has never been disappeared even in the infinite acceleration limit $r \to \pi/4$. Moreover, we find that $N_{\mathrm{A(BCI)}}$ is symmetric with respect to the angle $\theta = \pi/4$, but other negativities $N_{\mathrm{BI(ACI)}}$, $N_{\mathrm{CI(AB)}}$, $N_{\mathrm{A(BICI)}}$, $N_{\mathrm{BI(ACI)}}$ and $N_{\mathrm{AI(BICI)}}$ do not exist such symmetry. By tracing out some necessary qubit to generate bipartite subsystems with all possible combinations, we find that all 1–1 tangles such as $N_{\kappa\zeta}$, $N_{\kappa I\zeta}$ and $N_{\kappa I\zeta I}$ $[\kappa,\zeta \in (\mathrm{A},\mathrm{B},\mathrm{C})]$ are equal to zero, which implies that their degrees of entanglement do not exist anymore and becomes zero. This is different from the initial entangled W-class state,[22,40] in which the entanglement 1–1 tangles always exist.

Another quantification of tripartite entanglement is the algebraic average $\pi$-tangle, which has the following form[52]

$$
\begin{aligned}
\pi_\kappa &= N^2_{\kappa(\zeta\varnothing)} - N^2_{\kappa\zeta} - N^2_{\kappa\varnothing} = N^2_{\kappa(\zeta\varnothing)},\\
\pi_\zeta &= N^2_{\zeta(\kappa\varnothing)} - N^2_{\zeta\kappa} - N^2_{\zeta\varnothing} = N^2_{\zeta(\kappa\varnothing)},\\
\pi_\varnothing &= N^2_{\varnothing(\kappa\zeta)} - N^2_{\varnothing\kappa} - N^2_{\varnothing\zeta} = N^2_{\varnothing(\kappa\zeta)},
\end{aligned} \qquad (10)
$$

from which we can calculate the whole entanglement $\pi_3$-tangle by the following formula

$$
\pi_3 = \frac{1}{3}(\pi_\kappa + \pi_\zeta + \pi_\varnothing). \qquad (11)
$$

Moreover, we may use another whole entanglement measure named as geometric average $\Pi_3$[53]

$$
\Pi_3 = (\pi_\kappa\pi_\zeta\pi_\varnothing)^{\frac{1}{3}}. \qquad (12)
$$

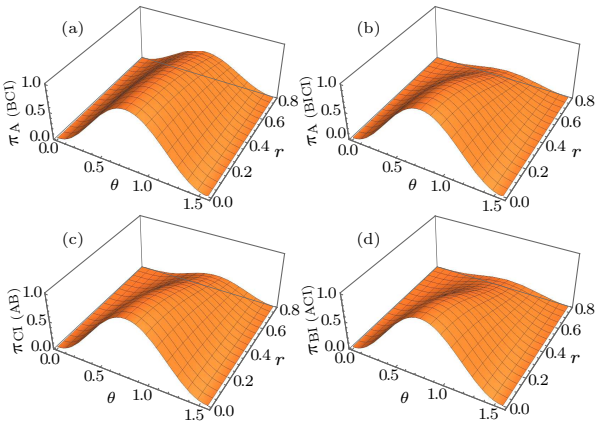


**Fig. 2.** Algebraic averages $\pi_{\mathrm{A(BCI)}}$, $\pi_{\mathrm{A(BICI)}}$, $\pi_{\mathrm{CI(AB)}}$ and $\pi_{\mathrm{BI(ACI)}}$ tangles versus both angle $\theta$ and parameter $r$ for different cases.

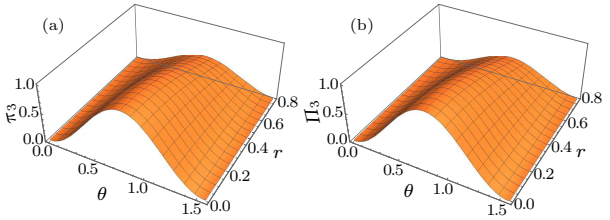


**Fig. 3.** Algebraic average $\pi_3$ and geometric average $\Pi_3$ versus both angle $\theta$ and parameter $r$ when only one observer is accelerated.

We illustrate the respective $\pi$-tangle in Fig. 2. It is found that all $\pi$-tangles first increase with the angle

$\theta$ and then decrease with it, but all of them decrease with the acceleration parameter $r$. Likewise, the algebraic average $\pi_{\mathrm{A(BCI)}}$ is also symmetric to the angle $\theta$ as for the negativity $N_{\mathrm{A(BCI)}}$. In Figs. 3 and 4, we show the algebraic average and geometric average for different cases; i.e., when only one observer is accelerated and when two observers are accelerated. All of these whole entanglement measures first increase with the angle $\theta$ and then decrease with it, while they decrease with the parameter $r$. We also find $\pi_{\mathrm{AI(BICI)}} = \pi_{\mathrm{BI(AICI)}} = \pi_{\mathrm{CI(AIBI)}} = \pi_3 = \Pi_3$, where $\pi_3$ refers to the case that all observers are accelerated simultaneously (see Fig. 5).

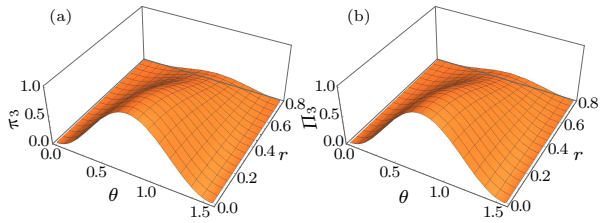


**Fig. 4.** Algebraic average $\pi_3$ and geometric average $\Pi_3$ versus both angle $\theta$ and parameter $r$ when two observers are accelerated.

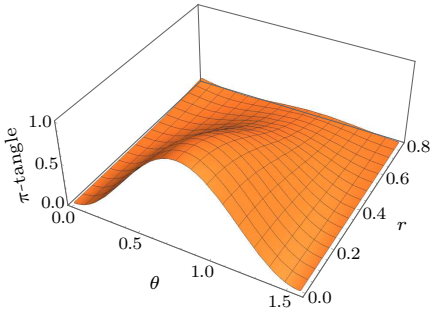


**Fig. 5.** Algebraic average $\pi_3$ and geometric average $\Pi_3$ versus both angle $\theta$ and parameter $r$ when all the three observers are accelerated.

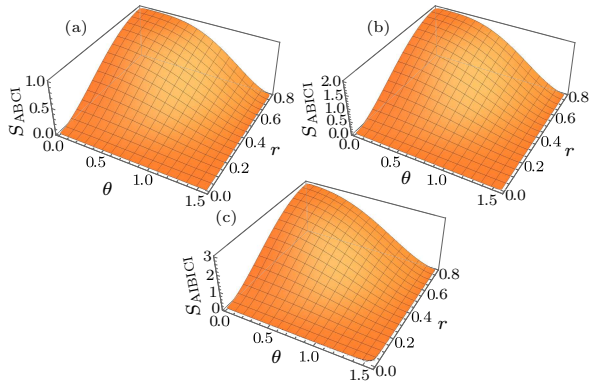


**Fig. 6.** The von Neumann entropies $S_{\mathrm{ABCI}}$, $S_{\mathrm{ABICI}}$ and $S_{\mathrm{AIBICI}}$ versus both parameter $r$ and the angel $\theta$.

To study the degree of the disorder of an entangled system, it is necessary to study the von Neumann entropy defined as[54]

$$S = -\mathrm{Tr}(\rho \log_2 \rho) = -\sum_{i=1}^{n} \lambda^{(i)} \log_2 \lambda^{(i)}, \quad (13)$$

where $\lambda^{(i)}$ denotes the $i$th nonzero eigenvalue of the density matrix $\rho$. It should be emphasized that the density matrix is not taken as its partial transpose. Based on this we can describe the degree of the satiability of the entangled system.

In what follows, we are going to present eigenvalues of all bipartite subsystems and tripartite systems; i.e.,

$$\begin{aligned}
&\lambda^{(1)}_{\mathrm{AB}} = \cos^2\theta,\ \lambda^{(2)}_{\mathrm{AB}} = \sin^2\theta,\ \lambda^{(1)}_{\mathrm{AC_I}} = \cos^2\theta\cos^2 r,\\
&\lambda^{(2)}_{\mathrm{AC_I}} = \cos^2\theta\sin^2 r,\ \lambda^{(3)}_{\mathrm{AC_I}} = \sin^2\theta, \qquad (14)\\
&\lambda^{(1)}_{\mathrm{ABC_I}} = \cos^2\theta\sin^2 r,\\
&\lambda^{(2)}_{\mathrm{ABC_I}} = \frac{1}{4}(-2\cos(2\theta)\sin^2 r + \cos(2r) + 3),\\
&\lambda^{(1)}_{\mathrm{BICI}} = \cos^2\theta\cos^4 r,\ \lambda^{(2,3)}_{\mathrm{BICI}} = \cos^2\theta\sin^2 r\cos^2 r,\\
&\lambda^{(4)}_{\mathrm{BICI}} = \sin^2\theta + \cos^2\theta\sin^4 r, \qquad (15)\\
&\lambda^{(1)}_{\mathrm{ABICI}} = \cos^2\theta\sin^4 r,\ \lambda^{(2,3)}_{\mathrm{ABICI}} = \cos^2\theta\sin^2 r\cos^2 r,\\
&\lambda^{(4)}_{\mathrm{ABICI}} = \frac{1}{16}[-4\cos(2\theta)\sin^2 r(\cos(2r)+3)\\
&\quad + 4\cos(2r) + \cos(4r) + 11],\\
&\lambda^{(1,2)}_{\mathrm{AIBICI}} = \frac{1}{128}\{-12\cos(2\theta) + 24\cos^2\theta\cos(4r)\\
&\quad \pm [-1144\cos(2\theta) + 98\cos(4\theta)\\
&\quad + 8\cos^4(\theta)(30\cos(8r) + \cos(12r))\\
&\quad + 12(213 - 43\cos(2\theta))\cos^2\theta\cos(4r)\\
&\quad + 2854]^{1/2} + 52\},\\
&\lambda^{(3,4,5)}_{\mathrm{AIBICI}} = \cos^2\theta\sin^2 r\cos^4 r,\\
&\lambda^{(6,7,8)}_{\mathrm{AIBICI}} = \cos^2\theta\sin^4 r\cos^2 r, \qquad (16)
\end{aligned}$$

where symbols $\pm$ appearing in $\lambda^{(1,2)}_{\mathrm{AIBICI}}$ correspond to $\lambda^{(1)}_{\mathrm{AIBICI}}$ and $\lambda^{(2)}_{\mathrm{AIBICI}}$, respectively. The eigenvalues of the bipartite subsystems $S_{\mathrm{AIBI}}$ and $S_{\mathrm{AICI}}$ are the same as those given by $\rho_{\mathrm{BICI}}$. The nonzero eigenvalues of $\rho_{BC_{\mathrm{I}}}$ are the same as those of $\rho_{AC_{\mathrm{I}}}$.

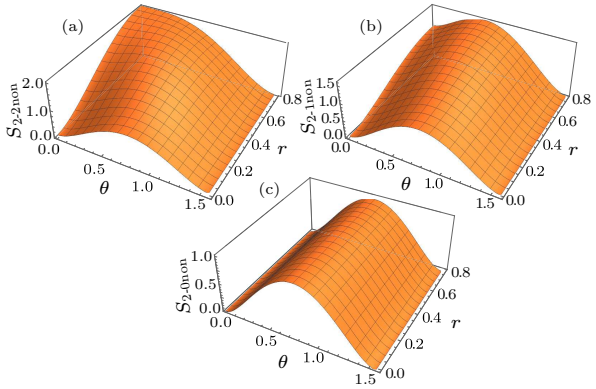


**Fig. 7.** The von Neumann entropies $S_{2-2_{\mathrm{non}}}$ (with two accelerations), $S_{2-1_{\mathrm{non}}}$ (with only one acceleration) and $S_{2-0_{\mathrm{non}}}$ (without acceleration) versus both parameter $r$ and the angle $\theta$.

We show the behavior of the von Neumann entropy in Fig. 6. It is interesting to see that the von Neumann entropy becomes larger with the increase of the number of accelerated observers. However, we find that the von Neumann entropy of whole bipartite system does not always increase with the acceleration $r$. For

example, as shown in Fig. 7, it is very interesting to observe that the entropy $S_{2-0_{\rm non}}$ (without acceleration) for two accelerated observers is independent of the variable $r$, and $S_{2-1_{\rm non}}$ (with one acceleration) and $S_{2-2_{\rm non}}$ (with two accelerations) first increase with the parameter $r$ and then decrease with it. We also notice that $S_{2-0_{\rm non}}$, $S_{2-1_{\rm non}}$ and $S_{2-2_{\rm non}}$ first increase with the angle $\theta$ and then decrease with it, but the former $S_{2-0_{\rm non}}$ is symmetric with respect to the angle $\theta = \pi/4$.

In summary, we first calculated the negativity of the generalized GHZ state such as the entanglement measures 1–1 and 1–2 tangles, which are used to calculate the whole entanglement average measures $\pi_3$ and $\Pi_3$ tangles. We have noticed that the 1–1 tangles for all possibilities are equal to zero. This is different from the entangled W-class case,[22,40] in which there exists an entanglement sudden death. We also have verified again the fact that the entanglement for this generalized GHZ entangled state is also observer-dependent in noninertial frame. When we compare the whole entanglement measures such as the arithmetic average $\pi_3$ and geometric average $\Pi_3$, it is seen that the algebraic average $\pi_3$ is almost the same as the geometric average $\Pi_3$. This means that we may make use of either $\pi_3$ or $\Pi_3$ to describe this entangled system.

For the von Neumann entropy, we observed that the entropy increases with the number of accelerated qubits in the entangled system. Moreover, we have noticed that the von Neumann entropies for tripartite systems always decrease with the angle $\theta$ and increase with the parameter $r$. However, the von Neumann entropies $S_{2-2_{\rm non}}$, $S_{2-1_{\rm non}}$ and $S_{2-0_{\rm non}}$ first increase with the angle $\theta$ and then decrease with it, but the former two entropies $S_{2-2_{\rm non}}$, $S_{2-1_{\rm non}}$ increase with the parameter $r$ and the last one $S_{2-0_{\rm non}}$ is independent of the parameter $r$ and symmetric to variable $\theta$. This implies that the subsystems $\rho_{\kappa\zeta_{\rm I}}$ and $\rho_{\kappa_{\rm I}\zeta_{\rm I}}$ are first more disorder and then the disorder is reduced with increasing the acceleration $r$. This means that the entangled system starts to become stable when the acceleration $r$ goes through some fixed acceleration values.

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