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Localization and Steering of Light in One-Dimensional Parity-Time Symmetric Photonic Lattices *

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We theoretically study the propagation dynamics of input light in one-dimensional mixed linear-nonlinear photonic lattices with a complex parity-time symmetric potential. Numerical computation shows simultaneous localization and steering of the optical beam due to the asymmetric scatter and interplay between Kerr-type nonlinearity and PT symmetry. This may provide a feasible measure for manipulation light in optical communications, integrated optics and so on.

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The non-Hermitian optics and photonics based on the quantum-inspired paradigm of parity-time (PT) symmetry[1,2] have been witnessed to be a substantial growth and its ramifications have passed on into a wide range of subjects in classical and quantum regimes because it was proposed[1] that non-Hermitian Hamiltonians respecting the weaker condition of PT-symmetry counter-intuitively could give rise to an entirely real eigenspectrum. Moreover, the fact that the Schrödinger equation in quantum mechanics is isomorphic to the paraxial wave equation in optics paves way for its practical realization in optical systems.[3,4] The judicious implementation of the complex refractive index profile, i.e., the real index-guiding and imaginary gain-loss distributions, can be used to realize a PT-symmetric system. This novel idea has resulted in a myriad of novel and exotic physical effects and phenomena that are otherwise unattainable in conventional Hermitian counterparts; for example, variable optical isolators,[4−6] coherent perfect laser absorbers,[7] optical switches,[8] optical couplers,[9] single-mode amplifiers,[10] unidirectional light transport,[11,12] nonreciprocal light transmission,[13] and PT nonlinearities.[14,15] In addition, PT-symmetric photonic lattices show great promise because of their hosting intriguing effects and properties, including gap solitons,[16−18] Anderson localization,[19−22] stable nonlocal solitons,[23−25] and so on, especially after the experimental observation of PT optical solitons.[26,27] In particular, the uniformly disordered two-dimensional PT-symmetric optical lattices have been proposed by Dragana,[28−31] in which they have discussed the roles of the PT-symmetric potential in the Anderson localization. Now localization of the optical signal plays an important role in both fundamentally and technological fields of optical communications and integrated devices. In the present work, we show that both localization and steering of light could be actualized in one-dimensional (1D) PT-symmetric optical lattices governed by a Schrödinger-type equation. Now, 1D waveguide systems reveal possibilities of realization in the experiment compared to the 2D systems because of the configuration advantages of this type of system. For example, optical beam steering in 2D often gives rise to the catastrophic collapse as is known in the theory of self-focusing, which is opposed to its 1D counterpart.[32]

In this work, to show the localization and steering of light in the 1D PT-symmetric photonic lattices, a detailed numerical simulation has been investigated for Gaussian beams. The results indicate that the light localization and steering can be controlled by changing some of the system parameters of the photonic lattices. For example, localization of light can be achieved by changing the nonlinear strength operating near the PT-symmetric exceptional point (EP) of the linear spectrum. Interestingly, it is found that light can be manipulated and tuned from a highly diffracted delocalized beam to a localized one that is steered along the propagation axis. The tilt angle of oblique propagation can be changed by varying strength of nonlinearity and linear coupling strength. These effects shed light on the optical beam propagation dynamics in 1D photonic lattices characterized by linear and nonlinear PT symmetric gain-loss modulations. It should also promote further understanding of optical wave manipulation in such photonic systems.

To construct non-Hermitian but PT-symmetric

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photonic lattices characterized by the refractive index distribution $n(x) = n_R(x) + in_I(x)$ the necessary condition of PT symmetry (the Hamiltonian commutes with the PT operator $[H, PT] = 0$) implies that $n^*(-x) = n(x)$ or equivalently $n_R(x) = n_R(-x)$ and $n_I(x) = -n_I(-x)$) where $n_R(x)$ is the real part of the refractive index, and $n_I(x)$ is the imaginary part.[3,33] In this work, we consider the PT optical potential along the transverse x axis. The paraxial optical field amplitude $E(z, x)$ can be expressed by the following Schrödinger-like equation[34]

$$ik_0 n_0 \frac{\partial E(z,x)}{\partial z} + \frac{1}{2}\frac{\partial^2 E(z,x)}{\partial x^2} + {k_0}^2 \Delta n(x) E(z,x) = 0, \tag{1}$$

where

$$\Delta n(x) = (1 + gV(x))|E(x,z)|^2 + pV(x) \tag{2}$$

represents the effective potential comprising mixed linear-nonlinear lattices in which the modulated nonlinearity is achievable via judicious spatial modulation of nonlinear gain and loss. The parameters g and p are the nonlinear and linear modulation strengths of the refractive index, respectively,[26,35] with the variable g acting as perturbation to affect the band structure of the PT system. Here, g is assumed to be positive, so that the first term of Eq. (2) represents self-focusing nonlinearity. In addition, the third term is proportional to the linear PT potential, which plays pivotal roles in the PT-symmetric optical medium.[26,36] Here $n(x)$ is the transverse refractive index ($n(x) = \Delta n(x) + n_0$), with n_0 representing the substrate index. It is worthwhile to mention that the optical system under consideration modeled by Eq. (1) can be realized via proper nonlinear waveguide settings of concatenated semiconductor amplifier and two-photon absorber cavities.[37–39] The transverse refractive index is modulated by the following PT-symmetric potential

$$V(x) = V_0[\cos^2(\Omega x/2) + iW_0 \sin(\Omega x)], \tag{3}$$

where V_0 and W_0 are the depths of the real component and the relative magnitude of the imaginary component of the potential, respectively. The condition $W_0 \sin(\Omega x) > 0$ represents loss while $W_0 \sin(\Omega x) < 0$ refers to gain,[3] where Ω is the spatial frequency of the PT potential. The spatial period of the real part of the potential $V(x)$ is $T_\mathrm{r} = 4\pi/\Omega$, and the period of the imaginary part is $T_\mathrm{i} = 2\pi/\Omega$.

A common 2D split-step-Fourier method is used to carry out the numerical simulations. A Gaussian beam centered at 780 nm is used to illuminate the photonic lattices. If we use the scaling transformation $z \sim Zk_0 n_0 (T_\mathrm{r}/4)^2$, $x \sim XT_\mathrm{r}/4$, then Eq. (1) can be simply expressed in the normal form

$$\begin{aligned} i\frac{\partial E(Z,X)}{\partial Z} = &-\frac{1}{2}\frac{\partial^2 E(Z,X)}{\partial X^2} - |E(Z,X)|^2 E(Z,X) \\ &- gV(X)|E(Z,X)|^2 E(Z,X) \\ &- pV(X)E(Z,X), \end{aligned} \tag{4}$$

where $k_0 n_0 (T_\mathrm{r}/4)^2$ and $T_\mathrm{r}/4$ are regarded as the unit lengths along Z and X-directions, respectively, so that all lengths are dimensionless in this study, and $k_0 = 2\pi/\lambda_0$,[40] with λ_0 being the vacuum wavelength.[36] Then Eq. (4) can be rewritten as

$$\frac{\partial E(Z,X)}{\partial Z} = (\hat{D} + \hat{N})E(Z,X), \tag{5}$$

where the linear and nonlinear operators can be described by $\hat{D} = (i/2)(\partial^2/\partial X^2)$ and $\hat{N} = i(|E(Z,X)|^2 + gV(X)|E(Z,X)|^2 + pV(X))$, respectively. The linear and nonlinear effects can be individually studied by the split-step-Fourier method. Thus, Eq. (5) can be expressed as

$$\frac{\partial E_N(Z,X)}{\partial Z} = \hat{N}E_N(Z,X), \tag{6}$$

$$\frac{\partial E_D(Z,X)}{\partial Z} = \hat{D}E_D(Z,X). \tag{7}$$

Equation (6) only contains the nonlinearity effects, while Eq. (7) only contains the linear effects. Then the solution to Eq. (1) can be written as

$$E(Z,X) = \exp[h(\hat{D} + \hat{N})]E(Z,X). \tag{8}$$

Here h represents a small distance. When light travels h in the medium, according to the Split-Step-Fourier method, the approximate solution to Eq. (1) can be obtained as follows:

$$\begin{aligned} E(Z+h,X) = &\exp\left(\frac{h}{2}\hat{D}\right)\exp\left[\int_Z^{Z+h}\hat{N}(Z')dZ'\right] \\ &\cdot \exp\left(\frac{h}{2}\hat{D}\right)E(Z,X). \end{aligned} \tag{9}$$

We can simulate the propagation of the signal light in the PT-symmetric photonic lattices using Eq. (9) with different parameters. Through scaling transformation, the gradient of the refraction index can be transformed into

$$\Delta n(X) = (1 + gV(X))|E(X,Z)|^2 + pV(X). \tag{10}$$

For the first scheme, we choose $\Omega = 3(N\lambda)^{-1}$, where λ is the wavelength of the beam, and N is an arbitrary integer. Here, we set $N = 100$, $V_0 = 1$, and $W_0 = 0.5$. The values of these parameters are arbitrary in our nonlinear structure, which essentially corresponds to the exceptional points of the linear system. In the calculation, the wavelength of signal light is $\lambda_0 = 7.8 \times 10^{-7}$ m, $T_\mathrm{r}/4 = 12.4\,\mu$m, $n_0 = 1$ and $p = 4$.

To understand the physical essence of the beam propagation in the photonic lattices, further simulations and calculations are carried out. The simulation results of the energy band and potential function have been illustrated in Fig. 1. From Figs. 1(a)

and 1(b), the band gap becomes narrower as W_0 increases, and closes completely when W_0 crosses the critical transition value of 0.5. Here, $W_0 = 0.5$ is the phase transition point of the linear spectrum.[4,41] More specifically, for $W_0 < 0.5$, the band structure is real, while for $W_0 > 0.5$, it becomes complex (starting from the lowest bands), meaning that spontaneous PT-symmetry breaking has occurred. Assuming that the original beam width of the incident light is $T_r/4$, and the gradient of the refractive index $\Delta n \sim 10^{-4}$. The PT-symmetric potential is presented in Fig. 1(c).

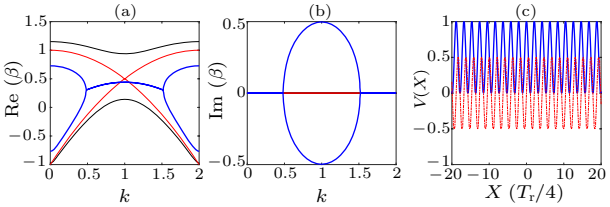

Fig. 1. (a) The real part of the bands of the same potential for different values of gain/loss amplitudes $W_0 = 0.3$ (black lines), $W_0 = 0.5$ (red lines), and $W_0 = 0.7$ (blue lines). (b) The imaginary part of the bands of the same potential for different values of gain/loss amplitudes $W_0 = 0.3$ (black lines), $W_0 = 0.5$ (red lines), $W_0 = 0.7$ (blue lines). (c) The parity-time-symmetric potential for $\Omega = 3$. The solid curve denotes the real part of $V(X)$, and the dashed curve denotes the imaginary part of $V(X)$. The system parameters have been given in the text, and X is in units of $T_r/4 = 12.4\,\mu$m.

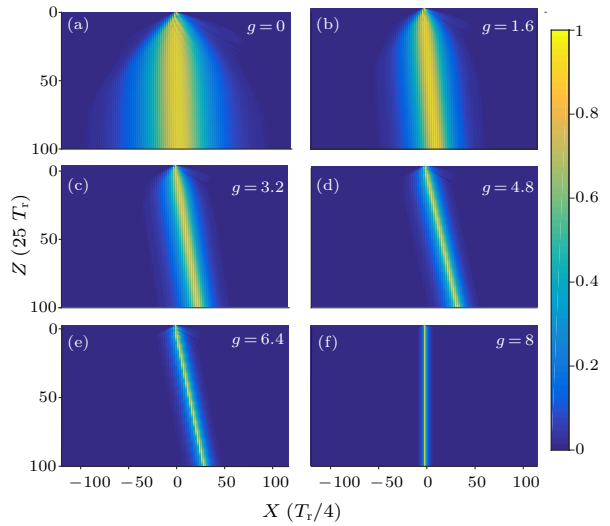

Fig. 2. The intensity distribution of the beam $|E(Z, X)|$ as a function of z with different nonlinear strengths: (a) $g = 0$, (b) $g = 1.6$, (c) $g = 3.2$, (d) $g = 4.8$, (e) $g = 6.4$, and (f) $g = 8$. The transverse amplitude is normalized to the corresponding maximum value at each Z. Here $W_0 = 0.5$, $E_0 = 1$, $\lambda_0 = 7.8 \times 10^{-7}$ m, and $T_r/4 = 12.4\,\mu$m.

In this work, the optical beam is considered to be the Gaussian beam $E(0, X) = E_0 \exp(-X^2)$ (E_0 is the maximum amplitude of the incident field). The beam is initially localized tightly in a quite small spatial region.

For different g, we can simulate the propagation of the beam in the PT-symmetric photonic lattices using Eq. (9). The propagation of the beam under different conditions is shown in Fig. 2 which reveals that different nonlinear strengths g essentially result in the altogether different properties. For $g = 0$, the beam propagates along the Z axis with its intensity highly diffracted. As g increases, with the signal refracted to the right domain, the diffraction trend reduces, and the spatial width decreases gradually, as shown in Figs. 2(a), 2(c) and 2(e). For $g = 4.8$ (which means self-focusing nonlinear photonic lattices), the beam is highly localized within a small width, and off-centered to the right-side oblique localization. When raising g, the oblique localization gradually becomes stationary vertical localization with no steering around the $x = 0$ axis. It can be argued from Eq. (10) that for $g > 8.5$, the beam will be absorbed by the PT-symmetric photonic lattices, and it cannot propagate along the Z axis. The transverse displacement can be quantitatively represented as ΔX. From Fig. 3(a), we can see that the changes in the transverse displacement are similar to a parabolic shape, with which the nonlinear strength g increases, showing that the beam deviates from the center gradually. When the nonlinear strength $g = 4.8$, the transverse displacement reaches a peak. When $8 < g < 8.5$, the transverse displacement remains at a small value and almost invariant, which implies vertical localization without steering.

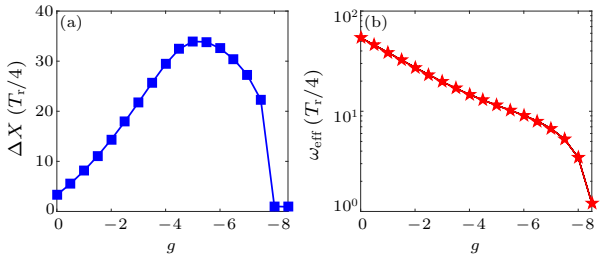

Fig. 3. (a) The transverse displacement as a function of nonlinear strength with the beam propagating in the photonic lattices. (b) Effective beam width of beam in the output surface as a function of g. Here $W_0 = 0.5$ and $\lambda_0 = 7.8 \times 10^{-7}$ m.

To analyze the phenomenon during propagation in the photonic lattices quantitatively, we can describe the width of the beam wave packet by the effective beam width ω_{eff}, given by[42]

$$\omega_{\text{eff}} = \frac{[\int I(Z, X)dX]^2}{\int I^2(Z, X)dX}, \tag{11}$$

which are the functions of the nonlinear strength g and propagation distance Z, and $I(Z, X)$ represents the intensity of the beam. From Eq. (11), we can see that ω_{eff} increases as the wave expands, and decreases as the wave converges. Figure 3(b) shows the effective beam width of the beam as a function of nonlinear strength g with $V_0 = 1$, $W_0 = 0.5$ and $g = 5$.

As a result, in Figs. 4(a)–4(d), the localization and steering of signal light have been elucidated for different linear modulation strengths of refractive index

at $g = 5$. It can be seen that for shallow linear lattices when $p = 1$, the beam is highly diffracted in intensity. With increasing p as the linear lattices become deeper and deeper, the beam is gradually localized and steers to the right which could be attributed to the interaction between the Kerr nonlinearity and the PT-symmetry of the gain-loss distributions of the photonic lattices,[43,44] which induce a transition from full PT symmetry to broken PT symmetry. When $p > 3$, the steering of the beam gets reduced to result in the in-axis localization around $x = 0$. The localization of light stems comes from the interplay between self-focusing nonlinearity and diffraction whereas the steering is a result of the asymmetric scattering from mixed linear-nonlinear lattices with PT-symmetric gain-loss distributions.[45–47]

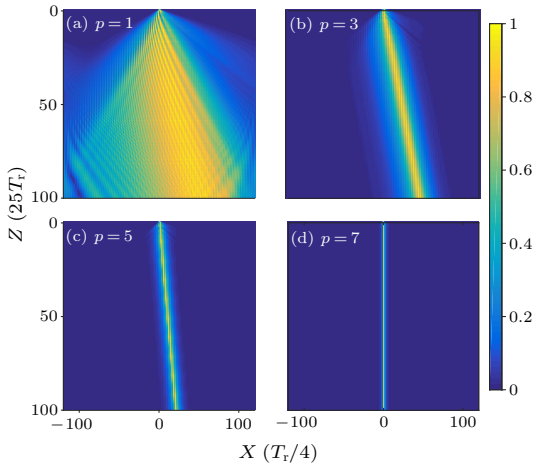

Fig. 4. The intensity distribution of signal light $|E(Z, X)|$ as a function of Z with different linear modulation strengths of refractive index p: (a) $p = 1$, (b) $p = 3$, (c) $p = 5$, and (d) $p = 7$. The transverse amplitude is normalized to the corresponding maximum value at each Z. Here $V_0 = 1$, $W_0 = 0.5$, $E_0 = 1$, and $g = 5$.

From Fig. 5(a) it can be seen that the effective beam width of the signal light ω_{eff} in the output surface decreases as p is enhanced (i.e., as the linear lattice becomes deeper) that gives rise to localization.[48] Meanwhile, in Fig. 5(b), we have depicted the dependence of the localization behavior on the gain-loss modulation parameter W_0 (the scale of y axis is all log coordinates). It can be seen that for the effective beam width of the signal light increases with W_0 until a certain value around $W_0 \approx 0.32$ and decreases afterwards, hence signifying localization of signal light. This could be attributed to the interplay between the self-focusing nonlinearity and diffraction in the PT-symmetric linear-nonlinear modulations. The transverse amplitude is normalized to the corresponding maximum value at each Z. As the strength of the self-focusing nonlinearity increases, the gradient of refractive index increases, and the beam is refracted in the region $X > 0$. In the gain region, the mutual interference effects of multiple scatterings of light become more prominent. The nonlinear absorption is enhanced as the nonlinear strength is gradually increased so that the weak light is absorbed by the photonic lattices around the center area. As a result, localization occurs as the nonlinear intensity grows. Even when the nonlinear intensity exceeds the threshold, the signal light is absorbed by the PT-symmetric photonic lattices so that it propagates only for a short distance.

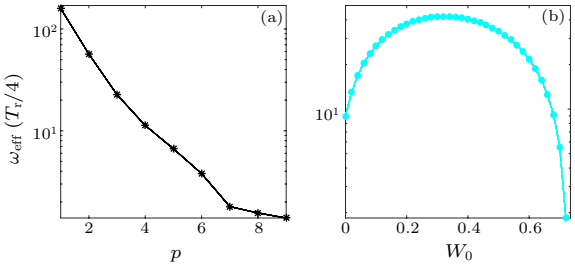

Fig. 5. (a) The effective beam width in the output surface as a function of p with the beam propagates in the photonic lattices. (b) The effective beam width in the output surface as a function of W_0. The transverse amplitude is normalized to the corresponding maximum value at each Z. Here $V_0 = 1$, $g = 5$, $p = 4$, $T_r/4 = 12.4\,\mu$m.

It is worthwhile to note that the interplay between nonlinearity and the PT-symmetric modulations could be pointed toward the nontrivial effect on the simultaneous localization and steering of the beam as shown in Figs. 2(a)–2(f). As the nonlinear strength g increases, it can nonlinearly affect the effective nonlinear lattices changing the PT threshold,[43,44] for which the photonic lattices can transform from full PT symmetry to PT symmetry broken. Thus the beam can be tuned and manipulated from a delocalized diffracted beam to an oblique (or even without steering) localized one.

In summary, we have shown that the paraxial beam can be simultaneously localized and steered along the propagation axis in 1D PT-symmetric photonic lattices by means of appropriately modulating the linear or nonlinear strength of the refractive index. Meanwhile, we also notice that the system can induce a PT phase transition from full PT symmetry to broken and vice versa via changing the system parameters. Due to the interaction between self-focusing Kerr nonlinearity and PT-symmetric potential, the light can be tuned from a diffraction to an obliquely (or even stationarily localized at the $x = 0$ axis for the specific value of the system parameters) localized beam. The beam steering and localization properties in the 1D photonic lattices are a feasible way to manipulate light in the field of optical communications and integrated optics to realize new types of directional optical elements, such as controlled optical isolator and optical solitons in open nonlinear non-Hermitian but PT-symmetric systems, and thus may open up an useful avenue for harnessing dissipative effects.

References

[1] Bender C M and Boettcher S 1998 *Phys. Rev. Lett.* **80** 5243
[2] Bender C M, Brody D C and Jones H F 2002 *Phys. Rev. Lett.* **89** 270401
[3] Guo A, Salamo G J, Duchesne D et al 2009 *Phys. Rev. Lett.* **103** 093902
[4] Makris K G, El-Ganainy R, Christodoulides D N and Musslimani Z H 2008 *Phys. Rev. Lett.* **100** 103904
[5] Rüter C E, Makris K G, El-Ganainy R, Christodoulides D N, Segev M and Kip D 2010 *Nat. Phys.* **6** 192
[6] Chang L, Jiang X S, Hua S Y, Yang C, Wen J M, Jiang L, Li G Y, Wang G Z and Xiao M 2014 *Nat. Photon.* **8** 524
[7] Chong Y D, Ge L and Stone A D 2011 *Phys. Rev. Lett.* **106** 093902
[8] Nazari F, Nazari M and Moravvej-Farshi M K 2011 *Opt. Lett.* **36** 4368
[9] Sukhorukov A A, Xu Z and Kivshar Y S 2010 *Phys. Rev. A* **82** 043818
[10] Miri M A, LiKamWa P and Christodoulides D N 2012 *Opt. Lett.* **37** 764
[11] Lin Z, Ramezani H, Eichelkraut T et al 2011 *Phys. Rev. Lett.* **106** 213901
[12] Feng L, Xu Y L, Fegadolli W S et al 2013 *Nat. Mater.* **12** 108
[13] Feng L, Ayache M, Huang J et al 2011 *Science* **333** 729
[14] Sarma A K, Miri M A, Musslimani Z H and Christodoulides D N 2014 *Phys. Rev. E* **89** 052918
[15] Gupta S K 2018 *Opt. Commun.* **411** 1
[16] Dong L W, Gu L L and Guo D C 2015 *Phys. Rev. A* **91** 053827
[17] Zhu X, Cao P, Song L et al 2014 *J. Opt. Soc. Am. B* **31** 2109
[18] Ge L J, Shen M, Ma C L et al 2014 *Opt. Express* **22** 29435
[19] Mejía-Cortés C and Molina M I 2015 *Phys. Rev. A* **91** 033815
[20] Vatnik I D, Tikan A, Onishchukov G et al 2017 *Sci. Rep.* **7** 4301
[21] Harter A K, Onanga F A and Joglekar Y N 2018 *Sci. Rep.* **8** 44
[22] Nguyen B P and Kim K 2016 *Phys. Rev. A* **94** 062122
[23] Xie J N, Chen W C, Lv J T et al 2013 *J. Opt. Soc. Am. B* **30** 1216
[24] Zhu X, Li H G, Wang H et al 2013 *J. Opt. Soc. Am. B* **30** 1987
[25] Yin C P, He Y J, Li H G et al 2012 *Opt. Express* **20** 19355
[26] Wimmer M, Regensburger A, Miri M A et al 2015 *Nat. Commun.* **6** 7782
[27] He Y, Zhu X, Mihalache D et al 2012 *Phys. Rev. A* **85** 013831
[28] Kramer B and MacKinnon A 1993 *Rep. Prog. Phys.* **56** 1469
[29] Sheng P 2006 *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena* (Berlin: Springer) pp 1–327
[30] Anderson P W 1958 *Phys. Rev.* **109** 1492
[31] Jović D M, Denz C and Belić M R 2012 *Opt. Lett.* **37** 4455
[32] Cao X D, Meyerhofer D D and Agrawal G P 1994 *J. Opt. Soc. Am. B* **11** 2224
[33] Wu Y and Yang X Y 2007 *Appl. Phys. Lett.* **91** 094104
[34] Kartashov Y V, Malomed B A and Torner L 2011 *Rev. Mod. Phys.* **83** 247
[35] Konotop V V, Yang J and Zezyulin D A 2016 *Rev. Mod. Phys.* **88** 035002
[36] Zhang Y L, Li X Y, Zhang Y, Sun P P and Zhu J Q 2010 *Chin. Opt. Lett.* **8** 210
[37] Abdullaev F K, Kartashov Y V, Konotop V V et al 2011 *Phys. Rev. A* **83** 041805
[38] Bender N, Factor S, Bodyfelt J D et al 2013 *Phys. Rev. Lett.* **110** 234101
[39] Huang C M, Li C Y and Dong L W 2013 *Opt. Express* **21** 3917
[40] Lu F F and Wang C F 2016 *Chin. Phys. Lett.* **33** 074202
[41] Musslimani Z H, Makris K G, El-Ganainy R and Christodoulides D N 2008 *Phys. Rev. Lett.* **100** 030402
[42] Schwartz T, Bartal G, Fishman S and Segev M 2007 *Nature* **446** 52
[43] Lumer Y, Plotnik Y, Rechtsman M C et al 2013 *Phys. Rev. Lett.* **111** 263901
[44] Ramezani H, Kottos T, Elganainy R et al 2010 *Phys. Rev. A* **82** 043803
[45] Kulishov M, Jones H F and Kress B 2015 *Opt. Express* **23** 18694
[46] Kulishov M, Jones H F and Kress B 2015 *Opt. Express* **23** 9347
[47] Zhu X Y, Xu Y L, Zou Y et al 2016 *Appl. Phys. Lett.* **109** 111101
[48] Zezyulin D A and Konotop V V 2012 *Phys. Rev. Lett.* **108** 213906