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Simultaneous Measurement of Fringe Visibility and Path Predictability of Wave-Particle Duality *

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An experimental scheme to simultaneously obtain the information of fringe visibility and path predictability is designed. In a modified Young's double-slit experiment, two density filters rotating at different frequencies are placed before the two pineholes to encode path information. The spatial and temporal distributions of the output provide us with the wave and particle information of the single photons, respectively. The simultaneous measurement of the wave and particle information inevitably disturbs the system and thus causes some loss of the duality information, which is equal to the mixedness of the photonic state behind the density filters.

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The wave-particle duality is an unavoidable topic when we discuss the nature of light. Young's double-slit experiment in the early nineteenth century initiated the acceptance of the wave theory of light. However, this does not imply the end of the particle description of light. Even in Young's double-slit experiment itself, the particle-like behavior of light can be demonstrated by turning the polarization of the light in the two paths perpendicular to each other, and the light's trajectory can then be deduced through the light polarization. The interference pattern disappears accordingly. Feynman proposed a gedanken experiment to find out which path an electron (replacing the light used in Young's double-slit experiment) has taken, and concluded that full knowledge of the path information causes the disappearance of the interference pattern.[1] An interesting phenomenon is that the interference pattern, destroyed by observing which path the photons have taken, can be recovered by 'erasing' the which-path information.[2–4] Recently, a concept of quantum wave-particle superposition for describing a photon behaving both as a particle and as a wave is proposed through a quantum beam splitter,[5] which has been experimentally demonstrated in a Mach–Zehnder interferometer,[6,7] in a superconducting Ramsey interferometer,[8] and also in a narrow-band single-photon wave packet.[9]

Since the wave nature of light is responsible for the interference in a two-path duality experiment, the visibility (V) of the interference pattern is used to quantify the wave-like behavior of light. On the other hand, the probability difference of the photon taking the two paths, called path predictability (P), can be used to quantify the particle-like behavior of light. The wave-like and particle-like behaviors exhibit a mutually exclusive relation,[10–16]

$$V^2 + P^2 \leq 1, \tag{1}$$

where the equal sign holds for pure states. The mutually exclusive relation between the wave- and particle-like behaviors has already been demonstrated in many experiments, such as in an atom interferometer,[17,18] in an electron interferometer,[19] in a system with a single neutral atom,[20,21] in an optical system,[22–25] in a delayed-choice experiment,[26–29] and between two twisted light beams.[30] However, the wave and particle information is measured separately in these duality experiments, where the photons participating the measurement of the wave information have no contribution on the measurement of the particle information at all, and vice versa. Since the fringe visibility is measured in an experiment, and the particle information is measured in another experiment, one may argue why the properties exhibited by two independent sets of photons can be said to belong to the same photons? Therefore, simultaneous measurement of fringe visibility and path predictability in fact plays a crucial role in verifying the wave-particle duality, which help close the loophole mentioned above and guarantee the wave-like and particle-like behaviors we observed belong to the same photons. Here we propose a feasible scheme to obtain the information of fringe visibility and path predictability simultaneously. In a modified Young's double-slit experiment, it is shown that the spatial and temporal distributions of the output beam provide us with the wave and particle information of the single photons, respectively, and the wave and particle behaviors we observed belong to the same photons.

Suppose we have two circular density filters, whose transmission coefficient t_j $(j = 1, 2)$, defined as the ratio between the amplitudes of the transmitted field and incident field,[31] depends on the central angle θ_j through the relation, $t_j = 1 - \alpha_j \cos^2\theta_j$. Here α_j is a small positive constant. At the same time, such a density filter could also be regarded as a special type of beam splitter, whose action on the incident light is described by a unitary transformation,

$$U(\theta_j) = \begin{pmatrix} r_j & t_j \\ -t_j & r_j \end{pmatrix}, \tag{2}$$

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with $r_j = (1-t_j^2)^{1/2}$. Now we place two such density filters before the two pinholes, S_1 and S_2, in the modified Young's double-slit experiment (see Fig. 1). The two pinholes, located at the positions $\boldsymbol{r}'_j$ in a horizontal line, are used to replace the two slits so that they can be fully covered by the two density filters, which rotate at different angular frequencies μ_1 and μ_2. According to quantum theory, a single microscopic quantum state is not enough to finish the measurement of a macroscopic observable, thus we need to prepare many copies of the photonic state, and make repeated measurements on them to achieve accurate results. In this scheme, a single photon source S_0 based on the spontaneous emission of an excited atom[32] is used for the state preparation, where the single photon field before the two filters can be described by a wave function $|\psi'\rangle = \sum_{\boldsymbol{k}} g_k[(\nu_k-\omega)+i\Gamma/2]^{-1}[A_1(k)\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}'_1)a^\dagger_{\boldsymbol{k}} + A_2(k)\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}'_2)a^\dagger_{\boldsymbol{k}}]|0\rangle$, where $\sum_{\boldsymbol{k}} g_k[(\nu_k-\omega)+i\Gamma/2]^{-1}\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}'_j)a^\dagger_{\boldsymbol{k}}|0\rangle$ represents the mode associated with the pinhole S_j, whose center frequency and width are ω and Γ, $a^\dagger_{\boldsymbol{k}}$ is the creation operator of the mode with wave vector $\boldsymbol{k}$. The two coefficients $A_1(k) = d_2[d_1^2+d_2^2]^{-1/2}\exp(ikd_1)$ and $A_2(k) = d_1[d_1^2+d_2^2]^{-1/2}\exp(ikd_2)$ are related to the geometrical structure of the setup, with d_j the distance between the single photon source S_0 and the pinhole S_j. The density filters turn the above wave function to

$$\begin{aligned}|\psi\rangle = \sum_{\boldsymbol{k}} & \frac{g_k}{(\nu_k-\omega)+i\Gamma/2} \\ & \cdot [A_1(k)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}'_1}(t_1 a^\dagger_{\boldsymbol{k}} + r_1 b^\dagger_{1,\boldsymbol{k}}) \\ & + A_2(k)e^{-i\boldsymbol{k}\cdot\boldsymbol{r}'_2}(t_2 a^\dagger_{\boldsymbol{k}} + r_2 b^\dagger_{2,\boldsymbol{k}})]|0\rangle, \end{aligned} \tag{3}$$

where $b^\dagger_{j,\boldsymbol{k}}$ denote creation operators of the modes reflected by the two density filters, which have no contribution to the interference behind the two pinholes. The parameters t_j and r_j vary with time τ because they depend on the central angles $\theta_j(\tau) = \mu_j\tau$ of the rotating density filters.

We use a charge-coupled device (CCD) to record the real time intensity distribution (RTID) of the interference pattern, which is just the first order correlation function[32]

$$\begin{aligned}\langle E^{(-)}(\boldsymbol{r},\tau)E^{(+)}(\boldsymbol{r},\tau)\rangle = \Big| & A_1\Big(\frac{\omega}{c}\Big)t_1\,\Psi_1(\boldsymbol{r},\tau) \\ & + A_2\Big(\frac{\omega}{c}\Big)t_2\,\Psi_2(\boldsymbol{r},\tau)\Big|^2, \end{aligned} \tag{4}$$

with $\Psi_j(\boldsymbol{r},\tau) = \varepsilon_0\,\Theta(\tau-|\boldsymbol{r}_j|/c)\exp[-(\tau-\frac{|\boldsymbol{r}_j|}{c})(i\omega+\frac{\Gamma}{2})]/|\boldsymbol{r}_j|$, where $E^{(+)}(\boldsymbol{r},\tau) = \sum_{\boldsymbol{k}}\varepsilon_k a_{\boldsymbol{k}}\exp[i(\boldsymbol{k}\cdot\boldsymbol{r}-\nu_k\tau)]$ with $\varepsilon_k = (\hbar\nu_k/2\epsilon_0 V)^{1/2}$ being the positive frequency part of the electric field, and the negative frequency part $E^{(-)}(\boldsymbol{r},\tau)$ is its Hermitian conjugate, the constant $\varepsilon_0 = iVk^2\varepsilon_k g_k/2\pi^2 c$ depends on the quantization volume V and the other two parameters ε_k and g_k at $k = (2\omega-i\Gamma)/2c$, and $\boldsymbol{r}_j = \boldsymbol{r}-\boldsymbol{r}'_j$ is the displacement vector from the pinhole located at $\boldsymbol{r}'_j$ to the point at $\boldsymbol{r}$. The step function $\Theta(\tau-|\boldsymbol{r}_j|/c)$ ensures that the signal cannot move faster than the speed of light. For the sake of simplicity, we make three assumptions in the following analysis. (i) We focus on the central bright spot with the highest intensity and its vicinity, and the maximum and minimum intensities in this small region are chosen to calculate the visibility. Otherwise, we always trivially obtain a vanishing value for the global minimum intensity, corresponding to the dark at infinity. This assumption is valid when the size of the CCD screen is much smaller than the distance between the CCD and the two pinholes, which could be regarded as a constant $|\boldsymbol{r}_0|$ now. (ii) The diffraction light far from the x-axis is ignored for simplicity, which enables us to concentrate on the interference pattern. (iii) We assume that the spontaneous linewidth is extremely small so that the source can be regarded as a monochromatic one with frequency ω_0. With these three assumptions, the interference under consideration becomes a one-dimensional problem. Based on Eq. (4), the RTID recorded by the CCD camera is simplified to

$$\begin{aligned} p(x,\tau) = \varepsilon_0^2\Big\{ & [E_1-\epsilon_1\cos(2\mu_1\tau)]^2 \\ & + [E_2-\epsilon_2\cos(2\mu_2\tau)]^2 \\ & + 2[E_1-\epsilon_1\cos(2\mu_1\tau)] \\ & \cdot [E_2-\epsilon_2\cos(2\mu_2\tau)]\cos\Big(\frac{k_0 dx}{L}+\phi_0\Big)\Big\}, \end{aligned} \tag{5}$$

with $k_0 = \omega_0/c$ being the wave number, d the distance between the two pinholes, and L the distance between the pinhole and the screen of the CCD camera. Here we have used the approximation $|\boldsymbol{r}_1|-|\boldsymbol{r}_2| \approx dx/L$ in the x-axis. The constant phase term ϕ_0 depends on the location of the original point in the x-axis. From a classical viewpoint, $\epsilon_j\cos(2\mu_j\tau)$ with $\epsilon_j = \alpha_j A_j(k_0)/2|\boldsymbol{r}_0|$ represents the instant amplitude of the lights reflected by the two rotating density filters, and $[E_j-\epsilon_j\cos(2\mu_j\tau)]$ with $E_j = (2-\alpha_j)A_j(k_0)/2|\boldsymbol{r}_0|$ stands for the amplitude of the transmitted ones.

The long-time average of RTID with $|\mu_1-\mu_2|T \gg 1$ gives out an interference pattern on the screen, which is

$$\begin{aligned} I(x) \equiv & \lim_{T\to+\infty}\frac{1}{T}\int_0^T p(x,\tau)d\tau \\ & \propto E_1^2 + \frac{1}{2}\epsilon_1^2 + E_2^2 + \frac{1}{2}\epsilon_2^2 \\ & + 2E_1E_2\cos\Big(\frac{kdx}{L}+\phi_0\Big), \end{aligned} \tag{6}$$

with the fringe visibility

$$V = \frac{4E_1E_2}{2E_1^2+\epsilon_1^2+2E_2^2+\epsilon_2^2}. \tag{7}$$

Note that this visibility can be measured in an experiment, because the maximum and minimum intensities, based on the time average of the RTID, i.e., Eq. (6), can be obtained in the experiment.

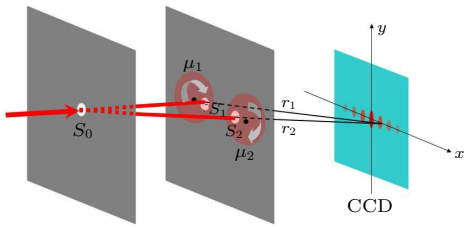

Fig. 1. A modified Young's double-slit experiment. Two density filters, rotating at angular frequencies μ_1 and μ_2, are used to periodically modulate the amplitude of light. A CCD camera is used to record the RTID of the interference pattern.

To obtain the path predictability, we need to know how much light passed through the two pinholes individually, and the spatial distribution plays no role in measuring the path predictability. Thus we can make an integral over the spatial coordinate to obtain the time evolution of the light intensity,

$$\begin{aligned} I(\tau) = &\left(E_1^2 + \frac{1}{2}\epsilon_1^2\right) + \left(E_2^2 + \frac{1}{2}\epsilon_2^2\right) \\ &- 2E_1\epsilon_1\cos(2\mu_1\tau) - 2E_2\epsilon_2\cos(2\mu_2\tau) \\ &+ \frac{1}{2}\epsilon_1^2\cos(4\mu_1\tau) + \frac{1}{2}\epsilon_2^2\cos(4\mu_2\tau). \end{aligned} \tag{8}$$

Taking the time average of Eq. (8), as the last four terms averaged out, we obtain

$$\begin{aligned} \bar{I} = &\left(E_1^2 + \frac{1}{2}\epsilon_1^2\right) + \left(E_2^2 + \frac{1}{2}\epsilon_2^2\right) \\ = &\bar{I}_1 + \bar{I}_2. \end{aligned} \tag{9}$$

It is clear that the two terms, $\bar{I}_{1(2)}$, correspond to the probabilities of the photon passing through path 1 and 2, respectively. Therefore, the theoretical value of the path predictability is

$$P \equiv \frac{|\bar{I}_1 - \bar{I}_2|}{\bar{I}_1 + \bar{I}_2} = \frac{|2E_1^2 + \epsilon_1^2 - 2E_2^2 - \epsilon_2^2|}{2E_1^2 + \epsilon_1^2 + 2E_2^2 + \epsilon_2^2}. \tag{10}$$

Now let us introduce how to obtain the information of path predictability experimentally. In Eq. (8), in addition to the first two terms, $\bar{I}_{1(2)}$, there are four terms that depend on the frequencies,

$$2E_j\epsilon_j\cos(2\mu_j\tau) \quad \text{and} \quad \frac{1}{2}\epsilon_j^2\cos(4\mu_j\tau), \tag{11}$$

at frequencies of $2\mu_j$ and $4\mu_j$, respectively. Comparing the above four terms with Eq. (10), we find that the above four terms are related to the four terms in Eq. (10). The coefficients of the four terms in Eq. (11) are the Fourier components of $I(\tau)$. Measuring the four peak heights of the Fourier transformation of $I(\tau)$, we obtain the four terms in Eq. (10). The heights, $H(\omega)$, are proportional to the corresponding coefficients of the cosine function in Eq. (8) (or Eq. (11)), $H(2\mu_j) = 2\beta E_j\epsilon_j$ and $H(4\mu_j) = \beta\epsilon_j^2/2$, with β a positive constant. Finally, the path predictability can be described in terms of the peak heights as

$$P = \frac{|p(\mu_1,\mu_2) - p_(\mu_2,\mu_1)|}{p(\mu_1,\mu_2) + p_(\mu_2,\mu_1)}, \tag{12a}$$

with

$$p(\mu_j,\mu_k) = [H^2(2\mu_j) + 8H^2(4\mu_j)]H(4\mu_k). \tag{12b}$$

Thus the knowledge of fringe visibility and path predictability is obtained simultaneously in this modified Young's double-slit experiment. The spatial and temporal distribution of RTID, obtained by integrating RTID over time and spatial coordinates, provides us with the wave and particle information, respectively.

At the same time, simultaneous measurement of the fringe visibility and path predictability causes some loss of duality information. According to Busch's theorem of 'no information gain without disturbance',[33] a non-trivial measurement must change the state of the system, no matter how weak this measurement is or what kind of ancilla is used. The expectation value of another observable may be affected as a consequence of the state change. In the current scheme, the two rotating density filters play the role of disturbing the interference system, so that the interference pattern exhibits a smaller visibility than the expected one, whose theoretical value is just Eq. (7) by setting $\epsilon_j = 0$, associated with the case of $\alpha_j = 0$ for no modulation. Based on Eqs. (7) and (10), the total duality information under the simultaneous measurement of the visibility and predictability is

$$V^2 + P^2 = 1 - \delta^2, \tag{13a}$$

with

$$\delta^2 = \frac{8E_1^2\epsilon_2^2 + 8E_2^2\epsilon_1^2 + 4\epsilon_1^2\epsilon_2^2}{(2E_1^2 + \epsilon_1^2 + 2E_2^2 + \epsilon_2^2)^2}. \tag{13b}$$

The total loss of the duality information δ^2 is equal to the mixedness of the state of the single photons behind the rotating density filters, defined as the linear entropy $M(\rho) \equiv 2 - 2\mathrm{Tr}(\rho^2)$ for a 2×2 density matrix ρ.[34] The state of the single photons behind the two rotating density filters can be obtained by tracing out the two reflected modes $b_{1(2)}^\dagger|0\rangle$ from the state in Eq. (3). Under the assumptions mentioned above, this reduced density matrix is

$$\rho = N\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \tag{14a}$$

with

$$\rho_{jj} = E_j^2 + \frac{1}{2}\epsilon_j^2, \ \rho_{12} = \rho_{21}^* = E_1E_2e^{ik_0(d_1-d_2)}, \tag{14b}$$

and a normalization constant $N = (\rho_{11} + \rho_{22})^{-1}$, in the basis $\{|\pm\rangle = \exp[-i\boldsymbol{k}_0 \cdot \boldsymbol{r}'_{1(2)}]a^\dagger|0\rangle\}$ for the two modes transmitted from the two pinholes. That is, the achievable duality information strongly depends on the measurement device we used for measuring it. A sensitive and accurate measurement device might provide us with the duality information through a weak disturbance on the interfering photon, so that its state changes slightly, and the mixedness of its density matrix remains small. In other words, the lost duality information can be considered as the price paid for the simultaneous measurement of the visibility and

predictability. Thus the equality in Eq. (1) cannot be satisfied even in the case of a pure state, which unavoidably turns to a mixed state if the fringe visibility and path predictability are measured simultaneously.

Wave-particle duality of elementary particles has been experimentally demonstrated in variant systems. However, the wave and particle information is measured separately in the previous experiments, which implies that the photons used for demonstrating the wave property and those used for demonstrating the particle property are independent. To demonstrate the wave and particle properties through the same photons, a loophole-free scheme to simultaneously obtain the information of fringe visibility and path predictability is proposed. In this scheme, we modify Young's double-slit experiment by placing two rotating density filters before the two pinholes (replacing the slits) and recording the RTID of the interference pattern via a CCD camera. It is found that the spatial and temporal distribution of the RTID carries the wave and particle information of the single photons, respectively. Making a time average of the RTID, we obtain the spatial distribution of the interfering photon, which presents an interfering pattern. At the same time, the Fourier spectrum of the intensity of the output beam brings out four peaks, whose heights could be used to work out the path information of the interfering photons. Since the measurement of the wave and path information is based on the same set of single photons, the properties illustrated in this scheme belong to the same photons, which is the key point for a loop-free experiment for verifying the wave-particle duality of single photons.

The simultaneous measure of visibility and predictability inevitably disturbs the system, and it causes some loss of wave-particle duality information. The loss of duality information is related to the measurement device. The more sensitivity and accuracy the device has, the less disturbance it causes, and the less wave-particle duality information the photon loses accordingly. It is shown that the loss of wave-particle duality information is equal to the mixedness of the density matrix of the photon after its interaction with the rotating density filters. The loss could not be zero, otherwise no path information is obtained. Thus the equality in the duality relation (1) can never be saturated under the simultaneous measurement of the fringe visibility and path predictability, even when the interfering photon is originally prepared in a pure state, which could be regarded as the price for the simultaneous measurement of the fringe visibility and path predictability. In other words, to realize the current scheme in an experiment and make the loss of the duality information small, a height standard on the spatial and temporal resolution of the measuring device is required, otherwise the wave and particle information encoded in the light beam is hard to extract. Another important point in this scheme is to prepare two circular density filters, whose transmission coefficient is related to the central angle. The quality of the two density filters determines the quality of the output light.

The demonstration of wave-particle duality of a single photon in an experiment depends not only on the state of the input photon, but also on what kind of measurement is made on it. This widely accepted statement is strongly supported by the experimental scheme proposed here. A full wave behavior may be observed if we only care about the interference, and a full particle behavior is illustrated if the photon propagates along a single line. However, if we intend to observe the wave and particle behaviors simultaneously, the wave-particle duality could not be fully demonstrated. Recently, the discussion on wave-particle duality has been extended to both a higher order case,[35,36] and a composite system.[37,38] It will be an interesting investigation to apply the current approach to those generalized duality experiments.

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