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Effect of Tidal Torques on Rotational Mixing in Close Binaries *

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The effect of tidal torques on rotational mixing in close binaries is investigated. It is found that spin angular momentum can attain a high value due to a strong tidal torque. Nitrogen and helium enrichment occurs early in the binary system that is triggered by tides. The stellar radius can reach a high value in the single star model with high initial velocities at the early stage of the evolution, but efficient rotational mixing can inhibit stellar expanding at the subsequent evolution. Central compactness is increased by the centrifugal force at the early stage of evolution but is reduced by rotational mixing induced by strong tides. The binary models with weak tides have high values of central temperature and stellar radius. Rotational mixing in single stars can slow down the shrinkage of convective cores, while convective cores can be expanded by strong tides in the binary system. Efficient rotational mixing induced by tides can cause the star to evolve towards high temperature and luminosity.

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Rotation is an important factor to be considered in the evolution of massive stars.[1,2] The centrifugal effect of rotation changes the shape of a star, deforming it to an oblate shape.[3–5] The radiative energy flux, therefore, and the effective temperature vary with the latitude.[6–8] In particular, rotation can trigger various instabilities inside the star. Rotational mixing is probably the most important effect of rotation in massive stars.[9–11] Fresh nitrogen and helium thus appear at the surface of the star and become continuously more enriched as a function of time during core hydrogen burning.[1,12]

In a close binary, angular momentum and kinetic energy can be exchanged between two stars and their orbits through tides. The system tends to evolve towards a state of minimum mechanical energy due to dissipative processes. This is an equilibrium state, where the orbit is circular, the spins of the stars are aligned and perpendicular to the orbital plane and the stars are in synchronous rotation with the orbital motion, such that $P_{\rm spin} = P_{\rm orbit}$. How quickly this equilibrium state is approached depends on the efficiency of the processes responsible for the energy dissipation. The most efficient form of energy dissipation takes place in turbulent regions of the star, such as convective layers, where the kinetic energy of the large-scale flow induced by the tides cascades down to smaller and smaller scales, until it is dissipated into heat. Zahn[13,14] estimated the timescale for synchronization due to this turbulent viscosity. In the absence of turbulent viscosity, another dissipative process is required to have efficient tides. If the star does not rotate synchronously, it experiences a varying gravitational potential, which triggers a large range of oscillations in the star. These oscillations are damped near the stellar surface by radiative dissipation. Zahn[15] derived the corresponding timescale for synchronization.

The early type massive stars considered in this work have radiative envelopes, and synchronization by radiative damping has been proposed to be the most efficient dissipation mechanism. However, in fast rotating stars, turbulence is induced by rotational instabilities and one may argue that the timescale that applies for turbulent viscosity can also be applied to these stars.[16,17] In addition, Witte and Savonije[18] showed that in some cases, resonance locking can contribute to even more efficient tidal dissipation. The aim of this work is to investigate how the tidal torque can affect rotational mixing in the early type massive binaries.

We consider a close binary consisting of two stars with masses M_1 and M_2 in a circular orbit with orbital period $P_{\rm orb}$. It is assumed that the spin axis of the model stars is perpendicular to the orbital plane.

For not yet synchronized binary systems, subjected to the effects of tides, the dissipation mechanisms play a major role. There are two different mechanisms to synchronize a star with the orbit: the equilibrium tide and the dynamical tide.[15] There are typically the viscous effect of the turbulence in stars with a convective envelope and the radiative damping for stars with a radiative envelope. A new expression for the tidal synchronization timescale due to a turbulent medium has been proposed as[16]

$$t_{\rm syn,turb} = f_{\rm turb} q^{-2} \left(\frac{a}{R_1}\right)^6 \text{year}, \qquad (1)$$

where a is the separation between two components in a binary system, the quantity q is $\frac{M_2}{M_1}$, R_1 is the radius of the star to be synchronized, and $f_{\rm turb} \sim 1.0$ depends on the structure of the star.

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The dynamical synchronization timescale due to radiative damping for stars with a radiative envelope has been given by[13,15]

$$\frac{1}{t_{\rm syn,dyn}} = 3\Big(\frac{GM_1}{R_1^3}\Big)^{1/2}\frac{M_1R_1^2}{I_1}E_2\Big[q^2\Big(\frac{R_1}{a}\Big)^6\Big]s_{22}^{5/3}, \quad (2)$$

where s_{22} is the tidal frequency $s_{22} = 2(\Omega_1 - \omega_{\rm orb})(\frac{R_1^6}{GM_1})^{1/2}$, Ω_1 and $\omega_{\rm orb}$ are the angular velocities of axial and orbital rotation, respectively, I_1 is the moment of inertia, and E_2 is $E_2 = 10^{-1.37}(\frac{r_{\rm conv,1}}{R_1})^8$ with $r_{\rm conv,1}$ being the radius of convective cores.

The change of the spin angular momentum of the accreting star due to tidal interaction is expressed as

$$\dot{J}_{1,\rm tide} = I_1\frac{d(\Omega_1 - \omega_{\rm orb})}{t_{\rm syn}}, \quad (3)$$

where $t_{\rm syn}$ can be calculated according to Eq. (1) (strong tides) or Eq. (2) (weak tides). Generally, Eq. (1) can be applied to the stars with an envelope hosting a strong turbulence due to convection. However, we also choose to use the ansatz for the equilibrium tide for determining $t_{\rm syn}$ in the star with the radiative envelope, although it was originally designed for convective stars, for two reasons. First, in the radiative envelopes of rapidly rotating stars, as considered here, rotation is supposed to produce a high level of turbulence, implying that turbulent viscosity dominates radiative viscosity.[11] Secondly, Toledano *et al.*[16] applying a kinematic model of tidal interaction to early type stars, found that the tidal time scale in these models indeed scales as $(a/R)^6$.

The loss of the spin angular momentum via stellar wind is given by

$$\dot{J}_{1,\rm wind} = \frac{2}{3}\dot{M}_{1,\rm w}\Omega_1R_1^2, \quad (4)$$

where $\dot{M}_{1,\rm w}$ is the mass-loss rate by stellar wind.[19]

The variation of spin angular momentum of the primary star can be expressed as

$$\frac{dJ_1}{dt} = \dot{J}_{1,\rm tide} - \dot{J}_{1,\rm wind}. \quad (5)$$

The variation of orbital separation in a non-synchronous binary can be written as

$$\frac{\dot{a}}{a} = \frac{4}{3}\Big(\frac{R_1}{a}\Big)^2\frac{M_1+M_2}{M_1M_2}\dot{M}_{1,\rm wind} - 2\frac{\dot{J}_{1,\rm tide}}{J_{\rm orb}} - \frac{\dot{M}_{1,\rm w}}{M_1+M_2}, \quad (6)$$

where $J_{\rm orb}$ is the orbital angular momentum and $J_{\rm orb} = M_1M_2(\frac{Ga}{M_1+M_2})^{1/2}$. For simplicity, in the calculation we focus on the evolution of the primary star. The companion star is treated as a mass point because the orbital variation induced by the secondary star is less than 2% of the initial orbital separation.

In a rotating star, the equation of transport of angular momentum can be written as

$$\rho\frac{\partial}{\partial t}(r^2\Omega)_{M_r} = \frac{1}{5r^2}\frac{\partial}{\partial r}(\rho r^4\Omega U_2(r)) + \frac{1}{r^2}\frac{\partial}{\partial r}\Big(\rho Dr^4\frac{\partial\Omega}{\partial r}\Big), \quad (7)$$

where the first term on the right-hand side is the divergence of the advected flux of angular momentum, while the second term is the divergence of the diffused flux, and $D = D_{\rm shear} + D_{\rm conv}$ is the total diffusion coefficient in the vertical direction, taking into account the various instabilities that transport angular momentum.[20,21] The convection coefficient $D_{\rm conv}$ is $D_{\rm conv} = \min(\frac{1}{2}H_P[\frac{3}{8}\frac{L}{4\pi r^2}g\nabla_{\rm ad}\frac{H_P}{P}]^{1/3}, \frac{\Delta r}{\Delta t})$. The shear mixing coefficient $D_{\rm shear}$ can be given by Ref. [22].

The meridional circulation U_2 is most efficient for angular momentum transports, and its velocity was determined by Refs. [11,23]. Note that tidal torques (i.e., Eq. (5)) have been included in the outer boundary condition of the equation for angular momentum transport.

Chaboyer and Zahn[24] showed that the horizontal turbulence completes efficiently with the advective term of the meridional circulation. The horizontal flow tends to homogenize the layer in such a way that the transport of chemical species by both meridional circulation and horizontal turbulence can be computed as a diffusive process,

$$\rho\frac{dX_i}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(\rho r^2[D+D_{\rm eff}]\frac{\partial X_i}{\partial r}\Big) + \rho\Big(\frac{dX_i}{dt}\Big)_{\rm nucl}, \quad (8)$$

where $D_{\rm eff} = \frac{1}{30}\frac{|rU_2|^2}{D_h}$ is the effective mixing coefficient for the chemical species.

The treatment of rotation was developed in a series of works published previously by the Geneva group.[1,12,22,25] The initial masses of the system components were set at $15M_\odot$ and $10M_\odot$ with the metallicity of the Sun $X = 0.71$ and $Z = 0.014$. The initial orbital period is chosen as 1.4 d and initial spin velocities are 300 km/s for the spin-down case in which the spin angular velocity is greater than the orbital one and 120 km/s for the spin-up case in which the spin angular velocity is smaller than the orbital one. The evolution is followed from the onset of central H burning to the end of the main sequence for single stars and to the moment of Roche lobe overflow (RLOF) for the evolution of binaries.

Figure 1 shows the spin angular momentum as a function of time for single stars and binary models. We can find that spin angular momentum for single stars approximately maintains a constant value in the main sequence because there is a little spin angular momentum loss due to weak stellar winds. For the spin-up case, spin angular momentum in model B2 with strong tides can increase faster than the one in B1 with weak tides. The result implies that the orbital separation is short in B2 because more orbital angular

momentums have been transformed into the spin angular momentum of two stars. However, the spin angular momentum of the primary star in B4 decreases faster before 2.0 Myr than the one in B3 because the dynamical synchronization timescale due to radiative damping is ∼3 orders of magnitude greater than the timescale for synchronization due to turbulent viscosity. The results indicate that the orbital separation is longer in B4 than the one in B3 because of the angular momentum transformation from the spin to the orbit. After that, the star in B4 has a larger spin angular momentum because strong tidal torques can cause the star to approach the orbital velocity.

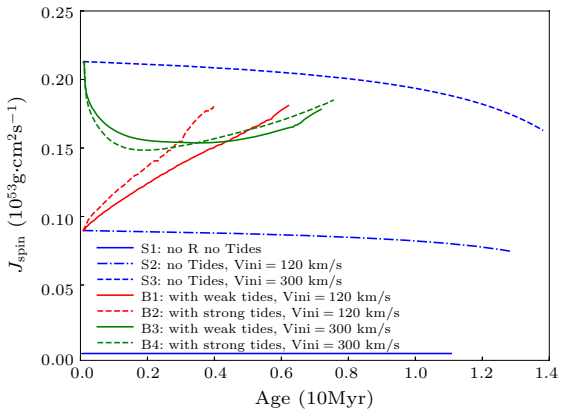

Fig. 1. Spin angular momentum as a function of time for three single models. The initial velocities for the models of a single star are set to be $V_{\rm ini} = 0$ km/s, $V_{\rm ini} = 120$ km/s, and $V_{\rm ini} = 300$ km/s, respectively. The initial velocities for binary models B1 and B2 are $V_{\rm ini} = 120$ km/s while the initial velocities for binary models B3 and B4 are $V_{\rm ini} = 300$ km/s. The primary star is imposed by a weak tidal torque (i.e., Eq. (2) is used) in models B1 (or B2) and B3 (or B4) while the primary star is imposed by a strong tidal torque (i.e., Eq. (1) is used).

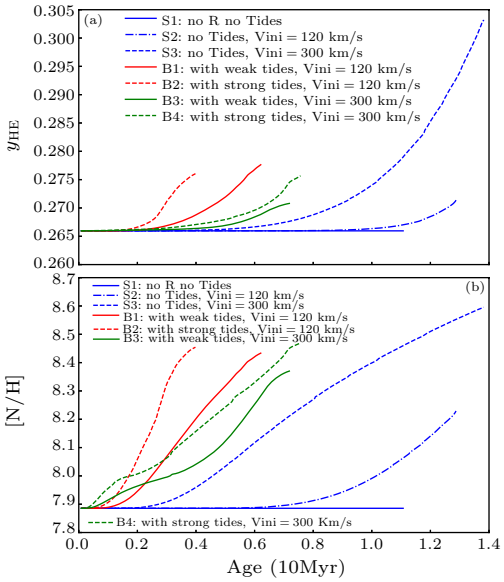

Fig. 2. (a) Surface helium abundance as a function of time for a single star and binaries. (b) Surface nitrogen abundance as a function of time for the single star and binary models. The lines have the same meaning as the ones in Fig. 1.

Figure 2(a) shows the surface helium abundance versus the evolutionary time. Figure 2(b) illustrates the surface nitrogen abundance versus the evolutionary time. The results indicate that helium and nitrogen enrichment go up with the increase of the initial velocity and evolutionary age in single stars. The nitrogen enrichment is stronger than the helium enrichment. For example, the helium enrichment factor is 0.14 whereas the nitrogen enrichment factor is 5.25 in S3 at the end of the main sequence. There are two reasons: firstly, the production of helium proceeds on the nuclear timescale. This reaction is slower than the production of nitrogen through the CN cycle, which proceeds a very short timescale. Secondly, with the production of helium, a steep gradient in mean molecular weight is established at the boundary between the core and the envelope, which can drastically inhibit the helium diffusion.

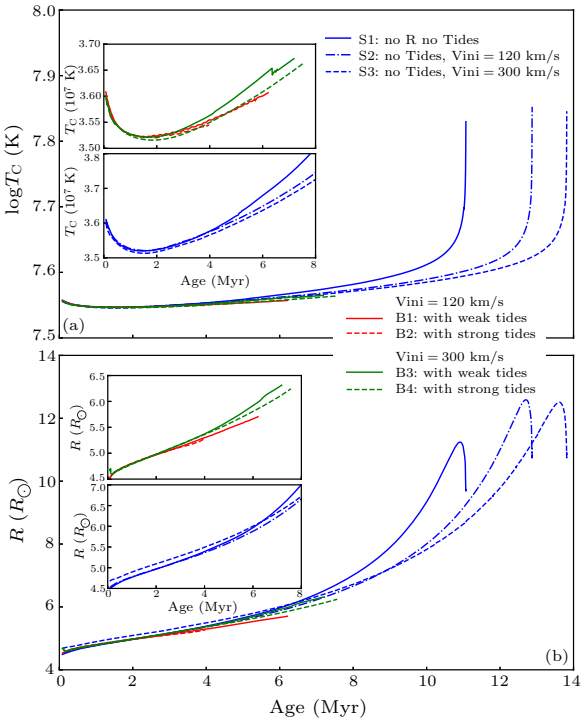

Fig. 3. (a) Central temperature as a function of time for a single star and binary models. (b) Stellar radius as a function of time for the single star and binary models. The lines have the same meaning as the ones in Fig. 1.

We also find that helium and nitrogen enrichment occurs earlier in binaries than the counterpart of single stars. Comparing B3 with B4, we find that helium and nitrogen enrichment occurs early in the star which is imposed by strong tidal torques. The reason is that shear turbulence induced by strong tidal torques during synchronization can efficiently transport chemical elements from the stellar core to the surface. Comparing B2 with B4, one can find that nitrogen enrichment occurs early in B4 at the beginning of evolution. The increase of helium induced tidal shear leads to the decrease of the opacity in the radiative envelope, resulting in more luminous and more compact stars. Comparing the star in B2 with the counterpart in B1,

we find that moderate different rotations can maintain in model B1 with weak tides after synchronization and then shear turbulence can efficiently transfer nitrogen to the surface.

Figure 3(a) shows the central temperature versus the evolutionary time. For single stars, the central temperature decreases with the increase of the initial rotational velocity. This can be understood by the fact that the centrifugal force can decrease stellar effective mass. Moreover, fresh hydrogen can be efficiently transferred from the envelope to the central core in the fast rotating model. Therefore, the lifetime of the main sequence is extended for rapidly rotating stars. Furthermore, the models with weak tides have a higher central temperature than the ones with weak tides.

Figure 3(b) illustrates the stellar radius versus the evolutionary time. For single stars, the stellar radius rapidly expands from zero age main sequence to the age of 6.0 Myr with the help of strong centrifugal forces (rapid rotation). At the subsequent stage, the stellar radius is larger in the non-rotating star because the high central temperature plays the most important role in stellar evolution. We also find that stellar radius is smaller in the binary model with strong tide because rotational mixing is efficient. The opacity is smaller in the radiative envelope, which decreases the expanding speed.

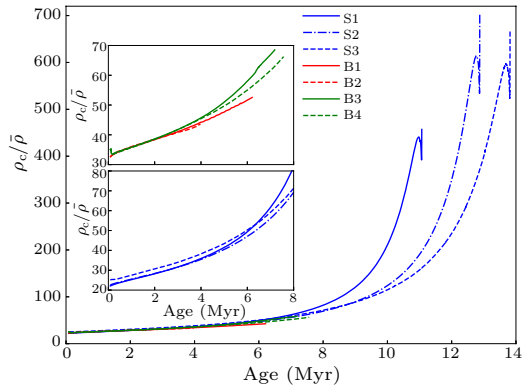

Fig. 4. Central compactness as a function of time for the single star and binary models. The lines have the same meaning as the ones in Fig. 1.

Figure 4 illustrates that the central compactness varies with the evolutionary time. For single stars, the central compactness increases with the initial rotational velocity from ZAMS to the age of 6.0 Myr. This can be attributed to the fact that rapid rotation can cause stars to become a central compact core because, to offset higher centrifugal forces, more matter tends to deposit towards the core for increasing the gravity.[3] Therefore, rapid rotation is not in favor of fast apsidal motion, which has been found by Claret.[26] At the subsequent stage, the central compactness has a lower value in the rotating model because rotational mixing tends to produce large values of mean density. At the subsequent stage, the central compactness is larger in the model with weak tides because strong rotational mixing leads to large mean density.

Figure 5(a) shows the mass for convective cores versus the evolutionary time. At the beginning of evolution, one finds that rapid rotation can decrease the convective core in single rotating stars. The reason is that radiative pressure can be greatly decreased by the centrifugal force. However, at the subsequent stage, the convective core is larger in the rapidly rotating model S3. The reason is that rotational mixing brings fresh H fuel into the convective core, slowing down its decrease in core mass during the main sequence. For a given value of central H content, the mass of convective cores in the rotating model is therefore larger than in the non-rotating one. Comparing B3 with S3 at the beginning of evolution, one can find that the model B3 with tides contributes to the high value of convective cores due to an efficient rotational mixing. This effect is similar to that of overshoot.

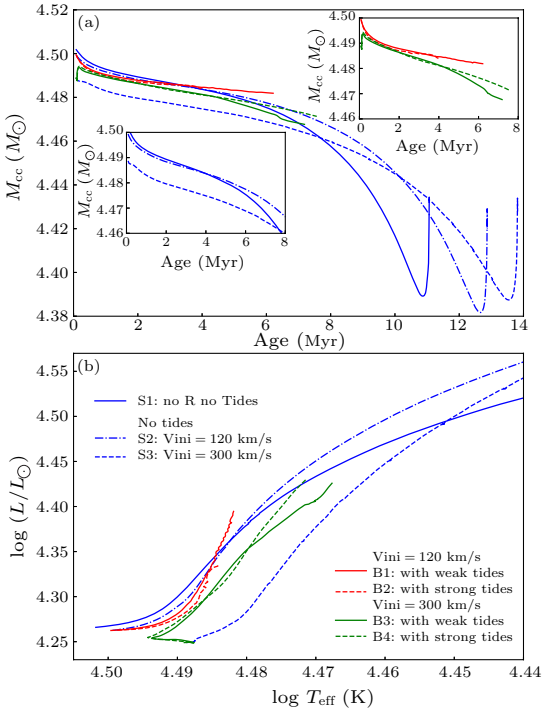

Fig. 5. (a) The mass of convective core as a function of time for the single star and binary models. (b) The evolution of the single star and binary models in HR diagram. The lines have the same meaning as the ones in Fig. 1.

Figure 5(b) shows the evolution of a single star and binary models in the HR diagram. For a single rotating star, rapid rotation produces a shift of tracks towards lower luminosities and effective temperature on and near the ZAMS, and this effect is due to the gravitational darkening. The lowering of effective gravity contributes to a low luminosity. When evolution proceeds, the luminosity of the non-rotating star is lower than the counterpart with rotation. The result indicates that rotating mixing can inhibit the shrinkage of convective cores. The evolutionary tracks B3 or B4 with high initial velocity (i.e., $v_{\rm ini}$ = 300 km/s) and

tidal interactions are shifted to the blue with respect to the track S3 without tidal interaction. The results indicate that the mass-age-luminosity relation is significantly influenced by tidal mixing, in the sense of an over-luminosity for a given mass in the case of tidal braking. There is a shift of tidally accelerated models B2 toward redder positions than for the model S2 without tidal interaction. This is because globally the stars are rapidly spun up and the centrifugal acceleration becomes considerable. The centrifugal acceleration balances the gravity. This tends to make the star follow an evolution corresponding to a lower initial mass, thus with a lower luminosity.

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