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Dipole Oscillation in a Bose–Fermi Superfluid Mixture of ^{41}K and ^{6}Li

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We study the relation between the bosonic frequency shift of a dipole oscillation and the fermionic compressibility in a Bose–Fermi superfluid mixture. Our results show that the Λ transition occurs in such a system, which interprets the perplexing phenomenon observed in the recent experiment [arXiv:1705.04496]. In that experiment, the frequency shift of the bosonic gas was measured with different fermion-fermion s-wave scattering lengths $a_{\rm f}$. The most striking feature of their measurement is that the frequency shift shows a puzzling non-monotonic behavior around $1/k_{\rm F}a_{\rm f} \simeq -0.2$ with $k_{\rm F}$ being the Fermi momentum. In the present work, the relation between the bosonic frequency shift and the compressibility of the Fermi gas is revealed by means of the equation of state of the superfluid mixture. Making use of the relation, we find that the compressibility of Fermi gas shows a Λ-like shape around $1/k_{\rm F}a_{\rm f} \simeq -0.2$. We would like to stress that no free parameter is used in our calculation and our results can be further testified in a future experiment.

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Realizing a Bose–Fermi superfluid mixture in bosonic ^{4}He and fermionic ^{3}He superfluid is still elusive,[1,2] although it was proposed over 40 years ago.[3,4] However, the achievement of realizing superfluid in atomic Bose–Fermi gases demonstrates that an ultracold atomic gas system is an ideal platform to study quantum matter, by means of which the knowledge on superfluids has been significantly enriched.[5,6] As a result, the enthusiasm of searching for Bose–Fermi superfluid mixtures has been rekindled by the rapid advances of ultracold atomic gases.

Recently, a Bose–Fermi superfluid mixture was firstly realized in quantum gas composed by ^{7}Li and ^{6}Li, which has aroused intensive theoretical and experimental interest.[7] A Bose–Fermi superfluid mixture composed by ^{6}Li and different bosonic gases has also been observed. For instance, dual superfluid was observed in the mixtures of ^{41}K-^{6}Li and ^{174}Yb-^{6}Li as well.[8,9] Furthermore, a coupled vortex lattice has been observed when the superfluid mixture is rotated, which has also been investigated theoretically.[10,11] The dual superfluid of ^{174}Yb-^{6}Li features different electronic structures of Boson and Fermion and a large mass ratio, which is proposed to facilitate enhancing the interaction effect.[12] The Bose–Fermi superfluid mixture in the ultracold atomic system is proposed to give rise to a plethora of interesting phenomena. On the few-body level, the investigation on atom-dimer scattering reveals a universal relation between the atom-dimer scattering length and the boson-fermion scattering length. This relation is strongly related to the dipole oscillation frequency shift which can be measured experimentally.[12–14] On the many-body level, various quasiparticle excitations are analyzed, which are related to the various collective modes observed in experiments.[15–17]

In all of the experiments, one of the most important measurements is the dipole oscillation frequency shift in the superfluid mixture. Both Bose–Fermi gases are trapped in an optical dipolar trap in the experiment. Near the minimum of the trap potential, it can be approximated as a harmonic trap. In general, the harmonic frequencies of Bose–Fermi gases are different and independent of each other in the absence of a Bose–Fermi interaction. However, under practice conditions, Bose–Fermi gases are interacting with each other, which leads to coupling between the bosonic and fermionic dipole modes. As a result, by measuring the dipole oscillation frequency shift of the mixture, we can reveal the interaction between Bose–Fermi gases. Furthermore, the exotic collective mode can be analyzed when the interaction strength is known. Thus the measurement of dipole oscillation frequency shift plays an important role in the study of Bose–Fermi superfluid mixture.

In the very recent experiment by Wu *et al.*,[18] a superfluid mixture of ^{41}K and ^{6}Li is prepared in a cigar-shaped optical dipolar trap. The frequency shifts of Bose–Fermi gases are measured on both the radial and axial directions with high precision. Thanks to the existence of the Feshbach resonance between the two components of the fermion, the s-wave scattering length $a_{\rm f}$ can be tuned. The most striking feature of the measurement is that around $1/k_{\rm F}a_{\rm f} \simeq -0.2$, the frequency shift shows a puzzling non-monotonic behavior, where $k_{\rm F}$ is the Fermi momentum. An immediate question is: what is the underlying mechanism for this phenomenon?

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To address this question, we start with the equation of state (EoS) of the superfluid mixture and reveal the relation between the frequency shift and the compressibility. By fitting the bosonic radial frequency shift data with the relation, we obtain the compressibility of Fermi gas as a function of the scattering length. Surprisingly, it is found that around $1/k_{\rm F}a_{\rm f} \simeq -0.2$, the compressibility of Fermi gas shows a Λ-like shape, which indicates the phase transition between the superfluid and the normal state.[19] We would like to emphasize that there is no free parameter in our calculation. As a result, we assert the underlying mechanism for the puzzling behavior of frequency shift is that, at a particular temperature, the system undergoes a transition from superfluid to the normal state when the scattering length is tuned from the BEC side to BCS side, as shown in Fig. 1. For this experiment, the phase transition point locates around $1/k_{\rm F}a_{\rm f} \simeq -0.2$. According to this interpretation, we can predict that the non-monotonic behavior of frequency shift is still retained for lower temperature, while the location will shift toward the BCS side. However, when the temperature is high enough, the non-monotonic behavior of frequency shift will disappear. In the phase transition between superfluid and the normal state, the specific heat and fraction of condensation show discontinuity as well and the discontinuity should also appear around $1/k_{\rm F}a_{\rm f} \simeq -0.2$ under the present experimental condition. Future experiments can test these predictions by choosing different temperatures.

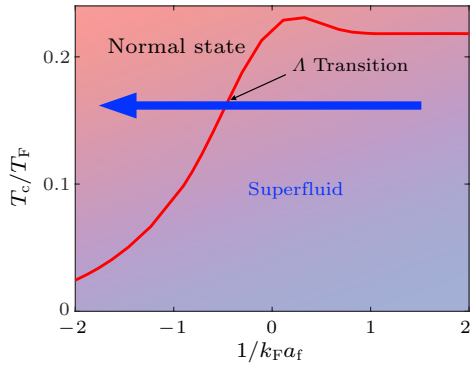

Fig. 1. Phase diagram of the two-component Fermi gas with s-wave interaction. The red solid curve is $T_{\rm c}$ between superfluid and the normal state. When the scattering length $a_{\rm f}$ is tuned from the BEC side to the BCS side at a particular temperature, as shown by the blue arrow, the Fermi gas undergoes a phase transition from the superfluid phase to the normal phase. At the transition point, the compressibility of the Fermi gas shows a Λ-like shape.

In the experiment,[18] both ^{41}K and ^{6}Li are trapped in a cigar-shaped optical dipolar trap. In the radial direction, the trap can be approximated as a harmonic trap. Due to the presence of the interaction between boson and fermion, the effective potential seen by the boson (fermion) is the sum of the bare trapping potential $V_{\rm b(f)}(r)$ and the interaction energy with fermion (boson) $g_{\rm bf}n_{\rm f(b)}(r)$. Thus the EoSs of boson and fermion can be written as

$$n_{\rm b}(r) = \frac{1}{g_{\rm bb}}\Big[\mu_{\rm b}^0 - \frac{1}{2}m_{\rm b}\omega_{\rm b}^2 r^2 - g_{\rm bf}n_{\rm f}(r)\Big], \quad (1)$$

$$n_{\rm f}(r) = n_{\rm f}\Big(\mu_{\rm f}^0 - \frac{1}{2}m_{\rm f}\omega_{\rm f}^2 r^2 - g_{\rm bf}n_{\rm b}(r)\Big), \quad (2)$$

where we have used the local density approximation (LDA), which is reasonable when the potential length is much larger than the cloud size, $n_{\rm b}(r)$ and $n_{\rm f}(r)$ are the spatial distributions of boson and fermion, $m_{\rm b}$ and $m_{\rm f}$ are the atom masses of boson and fermion, respectively, $g_{\rm bb} = \frac{4\pi\hbar^2 a_{\rm bb}}{m_{\rm b}}$ is the boson-boson interaction strength with the scattering length $a_{\rm bb}$, $g_{\rm bf} = \frac{2\pi\hbar^2 a_{\rm bf}}{m_{\rm bf}}$ is the boson-fermion interaction strength with $a_{\rm bf}$ being the scattering length, and $m_{\rm bf} = \frac{m_{\rm b}m_{\rm f}}{m_{\rm b}+m_{\rm f}}$ being the reduced mass of boson and fermion, $\omega_{\rm b(f)}$ is the frequency of boson (fermion) in the absence of the boson-fermion interaction, and $\mu_{\rm b(f)}^0$ is the chemical potential of boson (fermion) at the center of the trap. To obtain the solution of the EoS, we need to find the expression of $n_{\rm f}(r)$. However, there is no reliable method to determine $n_{\rm f}(r)$ for strongly interacting Fermi gas in the superfluid mixture system.

In the current experiment, the cloud size of the fermion is much larger than that of the boson. In practice, the trapping potential is smaller than $\mu_{\rm f}^0$ over the range of Bose gas. Thus, we can expand $n_{\rm f}(r)$ near the center of the trap to the linear order as

$$n_{\rm f}(r) \simeq n_{\rm f}^0 - \kappa_{\rm f}(n_{\rm f}^0)^2\Big[\frac{1}{2}m_{\rm f}\omega_{\rm f}^2 r^2 + g_{\rm bf}n_{\rm b}(r)\Big], \quad (3)$$

where $n_{\rm f}^0$ is the density of the fermion at the center of the trap, $\kappa_{\rm f} \equiv \frac{1}{(n_{\rm f}^0)^2}\frac{\partial n_{\rm f}^0}{\partial \mu_{\rm f}}|_T$ is the compressibility of Fermi gas. The compressibility of unitary Fermi gas has been studied experimentally and the Λ transition has been observed.[20] We find that the spatial distribution of fermion $n_{\rm f}(r)$ is related to the compressibility and that the frequency of fermion can be shifted by the boson-fermion interaction.

In an experiment, it is quite difficult to measure the interaction energy directly. However, the dipole oscillation of quantum gas can be well detected and the interaction energy can be deduced if the frequency shift is known. To this end, we first solve Eqs. (1) and (3) consistently and the solutions can be written as

$$n_{\rm b}(r) = \frac{C}{g_{\rm bb}}\Big([\mu_{\rm b}^0 - g_{\rm bf}n_{\rm f}^0(\mu_{\rm f}^0)] - \frac{1}{2}m_{\rm b}\Big[1 - g_{\rm bf}(n_{\rm f}^0)^2\kappa_{\rm f}\frac{m_{\rm f}}{m_{\rm b}}\frac{\omega_{\rm f}^2}{\omega_{\rm b}^2}\Big]\omega_{\rm b}^2 r^2\Big), \quad (4)$$

$$n_{\rm f}(r) = C\Big[n_{\rm f}^0 - \frac{g_{\rm bf}}{g_{\rm bb}}(n_{\rm f}^0)^2\kappa_{\rm f}\mu_{\rm b}^0\Big] - (n_{\rm f}^0)^2\kappa_{\rm f} \cdot \frac{1}{2}m_{\rm f}C\Big[1 - \frac{g_{\rm bf}}{g_{\rm bb}}\frac{m_{\rm b}}{m_{\rm f}}\frac{\omega_{\rm b}^2}{\omega_{\rm f}^2}\Big]\omega_{\rm f}^2 r^2, \quad (5)$$

where the expression of the coefficient C is

$$C = \Big[1 - \frac{g_{\rm bf}^2}{g_{\rm bb}}(n_{\rm f}^0)^2\kappa_{\rm f}\Big]^{-1}. \quad (6)$$

Being inspired by Eqs. (4) and (5), we introduce the concept of effective frequency for boson and fermion and recast the EoS in a more inspiring form

$$n_{\rm b}(r) = \frac{1}{g_{\rm bb}}\Big[\tilde{\mu}_{\rm b}^0 - \frac{1}{2}m_{\rm b}\tilde{\omega}_{\rm b}^2 r^2\Big], \quad (7)$$

$$n_{\rm f}(r) = \tilde{n}_{\rm f}^0 + \kappa_{\rm f}(n_{\rm f}^0)^2\frac{1}{2}m_{\rm f}\tilde{\omega}_{\rm f}^2 r^2, \quad (8)$$

where $\tilde{\omega}_{\rm b(f)}$ is the effective boson (fermion) frequency, which takes into account the effect of boson-boson interaction and boson-fermion interaction. It should be noticed that the boson density profile is reverse parabola-shaped, while the fermion density profile is parabola-shaped since the presence of bosonic gas is near the center of the trap. This configuration is consistent with that in the experiment. Comparing Eqs. (7) and (8) with Eqs. (4) and (5), the expression of the effective frequency can be written as

$$\tilde{\omega}_{\rm b} = \sqrt{C}\Big[1 - g_{\rm bf}\kappa_{\rm f}(n_{\rm f}^0)^2\frac{m_{\rm f}}{m_{\rm b}}\frac{\omega_{\rm f}^2}{\omega_{\rm b}^2}\Big]^{1/2}\omega_{\rm b}, \quad (9)$$

$$\tilde{\omega}_{\rm f} = \sqrt{C}\Big[\frac{g_{\rm bf}}{g_{\rm bb}}\frac{m_{\rm b}}{m_{\rm f}}\frac{\omega_{\rm b}^2}{\omega_{\rm f}^2} - 1\Big]^{1/2}\omega_{\rm f}. \quad (10)$$

It can be seen that in the absence of the boson-fermion interaction, the effective frequency is the same as the bare frequency. The frequency shift of Bose gas is defined as $\delta\omega_{\rm b} = \omega_{\rm b} - \tilde{\omega}_{\rm b}$, which can be expressed as

$$\frac{\delta\omega_{\rm b}}{\omega_{\rm b}} = 1 - \sqrt{C}\Big[1 - g_{\rm bf}\kappa_{\rm f}(n_{\rm f}^0)^2\frac{m_{\rm f}}{m_{\rm b}}\frac{\omega_{\rm f}^2}{\omega_{\rm b}^2}\Big]^{1/2}. \quad (11)$$

After some straightforward algebra, one immediately obtains the expression of compressibility of Fermi gas

$$\kappa_{\rm f} = \frac{\frac{\delta\omega_{\rm b}}{\omega_{\rm b}}\big(\frac{\delta\omega_{\rm b}}{\omega_{\rm b}} - 2\big)}{(n_{\rm f}^0)^2\big[\frac{g_{\rm bf}^2}{g_{\rm bb}}\big(\frac{\delta\omega_{\rm b}}{\omega_{\rm b}} - 1\big)^2 - g_{\rm bf}\frac{m_{\rm f}}{m_{\rm b}}\frac{\omega_{\rm f}^2}{\omega_{\rm b}^2}\big]}. \quad (12)$$

It can be seen that the compressibility of Fermi gas is relevant to the frequency of Bose gas. For a given system, such as the mixture of ^{41}K and ^{6}Li, the mass ratio $m_{\rm f}/m_{\rm b}$ is known and the boson-boson interaction $g_{\rm bb}$, the boson-fermion interaction $g_{\rm bf}$ can be tuned by the external magnetic field. Thus the compressibility of Fermi gas can be obtained by measuring the density of the Fermi gas and the frequency shift of the Bose gas.

In the experiment, the boson particle number is $N_{\rm b} = 2.3 \times 10^5$ and the fermion particle number is $N_{\rm f} = 1 \times 10^6$,[18] and the density of ^{6}Li at the trap center is about $3.04 \times 10^{12}\,{\rm cm}^{-3}$.[21] The scattering length of boson-boson is $a_{\rm bb} = 60.5a_0$, and the scattering length of boson-fermion is $a_{\rm bb} = 60.2a_0$, where a_0 is Bohr's radius. The bare frequency shifts of boson and fermion are $\omega_{\rm b} = 2\pi \times 170.7\,{\rm Hz}$ and $\omega_{\rm f} = 2\pi \times 295.4\,{\rm Hz}$, respectively. Making use of the bosonic frequency experimental data on the radial direction, one obtains the compressibility of Fermi gas with different scattering lengths. As shown in Fig. 2, the compressibility shows a conspicuous Λ-like shape around $1/k_{\rm F}a_{\rm f} \simeq -0.2$, which can serve as a smoking gun of the phase transition between superfluid and normal phase. It is pointed that no free parameter is used in our calculation. Another interesting feature of the result is that the compressibility of interacting Fermi gas is almost the same as that of the noninteracting Fermi gas. This is consistent with the experimental result in unitary Fermi gas.[20]

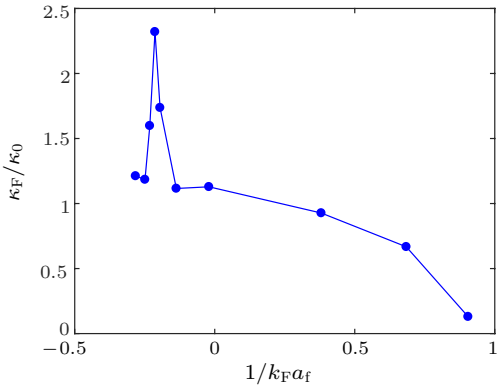

Fig. 2. The Λ transition in the Bose–Fermi superfluid mixture. When the scattering length is tuned near $1/k_{\rm F}a_{\rm f} \simeq -0.2$, the compressibility of Fermi gas shows a Λ-like shape, which denotes the phase transition from superfluid to the normal state. Here $\kappa_0 = \frac{3}{2}\frac{1}{n_{\rm f}^0 E_{\rm F}}$ is the compressibility of the noninteracting Fermi gas at $T = 0$ with $n_{\rm f}^0$ being the density of Fermi gas and $E_{\rm F}$ being the Fermi energy.

In our calculation, we only use the frequency shift of Bose gas in the radial direction. The reason is as follows: (1) because the fermion cloud size is larger than or comparable with the length of trap, the shape of the cloud will change significantly when they are oscillating, which makes the parabola-shaped density profile approximation work poorly. More precise descriptions of the fermion density profile are required. (2) On the axial direction, the repulsive interaction between fermion and boson makes the Bose gas off-centered, which makes the density profile of Bose gas not reverse parabola-shaped anymore. In the experiment, the temperature of the Fermi gas is about $0.06T_{\rm F}$. However, when the cloud is oscillating, the temperature will increase. Using our theory and experimental

data, we predict that the temperature of the oscillating Fermi gas is about $0.2T_{\rm F}$.

In summary, we have studied the dipolar oscillation frequency shift of a Bose–Fermi superfluid mixture. Starting with the EoS of the dual superfluid mixture, we reveal the relation between frequency shift of Bose gas on the radial direction and the compressibility of Fermi gas. Fitting the experimental data with our theory, we find the Λ-like shape of the compressibility near $1/k_{\rm F}a_{\rm f} \simeq -0.2$, which denotes the phase transition from the superfluid phase of Fermi gas to the normal state. Our theory interprets the puzzling non-monotonic behavior of the compressibility, and it can be further tested in future experiments.

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References

[1] Tuoriniemi J, Martikainen J, Pentti E, Sebedash A, Boldarev S and Pickett G 2002 *J. Low Temp. Phys.* **129** 531
[2] Rysti J, Tuoriniemi J and Salmela A 2012 *Phys. Rev.* B **85** 134529
[3] Andreev A and Bashkin E 1975 *Zh. Eksp. Teor. Fiz.* **69** 319
[4] Volovik G, Mineev V and Khalatnikov I 1975 *Sov. Phys. JETP* **69** 675
[5] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463
[6] Giorgini S, Pitaevskii L P and Stringari S 2008 *Rev. Mod. Phys.* **80** 1215
[7] Ferrier-Barbut I, Delehaye M, Laurent S, Grier A T, Pierce M, Rem B S, Chevy F and Salomon C 2014 *Science* **345** 1035
[8] Yao X C, Chen H Z, Wu Y P, Liu X P, Wang X Q, Jiang X, Deng Y, Chen Y A and Pan J W 2016 *Phys. Rev. Lett.* **117** 145301
[9] Roy R, Green A, Bowler R and Gupta S 2017 *Phys. Rev. Lett.* **118** 055301
[10] Jiang Y, Qi R, Shi Z Y and Zhai H 2017 *Phys. Rev. Lett.* **118** 080403
[11] Pan J S, Zhang W, Yi W and Guo G C 2017 *Phys. Rev.* A **95** 063614
[12] Zhang R, Zhang W, Zhai H and Zhang P 2014 *Phys. Rev.* A **90** 063614
[13] Cui X 2014 *Phys. Rev.* A **90** 041603(R)
[14] Wen W, Chen B and Zhang X 2017 *J. Phys. B: At. Mol. Opt. Phys.* **50** 035301
[15] Delehaye M, Laurent S, Ferrier-Barbut I, Jin S, Chevy F and Salomon C 2015 *Phys. Rev. Lett.* **115** 265303
[16] Zheng W and Zhai H 2014 *Phys. Rev. Lett.* **113** 265304
[17] Shen H and Zheng W 2015 *Phys. Rev.* A **92** 033620
[18] Wu Y P, Yao X C, Liu X P, Wang X Q, Wang Y X, Chen H Z, Deng Y, Chen Y A and Pan J W 2017 arXiv:1705.04496v1[cond-mat.quant-gas]
[19] Lipa J A, Nissen J A, Stricker D A, Swanson D R and Chui T C P 2003 *Phys. Rev.* B **68** 174518
[20] Ku M J H, Sommer A T, Cheuk L W and Zwierlein M W 2012 *Science* **335** 563
[21] Private communication with Yao Xingcan