Interaction between Breathers and Rogue Waves in a Nonlinear Optical Fiber *

Xiang-Shu Liu(刘祥树)[1,2], Li-Chen Zhao(赵立臣)[1,3], Liang Duan(段亮)[1,3], Peng Gao(高鹏)[1,3], Zhan-Ying Yang(杨战营)[1,3**], Wen-Li Yang(杨文力)[3,4]

[1]*School of Physics, Northwest University, Xi'an 710069*
[2]*Faculty of Science, Qinzhou University, Qinzhou 535000*
[3]*Shaanxi Key Laboratory for Theoretical Physics Frontiers, Xi'an 710069*
[4]*Institute of Modern Physics, Northwest University, Xi'an 710069*

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We study the interaction between breather and N-order rogue waves in a nonlinear optical fiber. The impacts of the relative phase and the interaction distance between breathers and rogue waves are discussed in detail. Specifically, the breather can reduce the maximum hump value of high-order rogue waves greatly in the cases of nonzero relative phase or nonzero interaction distance. The characteristic of exclusion between breathers and rogue waves is described qualitatively in the situation of different interaction distances, which can be used to change the temporal-spatial distribution of rogue waves. Their interaction properties are characterized by the trajectory of localized waves' valleys and humps. It is shown that the interaction changes the dynamical evolution trajectory of rogue waves and breathers. These results provide some possible ways to control high-order rogue waves.

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Nonlinear localized waves in nonlinear optical fibers have been a hot research topic in recent years.[1–8] Based on the nonlinear Schrödinger equation (NLSE), several basic localized waves such as solitons, breathers, and rogue waves (RWs), have been revealed theoretically[9–17] and observed experimentally.[2,3,18–25] Moreover, the studies of interaction between different localized waves can exhibit much richer localized wave structures and dynamic characteristics.[26–35] The interaction between solitons has been studied extensively,[36] which revealed the wave particle duality of the soliton.[37,38] Particle-like properties are shown in the elastic collision between solitons where the shape of the soliton remains unchanged before and after their collision.[39] Wave properties are presented when solitons interfere with each other in the interaction process,[40] which are used to design soliton interferometers.[41] Some inelastic interaction structures have been revealed in the interaction between breathers and solitons.[31] Recently, the interaction between breathers has caused great concern,[27,29,42,43] which is one possible way to excite high-order rogue waves (HRWs).[27,29] However, the interaction between breathers and RWs is rarely studied. Although RWs are considered as the limit of breathers, its dynamic properties are essentially different from breathers. Two well-known breathers, i.e., the Kuznetsov–Ma breathers (KMBs)[44,45] and the Akhmediev breathers (ABs)[46] are localized in time or space, respectively. RWs, on the one hand, are localized in both time and space. On the other hand, breathers and RWs can coexist on the continuous wave background (CWB), which provides the possibilities to study their interaction. We expect that some fascinating dynamic properties can be obtained in the interaction between breathers and RWs, which is different from the interaction between breathers.

In this work, we study the interaction between breathers and N-order RWs in a nonlinear optical fiber. It should be pointed out that the interactions between RWs and two different kinds of breathers (KMBs, ABs) follow a similar rule, thus we take the interaction between KMBs and RWs as an example to reveal their interaction properties. Our research indicates that the relative phase and interaction distance play key roles in the interacting process. The $(N+1)$-order RWs can be obtained from interaction between N-order RWs and one KMB, if their relative phase and interaction distance are both zero. However, a localized wave structure which is different from the HRWs pattern generated by breathers' interaction[27,29,43] can be gained under nonzero relative phase conditions. It can be used to reduce the maximum hump value of high-order rogue waves. In addition, there is an exclusion phenomenon between KMBs and RWs, which can be used to change the temporal-spatial distribution of rogue waves. The exclusive effect is insensitive to the relative phase, which stands in sharp contrast to the cases of intraspecific interaction between solitons or breathers.[36–39,43] Moreover, the dynamical evolution process of the interaction is characterized by the trajectory of localized waves' valleys and

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**Corresponding author. Email: zyyang@nwu.edu.cn

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humps. It is shown that the trajectory of the RWs and breathers has been completely changed by the interaction. These results further deepen our understanding on RW interaction dynamics and provide some possibilities to control HRWs.

The experiments on solitons, breathers and RWs[2,3,18,20,22,23,25] have shown that the simple scalar NLSE could describe the nonlinear localized waves well in nonlinear fibers. Therefore, we study the interaction between KMBs and RWs in a fiber system based on the following dimensionless NLSE,[3,47]

$$i\,\Psi_z + \frac{1}{2}\,\Psi_{\rm tt} + |\,\Psi|^2\,\Psi = 0, \tag{1}$$

where $\Psi(z,t)$ represents the slowly varying envelope field, z is the propagation distance, and t is the retarded time.[3] To obtain localized waves solutions of Eq. (1) on continuous wave background, we introduce the seed solution as follows:

$$\Psi[0] = s\exp[i\theta(t,z)], \tag{2}$$

where $\theta(t,z) = kz + \omega t$, $k = s^2 - \frac{1}{2}\omega^2$, s and ω denote the amplitude and frequency of the CWB. By means of the Darboux transformation method,[11] the interaction between KMBs and N-order RWs can be described by the analytical solution

$$\Psi[N+1] = \Psi[N] + 2(\lambda_2^* - \lambda_2)(P_2[N+1])_{21}, \tag{3}$$

where

$$P_2[N+1] = \frac{\phi_2[N]\phi_2[N]^\dagger}{\phi_2[N]^\dagger\phi_2[N]},$$
$$\phi_2[N] = T_2[N]T_2[N-1]\ldots T_2[1]\psi_2,$$
$$T_2[N] = \lambda_2 - \lambda_1^* + (\lambda_1^* - \lambda_1)P_1[N],$$
$$\Psi[N] = \Psi[0] + 2(\lambda_1^* - \lambda_1)\sum_{j=1}^{N}(P_1[N])_{21},$$
$$P_1[N] = \frac{\phi_1[N-1]\phi_1[N-1]^\dagger}{\phi_1[N-1]^\dagger\phi_1[N-1]},$$
$$\phi_1[0] = \psi_1(0),$$
$$\phi_1[1] = i\ \psi_1(0) + T_1[1]\ \psi_1^{[1]},$$
$$\phi_1[2] = -\psi_1(0) + i[T_1[2] + T_1[1]]\psi_1^{[1]} + T_1[2]T_1[1]\psi_1^{[2]},$$
$$\phi_1[m]|_{m-l\geqslant 0} = i^{(m-l)}|_{l=0}\psi_1(0)$$
$$+ i^{(m-l)}|_{l=1}[T_1[m] + \ldots + T_1[2] + T_1[1]]\psi_1^{[1]}$$
$$+ i^{(m-l)}|_{l=2}\sum_{m\geqslant i>j\geqslant 1} T_1[i]T_1[j]\psi_1^{[2]}$$
$$+ i^{(m-l)}|_{l=3}\sum_{m\geqslant i>j>k\geqslant 1} T_1[i]T_1[j]T_1[k]\psi_1^{[3]}$$
$$+ \ldots + i^{(m-l)}|_{l=m}[T_1[m]\ldots T_1[2]T_1[1]]\psi_1^{[m]},$$
$$\psi_1(f) = \psi_1(0) + \psi_1^{[1]}f^2 + \psi_1^{[2]}f^4$$
$$+ \ldots + \psi_1^{[N]}f^{2N} + \ldots,$$
$$\psi_1^{[n]} = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\psi_1(\lambda)|_{\lambda=\lambda_1},\quad n = 1,2,\ldots,$$
$$T_1[m] = \lambda_1 - \lambda_1^* + (\lambda_1^* - \lambda_1)P_1[m],\quad m = 1,2,\ldots.$$

Here $(P_j[n])_{21}$ represent the elements of matrices $(P_j[n])$ in the second row and first column at $\lambda = \lambda_j$, the star means complex conjugation, and a dagger denotes the matrix transpose and complex conjugation. The values of ψ_1 and ψ_2, two eigenfunctions corresponding to the Lax-pair of Eq. (1) with the eigenvalues λ_1 and λ_2, are given as

$$\psi_{j=1,2} = \begin{pmatrix} i(C_{j1}e^{A_j} - C_{j2}e^{-A_j})e^{-\frac{iz}{2}} \\ (C_{j2}e^{A_j} - C_{j1}e^{-A_j})e^{\frac{iz}{2}} \end{pmatrix},$$

where

$$C_{11} = \frac{\sqrt{h-\sqrt{h^2-1}}}{\sqrt{h^2-1}},\quad C_{12} = \frac{\sqrt{h+\sqrt{h^2-1}}}{\sqrt{h^2-1}},$$
$$C_{21} = \frac{\sqrt{-i\lambda_2-\tau}}{\tau},\quad C_{22} = \frac{\sqrt{-i\lambda_2+\tau}}{\tau},$$
$$\tau = \sqrt{-1-\lambda_2^2},$$
$$A_1 = \sqrt{h^2-1}((a_1+ib_1)f^2 + (a_2+ib_2)f^4 + ihz + t),$$
$$A_2 = \tau(t + \lambda_2 z - (r+id)),\ d = \frac{\Delta\varphi}{2Re[\sqrt{-\lambda_2^2-1}]},$$
$$h = 1 + f^2.$$

The parameters a_1, b_1, a_2 and b_2 determine the structure of HRWs, while r and $\Delta\varphi$ can be used to denote initial location difference and initial phase difference between KMBs and RWs. The solution enables us to investigate the interactions between a KMB and N-order RWs systematically.

The study on the intra-specific interaction of solitons and breathers shows the significant impact of relative phase on interaction.[27,29,36,41,42] For the inter-specific interaction between RWs and KMBs, we should also consider the influence of the relative phase. The relative phase ($\Delta\varphi$) here is the phase difference between KMBs and RWs at $z = 0$. Additionally, we use a variable (r) to describe the interaction, which is the distance of the initial locations between RWs and KMBs. In the following, we concentrate on the effect of the interaction distance (r) and the relative phase ($\Delta\varphi$). Then, the interaction characteristic between RWs and KMBs will be discussed in two cases. They are central collision ($r = 0$) and non-central collision ($r \neq 0$), respectively.

In the central collision case, different nonlinear localized wave structures can be obtained in the interaction area when the relative phase is zero or nonzero ($\Delta\varphi = 0$ and $\Delta\varphi \neq 0$). When $\Delta\varphi = 0$, the results of interaction between KMBs and first-order RWs, second-order RWs and third-order RWs are shown in Figs. 1(a)–1(c), respectively. It is shown that the N-order RWs can evolve to the $(N+1)$-order RWs in the interaction area when it collides with KMBs (see Figs. 1(a)–1(c)). As reported in Refs. [27,29], (N)-order RWs can be generated from the elastic collision of N breathers in the well-known scalar NLSE,

which has been experimentally demonstrated in optic fibers.[43] Thus there is a reason to believe that our result would be tested experimentally in the near future. Moreover, the maximum amplitude of the localized wave in the interaction area increases with the order of RWs (N). In theory, we can obtain very-high-energy pulses in the optics system in this way. As we well know, generation of high-energy pulses is one of the goals of modern optics.[20,48]

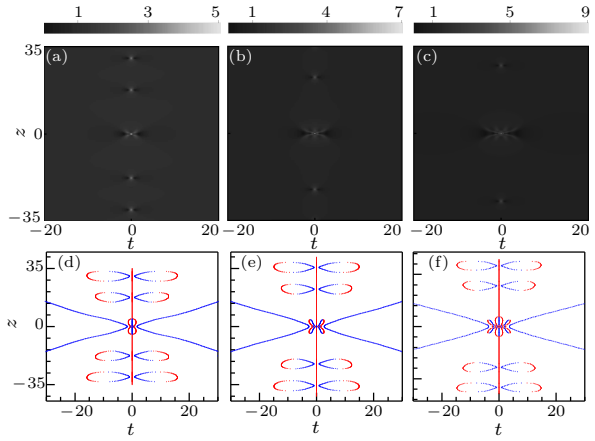

Fig. 1. The pattern of interaction between KMBs and N-order RWs in the case $r = 0$ and $\Delta\varphi = 0$ with (a) $N = 1$, (b) $N = 2$ and (c) $N = 3$. The trajectories of interaction corresponding to (a), (b) and (c) are presented in (d), (e) and (f). The parameters are $s = 1$, $\omega = 0$, $\lambda_1 = i$, $\lambda_2 = 1.03i$ and $a_1 = b_1 = a_2 = b_2 = 0$.

The study on RWs shows that the evolutive trajectory can help us to display the dynamical evolution process.[49,50] Accordingly, we present the evolutive trajectory of the localized waves' humps and valleys in Figs. 1(d)–1(f) corresponding to Figs. 1(a)–1(c), respectively. The red and blue lines denote the evolutive trajectory of humps and valleys, respectively. When N-order RWs interact with a KMB, we can see that the trajectory is the same as $(N + 1)$-order RWs in the central area[49] but becomes diverse in the non-central area. We take the interaction between a KMB and second-order RWs as an example to analyze the trajectory variation (see Fig. 1(e)) and there are similar physical features in other situations ($N \neq 2$).

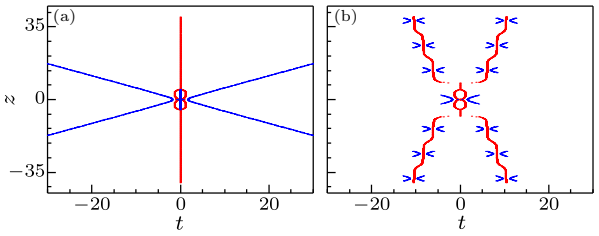

Fig. 2. (a) The trajectory of second-order RWs with $s = 1$, $\omega = 0$, $\lambda_1 = i$, and $a_1 = b_1 = 0$. (b) The trajectory of interaction between two breathers with $s = 1$, $\omega = 0$, $\lambda_2 = 0.01 + 1.03i$, $\lambda_3 = -0.01 + 1.03i$ and $c_1 = d_1 = c_2 = d_2 = 0$.

Compared with the trajectory of second-order RWs (see Fig. 2(a)), we can see that the X-shaped trajectory of the second-order RWs' valleys is maintained very well. In the central area, the trajectory characteristic of the second-order RWs is the same as the three-order RWs. In the non-central area, it is very interesting that some periodically oscillatory trajectories of humps appear in the areas near the unit structure of breathers. That is, the interaction changes the dynamical evolution trajectory of RWs and breathers. We should note that similar phenomena do not appear in the collision of two breathers as shown in Fig. 2(b).

When the relative phase is nonzero ($\Delta\varphi \neq 0$), some interesting physical phenomena will be observed. We take $\Delta\varphi = \pi$ as an example (other situations with arbitrary nonzero relative phase follow similar physical features) to demonstrate the result of interaction between KMBs and N-order RWs (see Figs. 3(a)–3(c)). When $N = 1$, there are only two peaks in the interaction area; when $N = 2$, there are five peaks which are assumed as a rectangle-shape distribution with a peak at the center; when $N = 3$, the inner structure is a second-order RW and the outer ring has six peaks. A systematic classification for HRW solutions to the NLSE has been presented and different patterns of HRWs have been listed in Ref. [51]. Obviously, the localized wave structures here are different from the previous HRWs.[51–53]

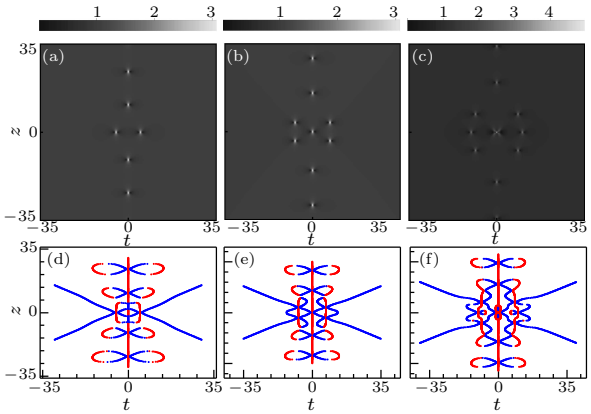

Fig. 3. The pattern of interaction between KMBs and N-order RWs under the case $r = 0$ and $\Delta\varphi = \pi$ with (a) $N = 1$, (b) $N = 2$ and (c) $N = 3$. The trajectory of interaction is shown in (d)–(f) corresponding to (a)–(c). The parameters are $s = 1$, $\omega = 0$, $\lambda_1 = i$, $\lambda_2 = 1.03i$ and $a_1 = b_1 = a_2 = b_2 = 0$.

Furthermore, the total number of peaks in the interaction area (see Figs. 3(a)–3(c)) is different from $(N + 1)$-order RWs.[51] The study on HRW solutions shows that if the HRWs are well separated, $(N + 1)$-order RWs will contain $\frac{1}{2}(N + 1)(N + 2)$ peaks.[52] That is, the number of peaks for $(N + 1)$-order RWs is one of the following numbers: 3, 6, 10, 15, etc.[51,53] However, the total number of peaks for the localized wave structure (see Fig. 3) in the interaction area is $\frac{1}{2}N(N + 1) + N$, namely, one of the following numbers 2, 5, 9, 14, etc. In short, when the relative phase is nonzero ($\Delta\varphi \neq 0$), compared with HRWs, the localized wave in the interaction area exhibits different

structures and total numbers of peaks. Figures 3(d)–3(f) show the trajectory corresponding to Figs. 3(a)–3(c), which are different from Figs. 1(d)–1(f). That is, they have different dynamical evolution processes for $\Delta\varphi = 0$ and $\Delta\varphi \neq 0$.

In the non-central collision case ($r \neq 0$), different physical phenomena can be observed for different interaction distances (r). When r is very small and $\Delta\varphi = 0$, the collision between N-order RWs and a KMB can generate a single-shell structure in the interaction area. When $N = 1$, three peaks form a ring without a central peak, which is the well-known triplet RWs[51] (see Fig. 4(a)). When $N = 2$, there is one central peak that is surrounded by a ring of five peaks (see Fig. 4(b)). When $N = 3$, the central structure is a second-order RW, which is surrounded by a ring of seven peaks (see Fig. 4(c)). Generally, N-order RWs colliding with a KMB form a structure that is composed of $(N-1)$-order RWs in the core and $2(N+1)-1$ peaks on the peripheral. The single-shell structures are reminiscent of an atom with a shell of electrons.[54] Compared with Refs. [51,54], their results come from nonlinear superposition of AB solutions, but our results originate from the interaction between KMBs and N-order RWs. It should be noted that our results are induced by relative phase $\Delta\varphi$ and interaction distance r. If r is very small and $\Delta\varphi \neq 0$, the case is similar to the situation of $r = 0$ and $\Delta\varphi \neq 0$ (see Fig. 3).

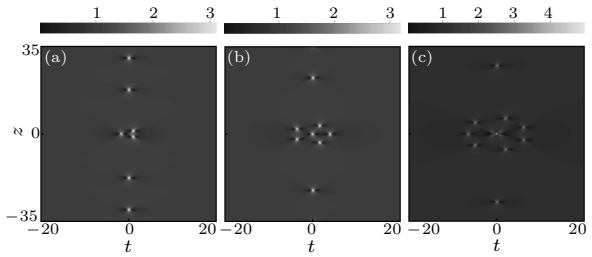

Fig. 4. The pattern of interaction between KMBs and N-order RWs under the case $r = 0.1$ and $\Delta\varphi = 0$ with (a) $N = 1$, (b) $N = 2$ and (c) $N = 3$. The parameters are $s = 1$, $\omega = 0$, $\lambda_1 = i$, $\lambda_2 = 1.03i$ and $a_1 = b_1 = a_2 = b_2 = 0$.

With the increase of r, the mutual repulsion between KMBs and HRWs becomes very obvious. Moreover, we notice that the interaction distance (r) has decisive influence on the repulsion effect, rather than the relative phase ($\Delta\varphi$). As an example, we demonstrate the result of interaction between a KMB and second-order RWs with different r in Fig. 5. In Figs. 5(a)–5(c), there are $r = 3$, $r = 10$ and $r = 20$, respectively. It is shown that the repulsion effect will weaken with the increase of r. In addition, the KMB is twisted by the repulsive interaction between KMBs and RWs. In particular, the repulsive effects are insensitive to the relative phase. This character is in sharp contrast to the cases for interactions of solitons and collision of breathers, of which the relative phase plays an important role in the repulsive or attractive effects.[36–39,43]

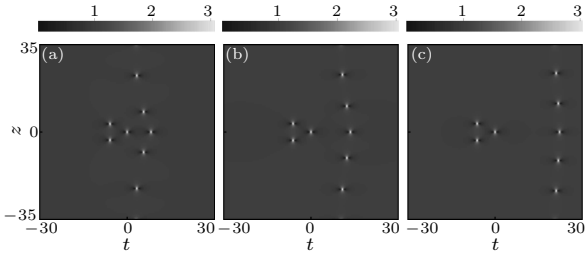

Fig. 5. The pattern of interaction between KMBs and second-order RWs under the case $\Delta\varphi = 0$ with different r: (a) $r = 3$, (b) $r = 10$ and (c) $r = 20$. The parameters are $s = 1$, $\omega = 0$, $\lambda_1 = i$, $\lambda_2 = 1.03i$ and $a_1 = b_1 = 0$.

In summary, we have investigated the interaction between KMBs and RWs in a nonlinear optical fiber. The results show that the relative phase and the interaction distance directly decide the localized wave structures in the interaction area. Different localized wave structures can be obtained from the interaction in different cases. In the central collision case, we can obtain $(N+1)$-order RWs with zero relative phase. At the same time, different localized wave structures as opposed to normal HRWs can be obtained with nonzero relative phase. In the non-central collision case and small r, single-shell structures are presented with zero relative phase. With the nonzero relative phase, the case is similar to the situation of $r = 0$ and $\Delta\varphi \neq 0$. The HRWs' structures have been changed by the breather under different r and $\Delta\varphi$. It is shown that the HRWs' structures are scattered in the case $r \neq 0$ or $\Delta\varphi \neq 0$. It can be used to control the maximum hump value and the temporal-spatial distribution of the HRWs by adjusting r and $\Delta\varphi$. In addition, we present qualitatively the repulsive effect between KMBs and HRWs, which shows that the repulsion decreases with the increase of the interaction distance r. In particular, the repulsive effect is insensitive to the relative phase, which is in sharp contrast to interaction of solitons and interaction of breathers. Moreover, the interaction properties are characterized by the trajectory analysis. It is shown that the trajectories of RWs and breathers are completely changed by the interaction. These results provide some possible ways to control high-order rogue waves.

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