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Thermal conductivity in near-nodal superconductors *

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The universal behavior of thermal conductivity at low temperatures is usually taken as the signature of gap nodes in superconductors. Here we show that in near-nodal superconductors the thermal conductivity obeys a two-parameter scaling law, and can develop super-universal behavior if the temperature is about half the gap minimum. However, when the temperature is fixed at about one quarter of the gap minimum, the thermal conductivity can develop a dip versus the scattering rate, which is in excellent agreement with the behavior of the experimental thermal conductivity in Sr_2RuO_4. Our theory is useful to correctly analyze the thermal conductivity in any near-nodal superconductor.

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Low-temperature thermal measurements can efficiently reveal the bulk nature of low-lying quasiparticles in superconductors. The specific heat effectively measures the density of states (DOS) of the quasiparticles. The thermal conductivity κ is slightly more involved. It depends on the temperature T, the DOS, the group velocity, and the scattering rate of the quasiparticles, and will be the focus of this work. In a fully gapped superconductor, quasi-particle excitations are suppressed exponentially at T much smaller than the energy gap, and one expects an exponential suppression of κ. However, this will be less obvious if the energy gap is anisotropic on the Fermi surface. For convenience, we define $\Delta_{\min}$ ($\Delta_{\max}$) as the gap minimum (maximum) along the Fermi surface. In a d-wave superconductor, such as cuprates, $\Delta_{\min} = 0$ is protected by symmetry. Consequently, in low-energy transports, the only energy scales are the temperature T and the scattering rate ζ, and previous theory shows that κ/T becomes independent of ζ (provided that $\zeta \gg T$).[1] This is referred to as the universal behavior of κ, following from the combination of the dual roles of quasiparticle scattering: on one hand, $\kappa/T \propto 1/\zeta$ from a simple Boltzmann equation point of view, and on the other hand it induces a finite zero-energy DOS $\rho_0 \propto \zeta$. The question is how κ/T would behave if $\Delta_{\min} \ll \Delta_{\max}$ but $\Delta_{\min} \neq 0$, i.e., the near-nodal case we will consider. Such an anisotropic gap structure is not rare. For example, it is observed quite generally in iron-based superconductors, where the $s_{\pm}$-wave pairing symmetry does not require $\Delta_{\min}$ to vanish.[2] A more interesting system is Sr_2RuO_4.[3] There are compelling evidences of chiral $p + ip'$-wave SC: for example, the lack of reduction of the Knight shift below T_c indicates spin-triplet pairing,[4–7] and the breaking of time-reversal symmetry is revealed by the signal of μ-on spin relaxation below T_c[8] and by the chirality in Kerr angle experiment.[9] The chiral p-wave gap is most likely fully gapped on the cylindrical Fermi surface of the layered Sr_2RuO_4.[10] However, low-energy quasi-particle excitations deep in the SC state are observed at low temperatures in specific heat,[11–13] superfluid density,[14] spin-lattice relaxation rate,[15] and thermal conductivity.[16–18] In particular, recent thermal conductivity measurements in Refs. [16–18] reveal universal behavior in κ/T, and the researchers strongly suggested true nodes from a d-wave or d-wave-like f-wave pairing function. However, taking the low but finite measurement temperature into account, we ask whether the d-wave-like feature in the thermal conductivity would rule out a near-node in the $p + ip'$-wave gap function. This is important to reconcile the earlier evidence of $p + ip'$-wave pairing symmetry. Recently, a comprehensive theoretical investigation of the superconductivity in Sr_2RuO_4 has been performed by a state-of-the-art singular-mode functional renormalization group,[19] taking into account all of the three orbitals and the atomic spin–orbital coupling. The physical properties calculated using the gap structure appear to be consistent, qualitatively and even quantitatively, with the results from the respective experiments.

In this Letter, we analytically investigate the behavior of thermal conductivity for any superconductors with near-nodes in the gap function in a more general context. We show that κ/T is a two-parameter scaling function of $\Delta_{\min}/T$ and ζ/T, interpolating the nodal limit $\Delta_{\min} = 0$ and the gapful limit where

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$\Delta_{\min}$ is large. We find that at a fixed temperature T with $\Delta_{\min}/T \sim 2$, κ/T could develop super-universality, namely, it is roughly a constant over an even larger range of ζ/T than that in the nodal case. As $\Delta_{\min}/T \sim 4$, κ/T is universal for $\zeta/T > 10$ but a mild dip occurs at $\zeta/T < 10$. The latter behavior is actually observed in Sr_2RuO_4, whereas the nodal d-wave case fails to reproduce the dip. Apart from the application in Sr_2RuO_4, our theory also suggests that to rule out a near-node in favor of a nodal gap, the measurement temperature must be sufficiently low so that $\Delta_{\min}/T \gg 1$ is satisfied. For a near-nodal superconductor, the latter condition might be a challenge in experiments.

The calculation of the thermal conductivity κ is standard. Since the thermal flux is carried by Bogoliubov–de Gennes (BdG) quasi-particles indifferent to spin, charge, and orbital, the calculation can be performed separately in each BdG band. We assume that the BdG spectrum is given by

$$E_{\boldsymbol{k}\nu} = \nu\sqrt{\epsilon_{\boldsymbol{k}}^2 + |\Delta_{\boldsymbol{k}}|^2}, \quad \nu = \pm 1, \tag{1}$$

where ν labels the BdG band, $\boldsymbol{k}$ is the momentum, $\epsilon_{\boldsymbol{k}}$ is the normal state dispersion, and $\Delta_{\boldsymbol{k}}$ is the gap function. This form of $E_{\boldsymbol{k}\nu}$ applies to all singlet superconductors, and to unitary triplet superconductors where the d-vector $\boldsymbol{d}_{\boldsymbol{k}}$ satisfies $\boldsymbol{d}_{\boldsymbol{k}} \times \boldsymbol{d}_{\boldsymbol{k}}^* = 0$. We are considering a single normal-state band (which gives rise to two BdG bands), but the discussion can be straightforwardly extended to multiple bands since the contributions to κ from different bands simply pile up. The thermal conductivity κ is given by[20]

$$\frac{\kappa}{T} = -\frac{1}{2T^2}\sum_{\nu}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}(\nabla_{\boldsymbol{k}}E_{\boldsymbol{k}\nu})^2 \cdot \int d\omega\omega^2\frac{\partial f}{\partial\omega}A_{\boldsymbol{k}\nu}^2(\omega), \tag{2}$$

where we assume two-dimensional momentum space and average over thermal current directions, $f(\omega) = 1/(1+e^{\beta\omega})$ is the Fermi function, and

$$A_{\boldsymbol{k}\nu}(\omega) = \frac{1}{\pi}\frac{\zeta}{\zeta^2 + (\omega - E_{\boldsymbol{k}\nu})^2} \tag{3}$$

is the spectral function of the BdG quasiparticle, smeared by the scattering rate ζ.

We consider the low-temperature limit so that we only need to consider the contributions from low-energy quasiparticle excitations. In such a limit, the normal state dispersion can be approximated as

$$\epsilon_{\boldsymbol{k}} \sim v_{\rm fn}q_x, \tag{4}$$

where $v_{\rm fn}$ is the Fermi velocity at the nth near-node, and q_x is the radial momentum deviation from the Fermi surface. For weak-coupling superconductors (in the sense that $\Delta_{\max} \ll E_{\rm f}$, where $E_{\rm f}$ is the Fermi energy), we may ignore the variation of the gap $|\Delta_{\boldsymbol{k}}|$ along the Fermi velocity. Since a near-node is a local gap minimum, we have

$$|\Delta_{\boldsymbol{k}}|^2 \sim \Delta_n^2 + (v_{\Delta n}q_y)^2 \tag{5}$$

for $\boldsymbol{k} = \boldsymbol{k}_n + \boldsymbol{q}$, where Δ_n is the local minimum at the nth near-node, and q_y is the component of $\boldsymbol{q}$ along the tangent of the Fermi surface. Note that $v_{\Delta n}$ becomes the usual gap velocity $|\nabla_{\boldsymbol{k}}\Delta_{\boldsymbol{k}}|$ in the case of a true node, but is otherwise just a parameter used to characterize the quadratic change of $|\Delta_{\boldsymbol{k}}|^2$. The BdG dispersion now becomes

$$E_{n\nu}(\boldsymbol{q}) \sim \nu\sqrt{(v_{\rm fn}q_x)^2 + (v_{\Delta_n}q_y)^2 + \Delta_n^2}. \tag{6}$$

To proceed, we notice that

$$(\nabla_{\boldsymbol{k}}E_{\boldsymbol{k}\nu})^2 \to [\nabla_{\boldsymbol{q}}E_{n\nu}(\boldsymbol{q})]^2 = \frac{v_{\rm fn}^2p_x^2 + v_{\Delta n}^2p_y^2}{p^2 + \Delta_n^2}, \tag{7}$$

where we define a rescaled vector (of the dimension of energy)

$$\boldsymbol{p} = (v_{\rm fn}q_x, v_{\Delta n}q_y). \tag{8}$$

Substituting everything into Eq. (2), we obtain

$$\frac{\kappa}{T} \sim -\frac{1}{2T^2}\sum_{n\nu}\int \frac{d^2\boldsymbol{p}}{(2\pi)^2 v_{\rm fn}v_{\Delta n}}\frac{v_{\rm fn}^2p_x^2 + v_{\Delta n}^2p_y^2}{p^2+\Delta_n^2} \cdot \int d\omega\omega^2\frac{\partial f}{\partial\omega}\frac{1}{\pi^2}\frac{\zeta^2}{[\zeta^2 + (\omega - \nu\sqrt{p^2+\Delta_n^2})^2]^2}. \tag{9}$$

The integration can not be performed analytically, but can be rescaled into the form

$$\frac{\kappa}{T} = \frac{1}{12\pi}\sum_n\left(\frac{v_{\rm f}}{v_\Delta} + \frac{v_\Delta}{v_{\rm f}}\right)_n g\left(\frac{\Delta_n}{T}, \frac{\zeta}{T}\right), \tag{10}$$

where g is a universal scaling function

$$g(x,y) = -\frac{6}{\pi^2}\int_0^\infty \frac{duu^3}{u^2+x^2} \cdot \int_{-\infty}^{\infty} dvv^2\frac{\partial f}{\partial v}\frac{y^2}{[y^2 + (v - \sqrt{u^2+x^2})^2]^2}, \tag{11}$$

with $f(v) = 1/(e^v+1)$. This is the central result of the present work.

Clearly, the behavior of κ/T is completely determined by the two-parameter scaling function g. Let us concentrate on the contribution from quasiparticle excitations near the nth gap minimum. We plot the universal function $g(x,y)$ in Fig. 1, where $x = \Delta_n/T$ and $y = \zeta/T$. In the nodal d-wave case, $x = 0$, and $g(0, y \gg 1) = 1$ corresponds to the universal behavior of κ/T discussed previously for the cuprates.[1] Naively, universal behavior should also appear in the near-nodal case if $\Delta_n \ll T \ll \zeta$, so that near-nodes behave as real ones. In fact, from Fig. 1 we see that,

up to $g = 1 \pm 0.1$ (allowing uncertainties in the experiment), the condition can be significantly relaxed, $\Delta_n/3 \leq T \leq \zeta/10$. Therefore, universal thermal conductivity will be expected quite loosely even if the gap is not exactly nodal. More interestingly, $g \sim 1$ is observed in a wider regime of ζ/T in the case of $\Delta_n/T \sim 2$ (thick line) than in the case of $\Delta_n/T \to 0$ (top line, appropriate for d-wave), thus displaying a super-universal behavior. However, near $\Delta_n/T \sim 4$, $g \sim 1$ for $\zeta/T > 10$, but it develops a mild dip where $\zeta/T < 10$. This behavior is in contrast to the nodal case $\Delta_n/T \to 0$ (the upper-most curve), where g increases monotonically as ζ decreases. Quite interestingly, the dip feature is actually observed in the measured thermal conductivity in Sr_2RuO_4[16−−18] without attention until [19]. The excellent agreement between theory and experiment suggests that instead of implying nodal d-wave pairing, the nodal-like feature in the experimental thermal conductivity actually implies near-nodes in the highly anisotropic $p + ip'$-wave gap function.[19] Of course, if $\Delta_n/T \gg 1$, g is suppressed (see, e.g. the bottom line in Fig. 1). At this stage the d-wave and chiral p-wave behave most differently. The lowest measurement temperature so far in the experiments for Sr_2RuO_4 is $T \sim 50\,\mathrm{mK}$, which is about $T_c/30$ with T_c being the superconducting transition temperature. If there is a near-node such that $\Delta_n \sim T_c/10$, the dip feature is visible at $T = 50\,\mathrm{mK}$. Indeed, there is a hierarchy of gap minima in the theoretical gap structure of [19], and that on the so-called α and β Fermi pockets is of the desired magnitude.

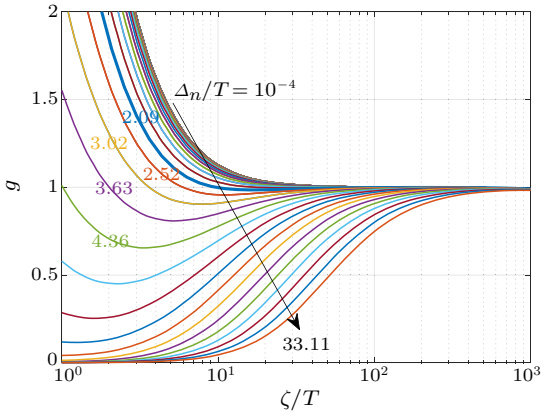

Fig. 1. Universal function $g(x, y)$ versus $y = \zeta/T$ for a range of $x = \Delta_n/T$, where Δ_n is the gap minimum at a quasi-node. The value of Δ_n/T increases exponentially from 10^{-4} up to 33.11 along the arrow. Several intermediate values of Δ_n/T are indicated explicitly.

To conclude, we have derived an analytic formula for the thermal conductivity of superconductors with near-nodes in the gap function, showing that κ/T is a two-parameter scaling function of $\Delta_{\min}/T$ and ζ/T, interpolating the nodal limit $\Delta_{\min} = 0$ and the gapful limit where $\Delta_{\min}$ is large. We find that at a fixed temperature T with $\Delta_{\min}/T \sim 2$, κ/T can develop super-universal behavior versus ζ/T, and as $\Delta_{\min}/T \sim 4$, κ/T is universal for $\zeta/T > 10$ but a mild dip occurs at $\zeta/T < 10$, which is in agreement with the experiments for Sr_2RuO_4. Our theory unravels the fact that near-nodal superconductors can also develop universal thermal conductivity at low but finite temperatures. To rule out a near-node in favor of a nodal gap, the measurement temperature must be sufficiently low so that $\Delta_{\min}/T \gg 1$ is satisfied.

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