CHIN. PHYS. LETT. Vol. 35, No. 10 (2018) 107201

Impurity Effects at Surfaces of a Photon-Dressed Bi_2Se_3 Thin Film *

Qiu-Shi Wang(王秋实)[1], Bin Zhang(张斌)[1], Wei-Zhu Yi(易伟柱)[1], Meng-Nan Chen(陈梦南)[2], Baigeng Wang(王伯根)[1,3], R. Shen(沈瑞)[1,3]**

[1]*National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093*
[2]*School of Science, Hangzhou Dianzi University, Hangzhou 310018*
[3]*Collaborative Innovation Center of Advanced Microstructures, Nanjing 210093*

(Received 2 July 2018)

We investigate the impurity effects on surfaces of a thin film topological insulator, applied by an off-resonant circular polarized light. It is found that the off-resonant driving induces a quantized total Hall conductivity, when the driving strength is larger than a critical value and the Fermi level lies in the band gap, indicating that our system is converted into the topological phase. We also find that with the increasing disorder strength, the Dirac masses of top and bottom surfaces are renormalized and then fixed to half of their initial values, respectively, which will shrink the widths of the half-integer plateau of anomalous Hall conductivities.

PACS: 72.80.Ng, 73.23.−b, 73.43.−f, 03.65.Vf DOI: 10.1088/0256-307X/35/10/107201

Recently, topological states have gained huge developments both theoretically[1−4] and experimentally.[5−7] The topological insulator (TI) is protected by time-reversal symmetry (TRS), with its intriguing gapless surface state[8−10] described by the Dirac equation. Breaking the TRS of the surface state, for example, by a Zeeman field, would drive it into an exotic half-integer quantum Hall state (QHS),[11−13] which may be potentially applicable in spintronics.

To overcome the limited controllability and tunability of static topological phases, there emerges one important way to generate a topological nontrivial state via periodically driving a trivial state out of equilibrium by applying time-periodic microwaves or other optical techniques.[14,15] Within the framework of the Floquet theory, these topologically nontrivial phases are termed as the Floquet topological phases. The off-resonant periodic driving is a widely adopted technique to modify the topological properties of real materials, which gives results in excellent agreement with experiments.[16] For example, when the HgTe quantum well is applied by the circular polarized light with the driving frequency of light much larger than the band width, the off-resonant approximation can be taken, through which the applied microwave can be shown to generate an effective Dirac mass term, which thus induces a topological phase transition from quantum spin Hall phase to quantum anomalous Hall phase.[17] Recently, there have been several breakthroughs about the Floquet Dirac states in real materials.[18,19]

Disorder effects also play an important role in characterizing topological phases. Though the gapless surface (edge) states of topological insulators are robust against weak nonmagnetic disorder, they may be destroyed under strong disorder strength, leading to interesting disorder-induced effects. In the previous studies, the ballistic edge states of two-dimensional TIs are robust under disorder as long as there is a bulk mobility gap,[2,20] while in the trivial 2D system, the disorder would lead to the localization of states. In the presence of spin-orbit coupling, the trivial phase can be transformed into a topological Anderson insulator.[21−23] These works reveal many important properties of the impurity induced system and develop very useful methods which are worthy of reference. However, most of them only consider the effects from the interaction of disorder with electrons near the fermi surface and ignore the contribution from the valence band. Here we consider the effects of both the fermi surface and band structure, and give an explicit description of not only the anomalous Hall conductance but also the correction of band gap. Moreover, we also calculate the critical point of the disorder's magnitude, which can describe a metal-insulator transition.

In this work, we consider a TI thin film, which can be described by the top and bottom surface $H = H_{\rm top} + H_{\rm bottom} + H_{\rm I}$. The first two terms describe the pure TI surface states as mentioned above, and $H_{\rm I}$ describes a weak hybridization between opposite surfaces states, which generates a gap in the spectrum. The whole system can be seen as a topologically trivial insulator.[3] Intriguingly, when introducing a circularly polarized light propagating perpendicularly to the surface, the time-dependent field would generate an effective mass term and would lead to a topological nontrivial phase with an anomalous Hall effect. We derive the condition for the occurrence of this topological phase transition explicitly through the anomalous Hall conductance.

Starting from the quasi two-dimensional TI system, we first derive the static effective Hamiltonian of the system[24] and further study the influence of onsite disorder. Using the self-consistent Born approximation method (SCBA),[25] we present the effects of disorder through the self-energy term, which corrects both the frequency and the band gap. We discuss the process of dynamical renormalization induced by the disorder for the frequency and Dirac mass term and solve the corresponding self-consistent equations in de-

*Supported by the National Key R&D Program of China under Grant No 2017YFA0303203, and the National Natural Science Foundation of China under Grant Nos 11574217 and 11474149.
**Corresponding author. Email: shen@nju.edu.cn
© 2018 Chinese Physical Society and IOP Publishing Ltd

tail. The long-ranged disorder term would not change the topological property of the system but only modulate the magnitude of the gap. With the increase of the disorder's strength, the system would have a metal-insulator transition under proper Fermi energy conditions. The critical value of disorder strength is carefully studied and presented through theoretical analysis on the effective continuum model. The lattice model realization for our model is also discussed.

Let us start from the surface Hamiltonian of a topological insulator Bi_2Se_3 thin film,[26]

$$H_0 = \frac{\Delta}{2}\hat{\tau}_z\hat{\sigma}_z + v_{\rm F}(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x), \tag{1}$$

where $\boldsymbol{k} = (k_x, k_y)$ is the electron momentum, $v_{\rm F}$ is the Fermi velocity, the Pauli matrices $\{\hat{\sigma}_i\}$ with $i = 1, 2, 3$ represent the real spin, $\{\hat{\tau}_z\}$ represents the bonding and antibonding of surface states on the two surfaces and will be referred to as 'pseudospin', and Δ is the hybridization gap coupling the two surfaces for an ultra-thin topological insulator film. Usually, Δ is much smaller than the energy scale of the system.

In the presence of circularly polarized light, the spatially uniform and time-dependent vector potential is given as

$$\boldsymbol{A}(t) = A_0(\gamma\sin(\omega t), \cos(\omega t)), \tag{2}$$

with $\boldsymbol{E}(t) = \partial\boldsymbol{A}(t)/\partial t$, where $\gamma = +(-)$ denotes the right- (left-) circularly polarized light, and A_0 is the intensity of the light.

According to the Floquet theory, we can obtain a static effective Hamiltonian in the off-resonant approximation, in which the driving frequency ω is taken to be large compared to other characteristic energy scales[15]

$$H_{\rm eff} \simeq \mathcal{H}_0 + \frac{[\mathcal{H}_{-1}, \mathcal{H}_{+1}]}{\omega} + \mathcal{O}\Big(\frac{1}{\omega^2}\Big), \tag{3}$$

where

$$\mathcal{H}_n = \frac{1}{T}\int_0^T dt e^{in\omega t}H(t) \tag{4}$$

is the nth Fourier harmonic of the time-periodic Hamiltonian, and $T = 2\pi/\omega$ is the time period.

We therefore obtain the static effective Hamiltonian as

$$H_{\rm eff}(\boldsymbol{k}) = \frac{\Delta}{2}\hat{\tau}_z\hat{\sigma}_z + \frac{\gamma(v_{\rm F}\tilde{A}_0)^2}{\omega}\hat{\sigma}_z + v_{\rm F}(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x). \tag{5}$$

We notice that the Dirac mass terms in Eq. (5) consist of two parts, the first term is the trivial mass and the second term is the topological mass. The latter would break the time reversal symmetry and would lead to the quantum anomalous Hall effect.[27]

We notice that Hamiltonian (5) is naturally diagonalized in the $\hat{\tau}$-subspace, the corresponding eigenvalues are given as

$$M_\pm = \frac{\gamma(v_{\rm F}\tilde{A}_0)^2}{\omega} \pm \frac{\Delta}{2}, \tag{6}$$

with the corresponding eigenvectors $|+\rangle = (1, 0)^{\rm T}$ and $|-\rangle = (0, 1)^{\rm T}$. In this case, the Hamiltonian has been projected to the top and bottom surfaces with the photon-dressed Dirac masses (6), respectively. The behavior of effective Dirac mass versus the intensity of light $\tilde{A}_0$ is shown in Fig. 1. This leads to two independent 2×2 blocks of the Hamiltonian, which read

$$H_{\rm eff}(\boldsymbol{k}) = \begin{pmatrix} h_+(\boldsymbol{k}) & 0 \\ 0 & h_-(\boldsymbol{k}) \end{pmatrix},$$
$$h_\tau(\boldsymbol{k}) = M_\tau\hat{\sigma}_z + v_{\rm F}(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x), \tag{7}$$

where we designate $\tau = \pm$ to stand for the pseudospin parallel (antiparallel) to the z axis. After a further diagonalization, we obtain the eigenvalues $\epsilon^{\rm c(v)}_{\boldsymbol{k},\tau} = +(-)\sqrt{v_{\rm F}^2k^2 + M_\tau^2}$ and wavefunctions in the $\hat{\sigma}$ space, which read

$$\varphi^{\rm v}_\tau(\boldsymbol{k}) = \begin{pmatrix} e^{-i\theta_k}\sin\alpha_\tau \\ -i\cos\alpha_\tau \end{pmatrix},$$
$$\varphi^{\rm c}_\tau(\boldsymbol{k}) = \begin{pmatrix} e^{-i\theta_k}\cos\alpha_\tau \\ -i\sin\alpha_\tau \end{pmatrix}, \tag{8}$$

where $e^{-i\theta_k} = (k_x - ik_y)/k$, and $2\alpha_\tau = \arccot(M_\tau/v_{\rm F}k)$ with $k = \sqrt{k_x^2 + k_y^2}$. The electron wavefunctions in the valence bands are given by

$$|\psi^{\rm v}_\tau(\boldsymbol{k})\rangle = |\tau\rangle \otimes |\varphi^{\rm v}_\tau(\boldsymbol{k})\rangle. \tag{9}$$

Clearly, the valence bands are decomposed into two isospin sectors.

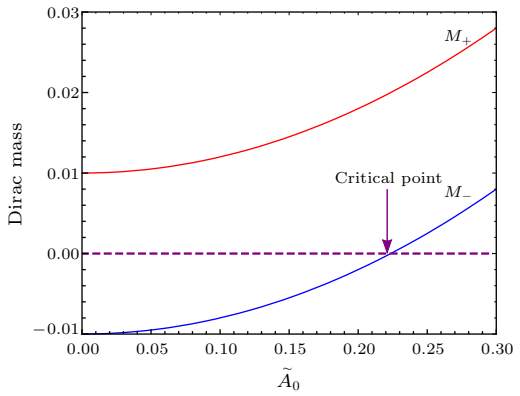

Fig. 1. The effective Dirac mass versus the intensity of light $\tilde{A}_0$. Here we have set $v_{\rm F} = 1.0$ and the momentum is set to be dimensionless, $\Delta = 0.02$ and $\omega = 5.0$.

According to the linear response theory, the intrinsic Hall conductance is given by the integral of the Berry curvature over occupied states in k space

$$\sigma^\tau_{xy} = -\frac{e^2}{h}\sum_i\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}f(E_i - E_{\rm F})F^i_\tau(\boldsymbol{k}), \tag{10}$$

where $f(\epsilon) = 1/(e^{\epsilon/k_{\rm B}T} + 1)$ is the Fermi function, $E_{\rm F}$ is the Fermi energy, and

$$F^i_\tau(\boldsymbol{k}) = i\hat{\boldsymbol{e}}_z\cdot\langle\nabla_{\boldsymbol{k}}\psi^i_\tau(\boldsymbol{k})| \times |\nabla_{\boldsymbol{k}}\psi^i_\tau(\boldsymbol{k})\rangle \tag{11}$$

denotes the Berry curvature with $i = {\rm v}, {\rm c}$ for the valence and conduction bands, respectively, and $\nabla_{\boldsymbol{k}} =$

$(\partial_{k_x}, \partial_{k_y})$. Using the polar coordinate system, it is straightforward to obtain $F^i_\tau(k) = \frac{1}{k}\frac{\partial}{\partial k}P^i_\tau(k)$ with

$$P^i_\tau(k) = i\langle\psi^i_\tau(\boldsymbol{k})|\partial_{\theta_k}|\psi^i_\tau(\boldsymbol{k})\rangle. \tag{12}$$

Substituting the expression for $P^i_\tau(k)$ into Eq. (10), we derive σ^τ_{xy} to be

$$\begin{aligned}\sigma^{\tau,\mathrm{v}}_{xy} &= -\frac{e^2}{h}[P^{\mathrm{v}}_\tau(\infty) - P^{\mathrm{v}}_\tau(0)] \\ &= -\frac{e^2}{2h}\mathrm{sgn}(M_\tau),\end{aligned} \tag{13}$$

where only the valence band is considered as the Fermi energy E_{F} is in the band gap. This is the half-integer quantized Hall conductivity, which is usually found in the 2D Dirac system. Just like Graphene which has two valleys, the Dirac bands for TI also come in pairs, i.e., the top surface and the bottom surface. Equation (13) also shows that the top surface ($\tau = +$) contributes $-(e^2/2h)\mathrm{sgn}(M_+)$ to the Hall conductance and the bottom surface ($\tau = -$) contributes $-(e^2/2h)\mathrm{sgn}(M_-)$ to the Hall conductance when the Fermi level is located in the gap. Therefore, the total Hall conductance is $\sigma_{xy} = -(e^2/2h)[\mathrm{sgn}(M_+) + \mathrm{sgn}(M_-)]$, which only depends on the sign of M_τ. For a small $\tilde{A}_0$, $M_+ > 0$ and $M_- < 0$, then $\sigma_{xy} = 0$. As $\tilde{A}_0$ increases, M_+ stays positive and M_- goes from negative to positive, i.e., $|\frac{\gamma(v_{\mathrm{F}}\tilde{A}_0)^2}{\omega}| > \frac{\Delta}{2}$, leading to a quantized plateau of σ_{xy}. In fact, there exists a topological phase transition because the first Chern number changes from 0 to 1.

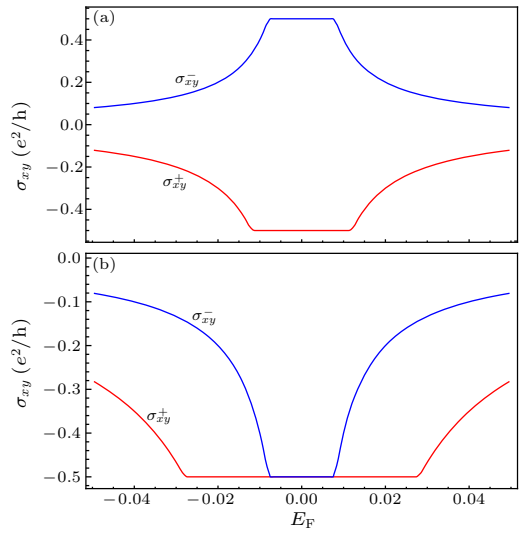

Fig. 2. The Hall conductivities as the function of the Fermi energy E_{F}. Parameters are taken to be the same as those in Fig. 1 unless for (a) $\tilde{A}_0 = 0.1$ and (b) $\tilde{A}_0 = 0.3$, respectively.

For the Fermi energy E_{F} lies in the conduction bands, we can find that the conduction bands also contribute to the Hall conductivities, which can be obtained as

$$\sigma^{\tau,\mathrm{c}}_{xy} = -\frac{e^2}{2h}\Big[\frac{M_\tau}{E_{\mathrm{F}}} - \mathrm{sgn}(M_\tau)\Big]. \tag{14}$$

Thus the Hall conductivity for a given surface is

$$\sigma^\tau_{xy} = \sigma^{\tau,\mathrm{v}}_{xy} + \sigma^{\tau,\mathrm{c}}_{xy} = -\frac{e^2}{2h}\frac{M_\tau}{E_{\mathrm{F}}}, \tag{15}$$

and the total Hall conductivity is

$$\sigma_{xy} = -\frac{e^2}{2h}\frac{M_+ + M_-}{E_{\mathrm{F}}} = -\frac{e^2}{h}\frac{\gamma(v_{\mathrm{F}}\tilde{A}_0)^2}{E_{\mathrm{F}}\omega}. \tag{16}$$

This is actually photoinduced anomalous Hall conductivity and is totally determined by the intensity of light $\tilde{A}_0$. The Hall conductivities of the two surfaces as functions of the Fermi level E_{F} for a given $\tilde{A}_0$ are plotted in Fig. 2, where the widths of half-integer anomalous Hall plateau are shown obviously.

In the Born approximation, we can obtain the δ-type correlated impurity white-noise spin-independent random potential[28]

$$\zeta(\boldsymbol{r} - \boldsymbol{r}') = \langle V(\boldsymbol{r})V(\boldsymbol{r}')\rangle_{\mathrm{imp}} = n_{\mathrm{i}}V_0^2\delta(\boldsymbol{r} - \boldsymbol{r}'), \tag{17}$$

and its form in the momentum space is given by

$$\zeta_{\boldsymbol{k}} = \int \zeta(\boldsymbol{r})e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}d\boldsymbol{r} = n_{\mathrm{i}}V_0^2, \tag{18}$$

where n_{i} is the impurity concentration, and V_0 is the impurity potential. In the Matsubara space, the disorder averaged Green's function can be given as

$$\hat{G}(\boldsymbol{k}, i\omega_n) = \frac{1}{i\omega_n - H_{\mathrm{eff}}(\boldsymbol{k}) - \hat{\Sigma}(\boldsymbol{k}, i\omega_n)}, \tag{19}$$

with the self energy defined as

$$\hat{\Sigma}(\boldsymbol{k}, i\omega_n) = n_{\mathrm{i}}V_0^2\int\frac{d^2\boldsymbol{k}'}{(2\pi)^2}\hat{G}(\boldsymbol{k}', i\omega_n). \tag{20}$$

We first take the first Born approximation and then carry out the analytical continuation $i\omega_n \to \Omega + i\eta$. The imaginary part of retarded self energy can be obtained as

$$\mathrm{Im}\hat{\Sigma}^{\mathrm{R}}_\tau = -\frac{i}{4\tilde{\tau}^{\mathrm{q}}}\Big(1 + \frac{M_\tau}{E_{\mathrm{F}}}\hat{\sigma}_z\Big), \tag{21}$$

where only the electron states close to the Fermi level are considered, as the Fermi level is in the conduction bands, and $\tilde{\tau}^{\mathrm{q}}$ is a quantum life time at the Fermi surface

$$\frac{1}{\tilde{\tau}^{\mathrm{q}}} = n_{\mathrm{i}}V_0^2\int kdk\delta(E_{\mathrm{F}} - \epsilon^{\mathrm{c}}_{\boldsymbol{k},\tau}) = \frac{n_{\mathrm{i}}V_0^2E_{\mathrm{F}}}{v_{\mathrm{F}}^2}, \tag{22}$$

where $\tilde{\tau}^{\mathrm{q}}$ determines the transport properties at the Fermi level.[29]

In the following we calculate the self energy in the self-consistent Born approximation to investigate the impurity induced gap renormalization, we mainly follow the method proposed by Rostami *et al.*[30] After carrying out the integration over $\boldsymbol{k}'$ and the disorder average, only diagonal terms mainly contribute to the self energy, therefore the self energy can be formally expanded as

$$\hat{\Sigma}(i\omega_n) = \Sigma_I\boldsymbol{I} + \Sigma_M\hat{\sigma}_z + \Sigma_\Delta\hat{\sigma}_z\hat{\tau}_z + \Sigma_0\hat{\tau}_z. \tag{23}$$

We can figure out the four equations

$$\begin{aligned}&\Sigma_I+\Sigma_0\\&=-\frac{n_{\rm i}V_0^2}{4\pi v_{\rm F}^2}(i\omega_n-\Sigma_I-\Sigma_0)\\&\times\log\Big[1+\frac{W^2}{(M_++\Sigma_M+\Sigma_\Delta)^2-(i\omega_n-\Sigma_I-\Sigma_0)^2}\Big],\end{aligned}\tag{24}$$

$$\begin{aligned}&\Sigma_M+\Sigma_\Delta\\&=-\frac{n_{\rm i}V_0^2}{4\pi v_{\rm F}^2}(M_++\Sigma_M+\Sigma_\Delta)\\&\times\log\Big[1+\frac{W^2}{(M_++\Sigma_M+\Sigma_\Delta)^2-(i\omega_n-\Sigma_I-\Sigma_0)^2}\Big],\end{aligned}\tag{25}$$

$$\begin{aligned}&\Sigma_I-\Sigma_0\\&=-\frac{n_{\rm i}V_0^2}{4\pi v_{\rm F}^2}(i\omega_n-\Sigma_I+\Sigma_0)\\&\times\log\Big[1+\frac{W^2}{(M_-+\Sigma_M-\Sigma_\Delta)^2-(i\omega_n-\Sigma_I+\Sigma_0)^2}\Big],\end{aligned}\tag{26}$$

$$\begin{aligned}&\Sigma_M-\Sigma_\Delta\\&=-\frac{n_{\rm i}V_0^2}{4\pi v_{\rm F}^2}(M_-+\Sigma_M-\Sigma_\Delta)\\&\times\log\Big[1+\frac{W^2}{(M_-+\Sigma_M-\Sigma_\Delta)^2-(i\omega_n-\Sigma_I+\Sigma_0)^2}\Big],\end{aligned}\tag{27}$$

where W is the energy cutoff. It is determined by the corresponding lattice model version of Hamiltonian[31]

$$\begin{aligned}H_0=&\frac{\Delta}{2}\sum_{\boldsymbol{i},\tau,\tau'}\hat{c}^\dagger_{\boldsymbol{i},\tau}(\hat{\tau}_z)_{\tau,\tau'}\hat{\sigma}_z\hat{c}_{\boldsymbol{i},\tau'}\\&+\sum_{\boldsymbol{i},\tau}\frac{i\hbar v_{\rm F}}{2a}(\hat{c}^\dagger_{\boldsymbol{i},\tau}\hat{\sigma}_y\hat{c}_{\boldsymbol{i}+\delta\hat{\boldsymbol{x}},\tau}\\&-\hat{c}^\dagger_{\boldsymbol{i},\tau}\hat{\sigma}_x\hat{c}_{\boldsymbol{i}+\delta\hat{\boldsymbol{y}},\tau})+\text{H.c.}\\=&\frac{\Delta}{2}\sum_{\boldsymbol{i}}\hat{c}^\dagger_{\boldsymbol{i}}\hat{\tau}_z\hat{\sigma}_z\hat{c}_{\boldsymbol{i}}+\sum_{\boldsymbol{i}}\frac{i\hbar v_{\rm F}}{2a}(\hat{c}^\dagger_{\boldsymbol{i}}\hat{\sigma}_y\hat{c}_{\boldsymbol{i}+\delta\hat{\boldsymbol{x}}}\\&-\hat{c}^\dagger_{\boldsymbol{i}}\hat{\sigma}_x\hat{c}_{\boldsymbol{i}+\delta\hat{\boldsymbol{y}}})+\text{H.c.},\end{aligned}\tag{28}$$

where $\hat{c}^\dagger_{\boldsymbol{i},\tau}$ and $\hat{c}_{\boldsymbol{i},\tau}$ are the annihilation and creation operators at site $\boldsymbol{i}$ in the surface τ, $\hat{c}_{\boldsymbol{i}}=(\hat{c}_{\boldsymbol{i},+},\hat{c}_{\boldsymbol{i},-})^{\rm T}$, $\hat{\boldsymbol{x}}$ ($\hat{\boldsymbol{y}}$) are the primitive vectors of the square lattice along the x (y) direction, and a is the lattice constant. The value of W can be given as $W=\hbar v_{\rm F}k_{\rm c}$. Here we introduce a momentum cutoff $k_{\rm c}$, which obeys $\pi k_{\rm c}^2=S_{\rm BZ}$ with $S_{\rm BZ}=(2\pi)^2/a^2$ denotes the area of the first Brillouin zone.

Denoting $\tilde{\Sigma}^0_\tau=\Sigma_I+\tau\Sigma_0$ and $\tilde{\Sigma}^z_\tau=\Sigma_M+\tau\Sigma_\Delta$ with $\tau=\pm$, the above equations are therefore written in a compact form as

$$\tilde{\Sigma}^0_\tau=-\frac{n_{\rm i}V_0^2}{4\pi v_{\rm F}^2}(i\omega_n-\tilde{\Sigma}^0_\tau)\alpha_\tau(i\omega_n),\tag{29}$$

$$\tilde{\Sigma}^z_\tau=-\frac{n_{\rm i}V_0^2}{4\pi v_{\rm F}^2}(M_\tau+\tilde{\Sigma}^z_\tau)\alpha_\tau(i\omega_n),\tag{30}$$

where

$$\alpha_\tau(i\omega_n)=\log\Big[1+\frac{W^2}{(M_\tau+\tilde{\Sigma}^z_\tau)^2-(i\omega_n-\tilde{\Sigma}^0_\tau)^2}\Big].\tag{31}$$

We can see that $\tilde{\Sigma}^0_\tau$ and $\tilde{\Sigma}^z_\tau$ provide a dynamical renormalization of the one particle excitations, $\tilde{\Sigma}^0_\tau$ corrects the Matsubara frequency by $i\tilde{\omega}_n=i\omega_n-\tilde{\Sigma}^0_\tau$, and $\tilde{\Sigma}^z_\tau$ means a correction for the Dirac mass of the gap function $\tilde{M}_\tau=M_\tau+\tilde{\Sigma}^z_\tau$.

Let $U=n_{\rm i}V_0^2/(4\pi v_{\rm F}^2)$ and $Z_\tau(i\omega_n)=1-\tilde{\Sigma}^0_\tau/(i\omega_n)$, we obtain

$$\tilde{\Sigma}^0_\tau(i\omega_n)=-(i\omega_n-\tilde{\Sigma}^0_\tau)U\alpha_\tau(i\omega_n),\tag{32}$$

$$\tilde{\Sigma}^z_\tau(i\omega_n)=-(M_\tau+\tilde{\Sigma}^z_\tau)U\alpha_\tau(i\omega_n),\tag{33}$$

where $Z_\tau(i\omega_n)$ is the quasiparticle renormalization function. After a few straightforward steps, we obtain

$$\tilde{\Sigma}^0_\tau(i\omega_n)=-i\omega_n\frac{U\alpha_\tau(i\omega_n)}{1-U\alpha_\tau(i\omega_n)},\tag{34}$$

$$\tilde{\Sigma}^z_\tau(i\omega_n)=-M_\tau\frac{U\alpha_\tau(i\omega_n)}{1+U\alpha_\tau(i\omega_n)}.\tag{35}$$

Equation (32) can also be written in terms of $Z_\tau(i\omega_n)$ as

$$\tilde{\Sigma}^0_\tau(i\omega_n)=-i\omega_nZ_\tau(i\omega_n)U\alpha_\tau(i\omega_n).\tag{36}$$

After an analytical continuation $i\omega_n\to\Omega+i\eta$ with $\eta\to0$, we obtain

$$\tilde{\Sigma}^0_\tau(\Omega)=-\Omega Z_\tau(\Omega)U\alpha_\tau(\Omega),\tag{37}$$

$$\tilde{\Sigma}^z_\tau(\Omega)=-\tilde{M}_\tau(\Omega)U\alpha_\tau(\Omega),\tag{38}$$

where $Z_\tau(\Omega)=1-\tilde{\Sigma}^0_\tau(\Omega)/\Omega$. Here the Ω dependent function $f(\Omega)$ represents the retarded part, i.e., $f(\Omega)=f(\Omega+i\eta)$.

We have thus explicitly

$$\alpha_\tau(\Omega)=\log\Big[1+\frac{W^2}{\tilde{M}^2_\tau(\Omega)-\Omega^2Z^2_\tau(\Omega)}\Big].\tag{39}$$

We are interested in the low energy limit, i.e., $\Omega=0$, from which we can find that $\tilde{\Sigma}^0_\tau(\Omega=0)=i\Gamma_\tau$ becomes purely imaginary, and $\delta_\tau=\tilde{\Sigma}^z_\tau(\Omega=0)$ remains a purely real quantity. We therefore can obtain a set of self-consistent equations for Γ_τ and δ_τ

$$\Gamma_\tau=\Gamma_\tau U\log\Big[1+\frac{W^2}{(M_\tau+\delta_\tau)^2+\Gamma^2_\tau}\Big],\tag{40}$$

$$\delta_\tau=-(M_\tau+\delta_\tau)U\log\Big[1+\frac{W^2}{(M_\tau+\delta_\tau)^2+\Gamma^2_\tau}\Big].\tag{41}$$

For small U, it is straightforward to figure out that the only possible solution is $\Gamma_\tau=0$. There exists a critical value $U^{\rm c}_\tau$, when $U>U^{\rm c}_\tau$, Γ_τ remains finite. The value of $U^{\rm c}_\tau$ can be calculated by

$$1=U^{\rm c}_\tau\log\Big[1+\frac{W^2}{(M_\tau+\delta_\tau)^2}\Big],\tag{42}$$

$$\delta_\tau=-(M_\tau+\delta_\tau)U^{\rm c}_\tau\log\Big[1+\frac{W^2}{(M_\tau+\delta_\tau)^2}\Big].\tag{43}$$

In fact, the critical value U_τ^c is referred to the band gap edge closing.[30] The solutions to the equations are $\delta_\tau = -M_\tau/2$ and

$$U_\tau^c = \frac{1}{\log(1+\frac{4W^2}{M_\tau^2})}. \tag{44}$$

Therefore, we can conclude that for $U \leq U_\tau^c$, $\Gamma_\tau = 0$ and $\delta_\tau \geq -M_\tau/2$. This can be referred to the weak scattering condition, under which the disorder-induced band smearing is much less than the energy separation between the valence and conduction bands, only the electron states close to the Fermi level should be considered. However, for $U > U_\tau^c$, as depicted in Fig. 3, δ_τ gets locked at $-M_\tau/2$ exactly and Γ_τ is finite, hence the effective Dirac masses are renormalized to half of their initial values, as well as the widths of the half-integer plateau of anomalous Hall conductivities.

This observation points out that the impurities do not induce the topological phase transition as it cannot change the signs of the effective Dirac masses. However, if we set the Fermi level in the gap and let its value vary in the range of $E_F \geq M_\tau/2$, a metal-insulator transition will happen with the increase of the disorder strength above a critical value $U \geq U_c$. The intrinsic total Hall conductivity is described by Eq. (16) with some disorder correction terms.[29]

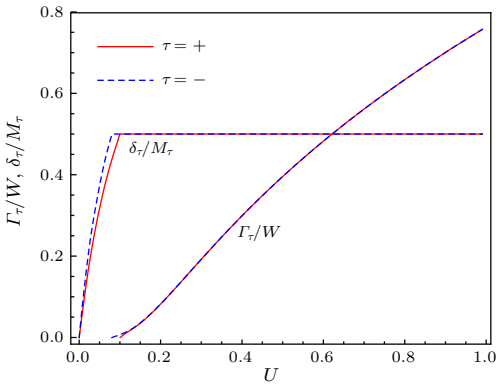

Fig. 3. Self consistent solutions for δ_τ and Γ_τ as the function of the disorder strength U for $\tilde{A}_0 = 0.3$ and $W = 2.0$, and other parameters are taken to be the same as those in Fig. 1.

In conclusion, we have systematically studied the impurity effects of the surface states in a Bi_2Se_3 thin film with applied off-resonant circular polarized light. We find that the total Hall conductivity is changed from 0 to the quantized value when the driving strength is larger than a critical value and the Fermi level lies in the band gap, indicating a topological phase transition. Based on the self-consistent Born approximation method, we find that such an impurity induced effect is surface-dependent and relies on the driving strength, the effective Dirac masses are reduced to their half values and then get fixed with the increasing disorder strength. The impurities do not affect the topological properties because the impurities cannot break any symmetry of the system. Impurity has effects on renomalizing the gap, leading to the changes of transport properties. These results are the same with graphene with a massive Hamiltonian. On the contrary, graphene with a massless Hamiltonian is different.[32] Our proposal can be confirmed using pump-probe-type measurements.[15]

Q. S. Wang and B. Zhang contributed equally to this work. Q. S. Wang would like to give the deepest thanks to Huaiqiang Wang and Mingxun Deng for their generous help. M. N. C is thankful to Habib Rostami, Wei Su on account of their fruitful discussions.

References

[1] Bernevig B A, Hughes T L and Zhang S C 2006 *Science* **314** 1757
[2] Fu L, Kane C L and Mele E J 2007 *Phys. Rev. Lett.* **98** 106803
[3] Kane C L and Mele E J 2005 *Phys. Rev. Lett.* **95** 146802
[4] Yang W M, Lin C J, Liao J and Li Y Q 2013 *Chin. Phys.* B **22** 097202
[5] Hsieh D, Qian D, Wray L, Xia Y, Hor Y S, Cava R J and Hasan M Z 2008 *Nature* **452** 970
[6] Roth A, Brune C, Buhmann H, Molenkamp L W, Maciejko J, Qi X L and Zhang S C 2009 *Science* **325** 294
[7] Feng Y, Wang Y Y, Feng X et al 2016 *Chin. Phys.* B **25** 087307
[8] Zhou B, Lu H Z, Chu R L, Shen S Q and Niu Q 2008 *Phys. Rev. Lett.* **101** 246807
[9] Lu M H, He C, Sun X C et al 2017 *Acta Phys. Sin.* **66** 224203 (in Chinese)
[10] König M, Buhmann H, Molenkamp L W, Hughes T, Liu C X, Qi X L and Zhang S C 2008 *J. Phys. Soc. Jpn.* **77** 031007
[11] König M, Buhmann H, Molenkamp L W, Hughes T, LiuC X, Qi X L and Zhang S C 2010 *Phys. Today* **63** 33
[12] König M, Wiedmann S, Brune C, Roth A, Buhmann H, Molenkamp L W, Qi X L and Zhang S C 2007 *Science* **318** 766
[13] Oka T and Aoki H 2009 *Phys. Rev.* B **79** 081406(R)
[14] Lindner N H, Refael G and Galitski V 2011 *Nat. Phys.* **7** 490
[15] Kitagawa T, Oka T, Brataas A, FuL and Demler E 2011 *Phys. Rev.* B **84** 235108
[16] Wang Y H, Steinberg H, Jarillo-Herrero P and Gedik N 2013 *Science* **342** 453
[17] Chen M N, Su W, Deng M X, Ruan J W, Luo W, Shao D X, Sheng L and Xing D Y 2016 *Phys. Rev.* B **94** 205429
[18] Liu H, Sun J T, Cheng C, Liu F and Meng S 2018 *Phys. Rev. Lett.* **120** 237403
[19] Hübener H, Sentef M A, Giovannini U D, Kemper A F and Rubio A 2017 *Nat. Commun.* **8** 13940
[20] Onoda M, Avishai Y and Nagaosa N 2007 *Phys. Rev. Lett.* **98** 076802
[21] Liu Q, Liu C X, Xu C K, Qi X L and Zhang S C 2009 *Phys. Rev. Lett.* **102** 156603
[22] Li J, Chu R L, Jain J K and Shen S Q 2009 *Phys. Rev. Lett.* **102** 136806
[23] Song J T, Liu H W, Jiang H, Sun Q F and Xie X C 2012 *Phys. Rev.* B **85** 195125
[24] Shon N H and Ando T 1998 *J. Phys. Soc. Jpn.* **67** 2421
[25] Titum P, Lindner N H, Rechtsman M C and Refael G 2015 *Phys. Rev. Lett.* **114** 056801
[26] Li H, Sheng L, Sheng D N and Xing D Y 2010 *Phys. Rev.* B **82** 165104
[27] Wang H Q, Yang Y Y, Ju Y, Sheng L and Xing D Y 2013 *Acta Phys. Sin.* **62** 037202 (in Chinese)
[28] Rammer J 2004 *Quantum Transport Theory* (Boulder: Westview Press)
[29] Sinitsyn N A, Hill J E, Min H, Sinova J and MacDonald A H 2006 *Phys. Rev. Lett.* **97** 106804
[30] Rostami H and Cappelluti E 2017 *Phys. Rev.* B **96** 054205
[31] March, D J J and Franz M 2012 *Phys. Rev.* B **86** 155146
[32] Zhai X and Jin G J 2014 *Phys. Rev.* B **89** 235416