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Surface Tension and Electrostriction in a Suspended Bridge of Dielectric Liquid

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The mechanism of the formation of a surprisingly long suspended liquid bridge subjected to a dc electric field has been intensively studied in the past few decades. However, the role of electrostriction and quantitative evaluation of surface tension in the bridge have not been evaluated. We present combined theoretical and experimental studies on this issue. Electrostriction is pointed out to be the driving force that pushes liquid upward against gravity and into the gap between two containers and forms the suspended bridge, which is within the framework of the Maxwell pressure tensor. Through a comparison between experiment and theory, the surface tension is found to play an important role in holding the long suspended bridge. Ignorance of the surface tension leads to much smaller bridge length than the experimental values. The dynamic stability of the bridge with respect to its diameter, length and conductance is also discussed.

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Water is a liquid full of mystery. It exhibits many extraordinary phenomena when subjected to a strong electric field.[1−3] Examples are emission of water jet and charged drops from Taylor cones,[2,4] non-coalescence of oppositely charged water droplets,[5−7] electro-wetting,[8] some of which have been successfully applied to the manipulation of a small amount of liquid in modern technology.[9−11] Another interesting phenomenon is suspension of an unexpectedly long water bridge, which connects two vessels, when an electric field of a few kV/cm or larger is applied (see Figs. 1(a) and 4(a)). Intuitively, one would expect the bridge collapses when the gap between the two vessels is larger than a few millimeters due to the cost of surface energy. However, the length of the bridge can extend more than a few centimeters when subjected to a large electric field.[12] This interesting phenomenon was first discovered a century ago by Armstrong,[13] and later demonstrated by Uhlig from the Laboratory of Inorganic Chemistry, ETH Honggerberg-HCI, Zurich in 2005 and Fuchs *et al.* in 2007.[12] Recently, a nanoscale water bridge restrained by an electric field was proposed for the manipulation of water in nanofluidics.[14]

A series of experiments have been carried out on characterizing the static and dynamic properties of the suspended water bridge.[12,15−18] Fuchs *et al.* suggested that the stable bridge was due to ordered dipole-dipole interaction in the highly ordered microstructure of water molecules arranged by a large electric field that compensates the cost of surface energy,[13,17] but it was later pointed out by Saija *et al.* that the micro-structures in water were not important in holding the bridge.[19] Widom *et al.* presented an elegant analysis based on the theory of the Maxwell pressure tensor in a dielectric fluid medium, and found that the floating water bridge was actually held by the electric field induced tension in water.[20] The model was then adopted by Fuchs *et al.* in their numerical simulation of the electric field in the bridge and confirmed that the gradient of the Maxwell pressure can support the bridge against gravity.[21] Under a similar theoretical framework, more efforts were involved in the evaluation of the force balance and the stability of a dielectric bridge.[22,23] In these analyses, the surface tension of the liquid was ignored. However, Aerov,[24] Skinner *et al.*[25] and Namin *et al.*[26] found that the surface tension of water plays an important role in the floating bridge. Nonetheless, an experimental evaluation of the surface tension versus the electric field induced tension in a suspended bridge is not satisfactory mainly because of the inaccuracy in measurement of electric potential on the suspended bridge in the experiment, which causes large uncertainty when comparing theory with experiment. On the other hand, although the above-mentioned theoretical analysis is within the framework of the Maxwell pressure tensor, the electrostriction term caused by the polarization of water molecules is often neglected.

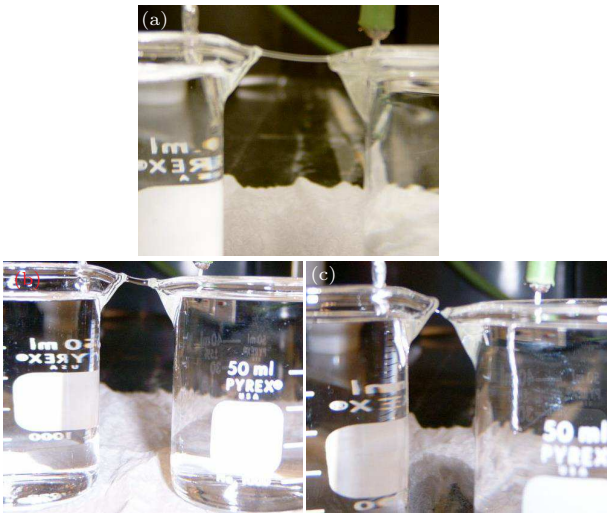

Fig. 1. Suspended liquid bridge formed by (a) water, (b) methanol, and (c) chloroform under a strong electric field.

In this Letter, we present experimental and theoretical studies on a quantitative evaluation of different contributions in a suspended dielectric liquid bridge.

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We show that the electrostrictive pressure term is crucial for the force balance in the bridge and should not be neglected. Actually, it is the electrostrictive pressure that pushes dielectric liquid into the space filled with the electric field. Using the four-probe scheme, the electric potential across the bridge is measured with high accuracy, which allows a quantitative comparison between experimental and theoretical results. We find that the surface tension is at least as important as the electrostatic pressure for dielectric liquid, such as de-ionized water, methanol, and ethanol. The stability of the bridge versus the dynamically varying radius, resistance, and the local voltage is also discussed.

We consider dielectric liquid subjected to an external electric field using the general electrostatic theory.[27] In equilibrium, the pressure on the surface and the force density in bulk must be in balance. For a curved surface, the surface tension shall be taken into account. Thus the balance equations are[27]

$$\begin{aligned} &p_0(\text{liqu}) - \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)E^2(\text{liqu}) \\ &- \frac{1}{2}\varepsilon_0\varepsilon[E_n^2(\text{liqu}) - E_t^2(\text{liqu})] \\ &= p_{\text{atm}} - \frac{1}{2}\varepsilon_0[E_n^2(\text{air}) - E_t^2(\text{air})] + \gamma\nabla\cdot\boldsymbol{n} \end{aligned} \tag{1}$$

on the surface, and

$$\boldsymbol{f} - \nabla p_0 + \rho\boldsymbol{g} = 0 \tag{2}$$

in bulk, with the electrostatic force density $\boldsymbol{f}$ for constant temperature given by[27]

$$\boldsymbol{f} = \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\nabla E^2 - \frac{1}{2}\varepsilon_0 E^2\nabla\varepsilon, \tag{3}$$

where γ is the surface tension, $\nabla\cdot\boldsymbol{n}$ is the curvature of the liquid surface, p_{atm} is the atmospheric pressure, p_0 is the pressure in liquid, $\boldsymbol{g}$ is the acceleration of gravity, ε is the dielectric constant, ρ is the mass density of liquid, and $\rho(\frac{\partial\varepsilon}{\partial\rho})$ is the electrostrictive coefficient. Note that the electrostatic force density consists of two origins. The first term in Eq. (3) is the electrostriction that is often ignored in a calculation of the electrostatic force, and the second is from the gradient of dielectric constant. We will show shortly that the former pushes liquid upwards against gravity.

The boundary condition requires continuity of $\varepsilon E_n(\text{liqu}) = E_n(\text{air})$ along the surface normal and $E_t(\text{liqu}) = E_t(\text{air})$ within the surface plane. To solve Eq. (1), one needs to know both tangential and normal components of the electric field at the boundary. Fortunately, there exist inevitably mobile ions dissolved in liquid. As a result, from $\nabla\times\boldsymbol{j} = 0$ with $\boldsymbol{j}_n = 0$ ($\boldsymbol{j}$ is the current density) and Ohm's law, the boundary condition can be simplified to $E_n = 0$ and $E_t(\text{liqu}) = E_t(\text{air}) = E$. In other words, bound charge on the surface is essentially screened by free ions in the liquid. Thus for dielectric liquid containing conductive ions, Eq. (1) can be reduced to

$$\begin{aligned} p_0(\text{liqu}) - p_{\text{atm}} = \frac{1}{2}\varepsilon_0 E^2\Big[\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big) - (\varepsilon-1)\Big] \\ + \gamma(\nabla\cdot\boldsymbol{n}). \end{aligned} \tag{4}$$

Along with the Poisson equation, Eqs. (2)–(4) lay the basics for a conductive dielectric liquid subjected to an electric field, and Eqs. (1)–(3) for an insulating liquid.

To understand why liquid is pushed upward against gravity, we first consider a well-known experiment as shown in Fig. 2(a). In this experiment, a pair of charged plates is placed partially in dielectric liquid, and then the height of liquid within the capacitor is elevated up to a certain level against gravity. In this case, the surface tension pressure is negligibly small due to the flat surface. Taking a reference point A right below the liquid surface where the electric field is absent, the pressure at point B right below the surface within the capacitor can be calculated using a line integration of Eq. (2) from A to B, which leads to

$$\begin{aligned} p_0(\text{B}) - p_0(\text{A}) &= \int_{\text{A}}^{\text{B}}\Big[\frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\nabla E^2 + \rho\boldsymbol{g}\Big]\cdot d\boldsymbol{l} \\ &= \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)E_{\text{B}}^2 - \rho g(z_{\text{B}} - z_{\text{A}}), \end{aligned} \tag{5}$$

where the gradient of the dielectric constant within liquid is neglected because the change of ε under the electric field is small. From Eq. (4), one finds that $p_0(\text{B}) > p_{\text{atm}} = p_0(\text{A})$ because the electrostriction $\rho(\frac{\partial\varepsilon}{\partial\rho})$ is larger than $\varepsilon - 1$.[27] Thus the electrostrictive pressure $\frac{1}{2}\varepsilon_0\rho(\frac{\partial\varepsilon}{\partial\rho})E_{\text{B}}^2$ is larger than $\rho g(z_{\text{B}} - z_{\text{A}})$ in Eq. (5). This means that the electrostrictive pressure pushes water upward against gravity in the capacitor. In terms of the energy argument, the electrostrictive force pushes water into the space subjected to an external electric field to better screen the field and lower the total free energy.

The above discussion demonstrates that electrostrictive force can effectively push dielectric liquid against gravity into the space filled with an electric field. In the formation of a suspended liquid bridge, the electrostrictive pressure first pushes a stream of liquid up to the edge of each container when subjected to an external electric field, and then the two streams meet and build up a suspended bridge. After formation, the stability of the bridge relies on the balance on the surface and in the bulk of the bridge simultaneously. The latter is governed by Eq. (2), while the former, after taking account of the surface tension term, can be derived from Eqs. (4) and (5) into

$$\begin{aligned} &\frac{1}{2}\varepsilon_0 E^2(\varepsilon-1) - \gamma(\nabla\cdot\boldsymbol{n}) \\ &= \frac{1}{2}\varepsilon_0 E^2\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big) - (p_0(\text{liqu}) - p_{\text{atm}}) \\ &= \frac{1}{2}\varepsilon_0 E^2\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big) - \Big[\frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)E^2 - \rho g\Delta z\Big] \\ &= \rho g\Delta z, \end{aligned} \tag{6}$$

where Δz is the height difference between a point on the surface of the bridge and a reference point at the level of the liquid surface in container where the electric field is negligible. Note that the electrostrictive term does not appear in Eq. (6). This may be the reason that the electrostrictive term is often overlooked,

because one can still achieve the same Eq. (6) at the boundary even if the electrostriction is missed.

To calculate the shape, diameter, and curvature of the suspended bridge, one needs to know the electric field distribution that is governed by the Poisson equation. Generally, there is no simple analytical solution for such coupled equations, and a numerical calculation is required. For simplicity, we assume that the cross section of the bridge is round and the electric field is uniformly distributed on the cross section.[19,24] Considering an infinitesimal slice of the liquid bridge as shown in Fig. 2(b), the electrostatic force along the tangential direction of the bridge is given by

$$\begin{aligned}\boldsymbol{F}_{\rm Elec} &= \int_V \boldsymbol{f} dV \\ &= \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\int_{\rm ends} E^2(\boldsymbol{n}\cdot dA) \\ &= \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)E_{\rm left}^2\boldsymbol{n}_{\rm left}A_{\rm left} \\ &\quad + \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)E_{\rm right}^2\boldsymbol{n}_{\rm right}A_{\rm right}, \end{aligned} \tag{7}$$

where $\boldsymbol{n}$ is the unit vector of the surface tangential of the suspended liquid bridge. The electric force along the round surface is ignored because the electric field is parallel to the surface.

Let $z = z(x)$ be the shape of the suspended liquid bridge (Fig. 2(b)). The net electrostatic force becomes

$$\begin{aligned}\boldsymbol{F}_{\rm Elec} &= \boldsymbol{F}_{\rm right} + \boldsymbol{F}_{\rm left} \\ &= \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\varphi(E_{\rm right}\cos\theta_{\rm right} - E_{\rm left}\cos\theta_{\rm left})\hat{\boldsymbol{x}} \\ &\quad + \frac{1}{2}\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\varphi(E_{\rm right}\sin\theta_{\rm right} \\ &\quad - E_{\rm left}\sin\theta_{\rm left})\hat{\boldsymbol{z}}, \end{aligned} \tag{8}$$

where θ is the angle between surface normal and the x axis. On the other hand, the forces by gravitation and surface tension are given as follows:

$$\begin{aligned} F_{\rm g} &= \rho g\int_{\rm volume} dV \\ &= \pi\rho g\int_x^{x+\delta x} r^2\sqrt{1+z'(x)^2}dx, \end{aligned} \tag{9}$$

$$F_\gamma = 2\pi r\gamma. \tag{10}$$

From Eqs. (8)–(10), the force balances along the x and z axes read

$$\begin{aligned}&\frac{1}{2}\pi\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\Big(E(x)^2r(x)^2\frac{1}{\sqrt{1+z'(x)^2}}\Big)'\delta x \\ &+ 2\pi\gamma\Big(r(x)\frac{1}{\sqrt{1+z'(x)^2}}\Big)'\delta x = 0, \end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{1}{2}\pi\varepsilon_0\rho\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)\Big(E(x)^2r(x)^2\frac{z'(x)}{\sqrt{1+z'(x)^2}}\Big)'\delta x \\ &+ 2\pi\gamma\Big(r(x)\frac{z'(x)}{\sqrt{1+z'(x)^2}}\Big)'\delta x = \rho g\delta V. \end{aligned} \tag{12}$$

To solve Eqs. (11) and (12), we also need to know the relation between $E(x)$ and $r(x)$, which can be deduced from Eq. (6). Knowing the constraint of potential on the bridge, $V_0 = \int_{-a}^{a} E(x)\sqrt{1+z'(x)^2}dx$, the three coupled equations can be solved numerically, and the calculated results are compared with the experimental ones.

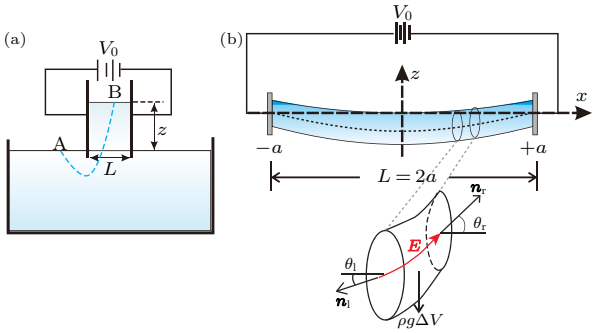

Fig. 2. (a) A plate capacitor is placed partially in a water container. The water level between plates is elevated by the electrostriction effect. (b) A suspended liquid bridge is held up by electrostriction against gravity in a capacitor with the two electrodes separated by $L = 2a$. An infinitesimal volume is enlarged in the inset with a schematic drawing of parameters used in the calculation of the balance of forces.

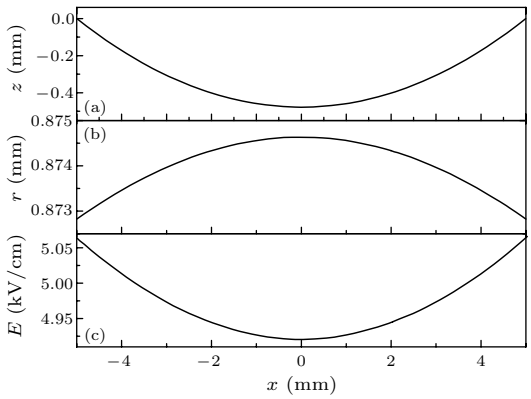

Fig. 3. Calculated curves of (a) the shape $z(x)$, (b) the radius $r(x)$, and (c) the electric field $E(x)$ of a suspended water bridge when $V_0 = 5000\,\text{V}$, $L = 10\,\text{mm}$, $\rho(\frac{\partial\varepsilon}{\partial\rho}) = 86$, $\varepsilon = 78$ and $\rho = 1.000\,\text{g/cm}^3$, and $\gamma = 0.072\,\text{N/m}$.

The calculated shape $z(x)$, radius $r(x)$ and electric field $E(x)$ of a suspended water bridge are plotted in Fig. 3, taking the input parameters of $V_0 = 5000\,\text{V}$, $L = 10\,\text{mm}$, $\rho(\frac{\partial\varepsilon}{\partial\rho}) = 86$, $\varepsilon = 78$, and $\rho = 1.000\,\text{g/cm}^3$. Under this condition, the bridge is very close to the maximum length under the given voltage. In Fig. 3(a), it shows that the bridge is slightly pulled downward by gravity, suggesting strong tangential tension along the bridge. The radius of the bridge depicted in Fig. 3(b) is 0.875 mm at the center and slightly decreases by ∼0.2% towards the two ends. Accordingly, the electric field (Fig. 3(c)) shows the opposite trend, because the total field flux (electric field times cross-section) should be constant along the bridge.

To verify the above theoretical analysis, the maximum length ($L_{\rm c}$) of the bridge versus the applied

voltage across the bridge was measured experimentally for a few dielectric liquids. Because the resistance at the two electrode/liquid interfaces is not negligible in comparison to that of the bridge, it is crucial to probe directly the voltage across the suspended bridge. We thus used the four-probe scheme as shown in Fig. 4(a), in which two electrodes are placed in the two beakers and the two voltage probes are inserted into the two ends of the liquid bridge. The inner resistance of our voltage meter is larger than 1000 MΩ. Figures 4(b)–4(d) show the results of the maximum lengths versus voltage across the bridge formed by water, methanol, and ethanol, respectively. Qualitatively, liquid with a larger dielectric constant holds a longer suspended bridge under the same of voltage. Moreover, the bridges of methanol and ethanol versus that of water also benefit from lower density in the former case.

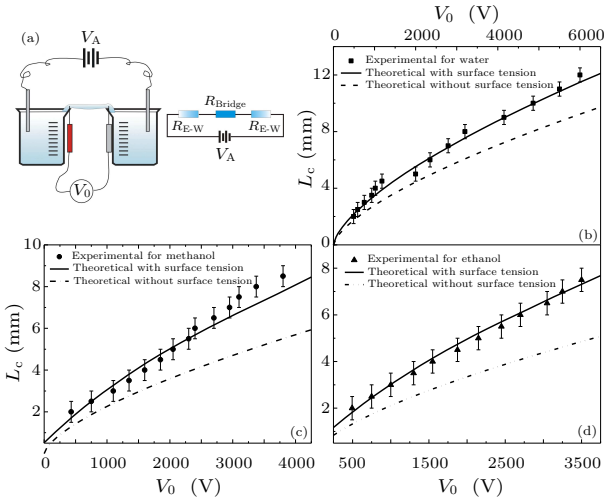

Fig. 4. (a) Schematic diagram of experimental arrangement. The experimental (dots) and calculated (solid curve) maximum length of liquid bridge versus the applied voltage for (b) water, (c) methanol, and (d) ethanol. The dashed curves represent the theoretical results when the surface tension term is ignored.

We calculate the voltage-dependent L_c based on Eqs. (6), (11), and (12), and compared it with the experiment. The theoretical results are shown in Figs. 4(b)–4(d) together with the measured data points. Taking into account the surface tension, the experiments agree nicely with the calculation for water, methanol, and ethanol. In contrast, if the surface tension term is ignored, although $z(x)$ and electric field $E(x)$ of the suspended bridge can adjust to a new balance condition, L_c (dashed curve in Fig. 4) is significantly smaller than the measured data. This result suggests that electrostatic force alone cannot hold the suspended bridge against gravitation up to the length observed in the experiment. Thus, the surface tension plays a very important role in balancing a suspended liquid bridge.

To better quantify the contribution of surface tension and electrostatic force against gravity when the bridge approaches the critical length, the ratio of the vertical force components by surface tension and electrostatic force under different voltages across the suspended bridge was calculated, which is presented in Fig. 5. Obviously, the surface tension contributes more than 60% in a wide range of applied voltage, larger than the electrostatic force.

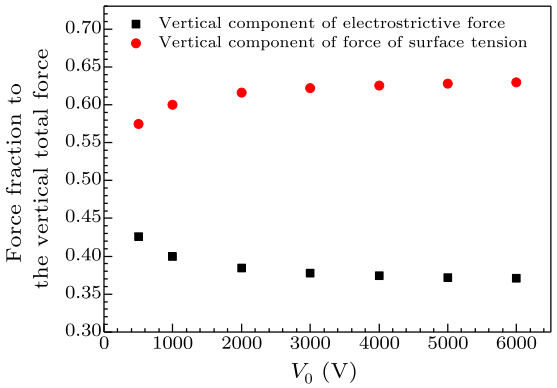

Fig. 5. The vertical force components due to surface tension (filled circles) and electrostriction (filled squares) versus the applied voltage V_0 when the bridge approaches the maximum length.

The dynamic stability of the bridge with respect to its diameter and conductivity can also be understood by taking into account the dimension parameters and resistivity of the bridge, as well as the voltage distributed on it. Because the contact resistances at the two liquid-electrode interfaces are comparable or even larger than that of the bridge, the voltage across the bridge is determined by the ratio between the former and the latter, which is given by

$$V_0 = V_A \frac{R_{Bridge}}{R_{Bridge} + 2R_{E-W}}, \tag{13}$$

where $R_{Bridge} = \int \frac{1}{\sigma A(x)} dl$ and R_{E-W} are resistances of the bridge and at the liquid-electrode interface, respectively, V_0 and V_A are voltages across the suspended bridge and output from the power supply, and σ is the conductivity of liquid. In an experiment, V_A from a voltage supplier is generally fixed, while V_0 depends on the diameter and length of the bridge, the conductivity of the liquid and R_{E-W}. Typically, R_{Bridge} and R_{E-W} are in the order of MΩ, and the ratio of the two varies between ∼0.1 and ∼1. In the initial formation process, the electrostrictive pressure pushes more liquid into the bridge. Accordingly, the diameter and the cross section increase, which results in the decrease of the bridge resistance and the smaller voltage V_0 and electric field on the bridge that weakens the electrostrictive pressure. Eventually, it reaches a balanced diameter, V_0, and shape of the bridge. When the suspended bridge is stretched longer in length, V_0 increases for its larger resistance, while its diameter adjusts so that the electric field is large enough to hold the new balance between surface tension and gravity. It is obvious that a suspended bridge breaks down when an appreciable amount of conductive salt is added into the liquid, because of the drastic decrease of R_{Bridge}.

In conclusion, we have presented a combined theoretical and experimental study and revisited the mechanism of the formation and stability of a suspended liquid bridge subjected to an electric field. The stability of the suspended bridge requires balance both

in the bulk and on the surface of the bridge. We clarify the role of electrostriction that is the driving force pushing liquid upward into the floated bridge and is the cause of inner tension to hold the balance inside the bridge. Through a comparison between experiment and theoretical analysis, surface tension is found to be crucial for the balance on the surface and in supporting the surprisingly long bridge, which is at least as important as the electrostatic force. The dynamic stability of the bridge strongly depends on its diameter, length, and the conductivity of the liquid by considering the real voltage applied on the bridge.

References

[1] Lord Rayleigh F R S 1879 *Proc. R. Soc. London Ser.* A **28** 405
[2] Taylor G 1964 *Proc. R. Soc. London Ser.* A **280** 383
[3] Ochs H T and Czys R R 1987 *Nature* **327** 606
[4] Collins R T, Jones J J, Harris M T et al 2008 *Nat. Phys.* **4** 149
[5] Bird J C, Ristenpart W D, Belmonte A et al 2009 *Phys. Rev. Lett.* **103** 164502
[6] Mugele F 2009 *Nature* **461** 356
[7] Ristenpart W D, Bird J C, Belmonte A et al 2009 *Nature* **461** 377
[8] Staicu A and Mugele F 2006 *Phys. Rev. Lett.* **97** 167801
[9] Calvert P 2001 *Chem. Mater.* **13** 3299
[10] Schneider J M 1999 *Recent Prog. Ink Jet Technol. IS&T* II 246
[11] Fenn J B, Mann M, Meng C K et al 1989 *Science* **246** 64
[12] Fuchs E C, Woisetschlager J, Gatterer K et al 2007 *J. Phys.* D **40** 6112
[13] Armstrong W G 1893 *Electr. Engineer.* **10** 154
[14] Chen J G, Wang C L, Wei N et al 2016 *Nanoscale* **8** 5676
[15] Fuchs E C, Baroni P, Bitschnau B et al 2010 *J. Phys.* D **43** 105502
[16] Fuchs E C, Bitschnau B, Woisetschlager J et al 2009 *J. Phys.* D **42** 065502
[17] Fuchs E C, Gatterer K, Holler G et al 2008 *J. Phys.* D **41** 185502
[18] Woisetschlager J, Gatterer K and Fuchs E C 2010 *Exp. Fluids* **48** 121
[19] Saija F, Aliotta F, Fontanella M E et al 2010 *J. Chem. Phys.* **133** 081104
[20] Widom A, Swain J, Silverberg J et al 2009 *Phys. Rev.* E **80** 016301
[21] Woisetschlager J, Wexler A D, Holler G et al 2012 *Exp. Fluids* **52** 193
[22] Marin A G and Lohse D 2010 *Phys. Fluids* **22** 122104
[23] Morawetz K 2012 *Phys. Rev.* E **86** 026302
[24] Aerov A A 2011 *Phys. Rev.* E **84** 036314
[25] Skinner L B, Benmore C J, Shyam B et al 2012 *Proc. Natl. Acad. Sci. USA* **109** 16463
[26] Namin R M, Lindi S A, Amjadi A et al 2013 *Phys. Rev.* E **88** 033019
[27] Landau L D, Lifshitz E M and Pitaevskii P 2007 *Electrodynamics of Continuous Media* 2nd edn (Singapore: Elsevier)