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Gravitational Perturbations in Einstein Aether Black Hole Spacetime *

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We study the gravitational perturbations in Einstein aether black hole spacetime and find that the quasinormal modes (QNMs) of the first kind of aether black hole are similar to that of a Lorentz violation (LV) model, the quantum electrodynamics (QED) extension limit of standard model extension. These similarities between completely different backgrounds may imply that LV in the gravity sector and LV in the matter sector have some connections: damping QNMs more rapidly and prolonging its oscillation period. Compared to the Schwarzschild case, the first kind of black holes have larger damping rates and the second ones have lower damping rates, and they all have smaller real oscillation frequency. These differences could be detected by the new generation of gravitational antennas.

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After the first discovery of a gravitational wave (GW) on September 14, 2015 (GW150914),[1] the laser interferometer gravitational wave observatory (LIGO) has detected GW from a binary black hole coalescence for the fourth time, on August 14, 2017 (GW170814).[2] Some of the GW detections are used to test alternative theories of gravity where Lorentz invariance (LI) is broken, which affects the dispersion relation for GW.[3] Why consider Lorentz violation (LV)? Because LI may not be an exact symmetry at all energies,[4] particularly when one considers the quantum gravity effect, it should not be applicable. The studies on LV include LV in the neutrino sector,[5] the standard-model extension,[6] LV in the non-gravity sector,[7] and the LV effect on the formation of atmospheric showers.[8] The Einstein aether theory can be considered as an effective description of Lorentz symmetry breaking in the gravity sector and has been extensively used to obtain quantitative constraints on Lorentz-violating gravity.[9]

In the Einstein aether theory, the background tensor fields break the Lorentz symmetry only down to a rotation subgroup by the existence of a preferred time direction established by aether vector u^{a}, which allows that matter fields can travel faster than the speed of light.[10] Due to the existence of the superluminal gravitational modes, thus the corresponding light cones can be completely flat, and the causality is more like that of Newtonian theory.[11] It is the universal horizons that can trap excitations traveling at arbitrarily high velocities. In 2012, two exact black hole solutions in this theory were found by Berglund *et al.*[12] In 2015, two exact charged solutions in four- and three-dimensional spacetimes were found by Ding *et al.*[13,14] Constraints on this theory were studied by Oost *et al.*[15] and, its gravitational wave was studied by Gong *et al.*[16] after GW170817.

Recently, I studied the scalar and electromagnetic perturbations of the Einstein aether black holes, and found[17] that their characteristic is similar to that of a LV model, the quantum electrodynamics (QED) extension limit of standard model extension (SME). There are two types of perturbations of a black hole: adding fields to the black hole or perturbing the black hole metric itself. The latter is gravitational perturbations. Then our goal here is to study on a gravitational perturbed black hole in the Einstein aether theory.

The static, spherically symmetric metric for the Einstein aether black hole spacetime can be written in the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

There are two kinds of exact solutions.[12,13,17] In the first case $c_{14} = 0$ and $c_{123} \neq 0$ (termed the first kind of aether black hole), we impose the following constraints[12] $0 \leq c_{14} < 2$, $2 + c_{13} + 3c_2 > 0$, $0 \leq c_{13} < 1$, where $c_{14} \equiv c_1 + c_4$, and $c_i (i = 1, 2, 3, 4)$ are coupling constants in the Einstein aether theory, thus the metric function is

$$f(r) = 1 - \frac{2M}{r} - I\left(\frac{2M}{r}\right)^4, \qquad I = \frac{27c_{13}}{256(1 - c_{13})}, \quad (2)$$

where M is the mass of the black hole spacetime. If the coefficient $c_{13} = 0$, then it reduces to the Schwarzschild black hole. In the second case $c_{14} \neq 0$, $c_{123} = 0$ (termed the second kind of aether black hole), the metric function is

$$f(r) = 1 - \frac{2M}{r} - J\left(\frac{M}{r}\right)^2, \quad J = \frac{c_{13} - \frac{1}{2}c_{14}}{1 - c_{13}}. \quad (3)$$

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If the coefficient $c_{13} = c_{14}/2$, it also reduces to the Schwarzschild black hole. It is interesting to note that it will be the Reissner–Nordström like solution with the mass charge M if J is negative. However, the condition $c_{13} \geq c_{14}/2$ should be imposed (cf. 4.26 in Ref. [13]) for the existence of a black hole solution. Plus the condition $0 \leq c_{13} < 1$, the quantity J is positive, and then the solution (3) is unlike to the Reissner–Nordström solution.

According to the Chandrasekhar designations,[18] the general form of the perturbed metric is

$$\begin{aligned} ds^2 =& e^{2\nu}dt^2 - e^{2\psi}(d\phi - \sigma dt - q_r dr - q_\theta d\theta)^2 \\ & - e^{-2\mu_2}dr^2 - e^{-2\mu_3}d\theta^2, \end{aligned} \tag{4}$$

where $e^{2\nu} = e^{2\mu_2} = f(r)$, $e^{2\mu_3} = r^2$, $e^{2\psi} = r^2\sin^2\theta$, and $\sigma = q_r = q_\theta = 0$ for the non-perturbed case. The perturbations will lead to non-vanishing values of σ, q_r, and q_θ and increments in ν, μ_2, μ_3, and ψ, which are corresponding to axial and polar perturbations, respectively. Here we consider the axial type ones, which can be reduced to the Schrodinger wave-like equations[18]

$$\frac{d^2\Psi}{dr_*^2} + [\omega^2 - V(r)]\Psi = 0, \quad dr_* = f(r)dr, \tag{5}$$

for gravitational field Ψ. The effective potentials take the form

$$V = f(r)\Big[\frac{(l+2)(l-1)+2f(r)}{r^2} - \frac{1}{r}\frac{df(r)}{dr}\Big]. \tag{6}$$

For the first kind of aether black hole, its potential is

$$\begin{aligned} V =& \Big(1-\frac{2M}{r}\Big)\Big[\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\Big] + \frac{16M^4 I}{r^6}\Big[\frac{18M}{r} \\ & + 96I\frac{M^4}{r^4} - l(l+1) - 6\Big], \end{aligned} \tag{7}$$

where the first two terms are the Schwarzschild potentials, and the rest are the aether modified terms, as shown in Fig. 1.

Obviously, if $c_{13} = 0$, the effective potentials can reduce to those of the Schwarzschild black hole. The peak value gets lower and the turning point shifts to the right with increasing c_{13}. It is similar to the Born–Infeld parameter b in the Einstein–Born–Infeld theory[19] and the Lorentz violation b_μ in the QED-extension limit of the SME theory,[20] which both decrease the potential peak. For the second kind of aether black hole, the potential shown in Fig. 1 is

$$\begin{aligned} V =& \Big(1-\frac{2M}{r}\Big)\Big[\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\Big] + \frac{M^2 J}{r^4}\Big[\frac{14M}{r} \\ & + 4J\frac{M^2}{r^2} - l(l+1) - 4\Big]. \end{aligned} \tag{8}$$

The Schrödinger-like wave Eq. (5) with the effective potential (6) containing the lapse function $f(r)$ related to the Einstein aether black holes is not solvable analytically. Then we use the sixth-order WKB approximation method[21] to evaluate the quasinormal modes (QNMs) of gravitational field perturbation. The formula for QNMs is given by

$$\begin{aligned} \omega^2 &= V_0 + \Lambda_2 - i\sqrt{-2V_0''}\Big(n + \frac{1}{2} + \sum_{i=3}^{6}\Lambda_i\Big), \\ V_0^{(m)} &= \frac{d^m V}{dr_*^m}\Big|_{r_*(r_{\rm p})}, \end{aligned} \tag{9}$$

where the constants Λ_2 and Λ_3 are obtained from Ref. [22], the constants Λ_4, Λ_5 and Λ_6 are obtained from Ref. [21] (the relation between Q_i there and V here is $Q_i = -V_0^{(i)}$), n is the overtone number, and $r_{\rm p}$ is the turning point value of polar coordinate r at which the effective potential (6) reaches its maximum.

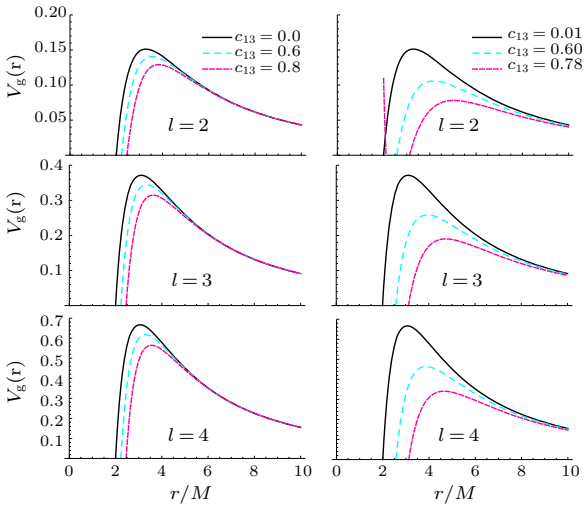

Fig. 1. The effective potentials $V_{\rm g}$ near the first kind (left) and second kind (right, with fixed $c_{14} = 0.02$) of aether black hole ($M = 1$).

The gravitational field perturbations to the first kind of aether black hole are shown in Table 1 and Fig. 2. Table 1 lists that the real and the absolute imaginary parts of frequencies are lower than the electromagnetical and scalar fields perturbations,[17] i.e., gravitational<electromagnetical<scalar. Table 1 also lists that, for fixed c_{13}, both the real part and the absolute imaginary part of frequencies increase with the angular quantum number l. For large l, the imaginary parts approach a fixed value. These properties are similar to the usual black holes, as also shown in Fig. 2. These increasing behaviors are similar to the electromagnetical field perturbations, but different from the scalar field ones whose absolute imaginary parts decrease with l.

Table 1 also lists the derivations from the Schwarzschild black hole. For $l = 2$, the decrease in Reω is about from 0.7% to 17%, while the increase in $-$Imω is about from 1% to 15%, and could be detected by the new generation of gravitational antennas. It will help us to seek LV information in nature in a low energy scale.

On the other hand, for the fixed angular number l, or overtone number n with different c_{13} (small c_{13}), Table 1 lists that the real parts of frequencies decrease, and the absolute imaginary ones increase with c_{13} firstly to $c_{13} = 0.9$ and then decrease on the contrary, which is similar to the scalar and electromagnetic perturbations.[17] It is different from that of the non-reduced aether black hole,[23] where both increase with c_1.

Table 1. The gravitational QNMs with $n = 0$ of the first kind of aether black hole.

c_{13}	$M\omega(l=2)$	$M\omega(l=3)$	$M\omega(l=4)$	$M\omega(l=5)$	$M\omega(l=6)$
0.00	$0.373619 - 0.088891i$	$0.599443 - 0.092703i$	$0.809178 - 0.094164i$	$1.012300 - 0.094871i$	$1.212010 - 0.095266i$
0.15	$0.371123 - 0.089949i$	$0.595985 - 0.093676i$	$0.804653 - 0.095142i$	$1.006720 - 0.095852i$	$1.205390 - 0.096250i$
0.30	$0.367765 - 0.091235i$	$0.591354 - 0.094848i$	$0.798599 - 0.096326i$	$0.999260 - 0.097042i$	$1.196530 - 0.097444i$
0.45	$0.363006 - 0.092838i$	$0.584808 - 0.096284i$	$0.790050 - 0.097785i$	$0.988731 - 0.098514i$	$1.184040 - 0.098924i$
0.60	$0.355705 - 0.094918i$	$0.574769 - 0.098074i$	$0.776938 - 0.099617i$	$0.972582 - 0.100371i$	$1.164880 - 0.100795i$
0.75	$0.342863 - 0.097769i$	$0.557015 - 0.100290i$	$0.753718 - 0.101918i$	$0.943965 - 0.102720i$	$1.130900 - 0.103173i$
0.90	$0.311907 - 0.101800i$	$0.513073 - 0.102246i$	$0.696009 - 0.104045i$	$0.872663 - 0.104961i$	$1.046130 - 0.105484i$

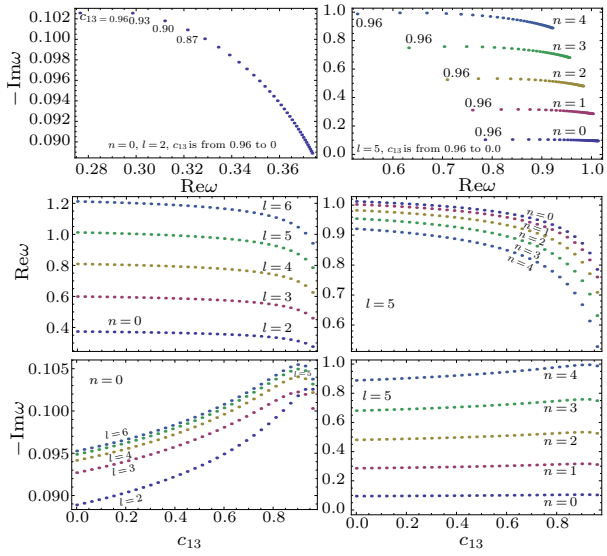

Fig. 2. The gravitational QNMs with fixed $n = 0$ (left) and $l = 5$ (right) of the first kind of aether black hole.

Compared to the LV model, the QED-extension limit of SME (for more details, see Ref. [17]), the above gravitational field QNM properties with c_{13} are similar to the Dirac field QNMs with LV coefficient b,[20] i.e., the real part decreases while the absolute imaginary part increases with the given LV coefficient, which are like their scalar and electromagnetical field counterparts.[17] These similarities between different backgrounds may imply some common properties of LV coefficient on QNMs, i.e., in the presence of LV, the perturbation field oscillation damps more rapidly, and its period becomes longer.

The gravitational field perturbations to the second kind of aether black hole with fixed $c_{14} = 0.02$ are shown in Table 2 and Fig. 3. Table 2 lists that, like the first kind of aether black hole, the real and the absolute imaginary parts of frequencies are lower than the electromagnetical and scalar fields perturbations.[17] Table 2 also lists that, for fixed c_{13}, both the real and absolute imaginary parts of frequencies increase with the angular quantum number l. For large l, the imaginary parts approach a fixed value, which is similar to the usual black holes, as also shown in Fig. 3. Therefore, these l-dependant behaviors are similar to the first kind of aehter black hole.

Table 2 also lists the derivations from the Schwarzschild black hole. For $l = 2$, the decrease in Reω is about from 3% to 45%, while the decrease in $-$Imω is about from 1% to 31%, both of which are larger than the first kind of aether black hole.

Table 2. The gravitational QNMs with $n = 0$ of the second kind of aether black hole with fixed $c_{14} = 0.02$.

c_{13}	$M\omega(l=2)$	$M\omega(l=3)$	$M\omega(l=4)$	$M\omega(l=5)$	$M\omega(l=6)$
0.01	$0.373619 - 0.088891i$	$0.599443 - 0.092703i$	$0.809178 - 0.094164i$	$1.012300 - 0.094871i$	$1.212010 - 0.095266i$
0.15	$0.363508 - 0.087996i$	$0.583585 - 0.091775i$	$0.787938 - 0.093240i$	$0.985828 - 0.093950i$	$1.180390 - 0.094348i$
0.30	$0.350114 - 0.086611i$	$0.562563 - 0.090330i$	$0.759762 - 0.091795i$	$0.950709 - 0.092508i$	$1.138430 - 0.092909i$
0.45	$0.332795 - 0.084518i$	$0.535354 - 0.088129i$	$0.723264 - 0.089587i$	$0.905196 - 0.090301i$	$1.084040 - 0.090703i$
0.60	$0.309073 - 0.081156i$	$0.498024 - 0.084570i$	$0.673136 - 0.086006i$	$0.842654 - 0.086713i$	$1.009280 - 0.087113i$
0.75	$0.273208 - 0.075123i$	$0.441421 - 0.078143i$	$0.597022 - 0.079520i$	$0.747622 - 0.080205i$	$0.895630 - 0.080593i$
0.90	$0.205326 - 0.061015i$	$0.333580 - 0.063088i$	$0.451685 - 0.064270i$	$0.565957 - 0.064866i$	$0.678232 - 0.065208i$

On the other hand, for the fixed angular number l, or overtone number n, Table 2 and Fig. 3 show that both the real and the absolute imaginary parts of frequencies decrease with c_{13}, which is completely different from that of the non-reduced aether black hole,[23] where both increase with c_1. This property of both decreasing is similar to that of the Einstein–Born–Infeld black hole.[19,24]

In summary, for the first kind of aether black hole, the real part of gravitational QNMs becomes smaller with increasing c_{13}. The absolute value of the imaginary part of QNMs becomes larger with smaller c_{13}. These properties with c_{13} are similar to the behavior of Dirac QNMs with LV coefficient b.[20] These similarities between different backgrounds may imply some connections between the Einstein aether theory and the QED-extension limit of SME.

For the second kind of aether black hole, both de-

crease with c_{13}, which are similar to the scalar field QNMs of the Einstein–Born–Infeld black holes with the Born–Infeld parameter b.[19]

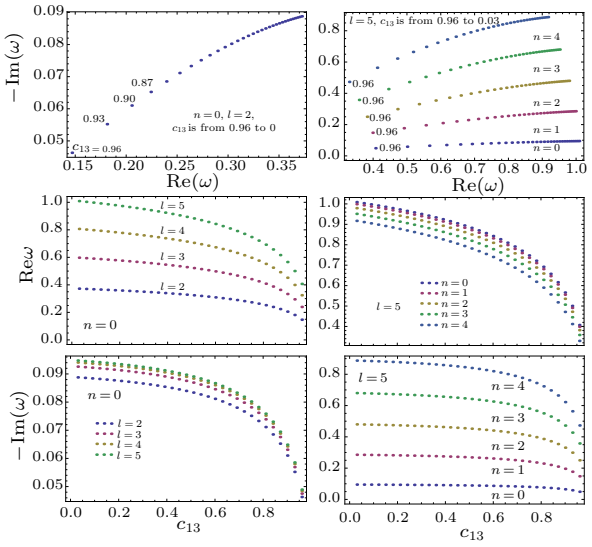

Fig. 3. The gravitational QNMs with fixed $n = 0$ (left) and $l = 5$ (right) of the second kind of aether black hole.

Compared to Ref. [17], the real and the absolute imaginary parts of QNMs are lower than the electromagnetical and scalar field perturbations, i.e., gravitational<electromagnetical<scalar. For fixed c_{13}, like the electromagnetical field, both the real part and the absolute imaginary part of frequencies increase with l.

Compared to the Schwarzschild black hole, the first kind of aether black holes have a larger damping rate and the second ones have a lower damping rate. They all have a smaller real oscillation frequency of QNMs. If LV is not very small, the derivation of QNMs from the Schwarzschild values may be observed in near future gravitational wave events.

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