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# Nonadiabatic Population Transfer in a Tangent-Pulse Driven Quantum Model

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*Fine control of the dynamics of a quantum system is the key element to perform quantum information processing and coherent manipulations for atomic and molecular systems. We propose a control protocol using a tangent-pulse driven model and demonstrate that it indicates a desirable design, i.e., of being both fast and accurate for population transfer. As opposed to other existing strategies, a remarkable character of the present scheme is that high velocity of the nonadiabatic evolution itself not only will not lead to unwanted transitions but also can suppress the error caused by the truncation of the driving pulse.*

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Dynamical control of quantum systems that undergo avoided level crossings plays an important role in many areas of physics as well as some of quantum chemistry.[1,2] A well-known paradigm is that of the Landau–Zener (LZ) model[3,4] and its multi-state extensions,[5–11] which describe the evolution of quantum states in the presence of a linearly changed external field. Owing to its form of the simplest driving field, the LZ model has become one of the mostly investigated explicitly time-dependent quantum systems and is exploited as a tool for controlling the population in various physical systems, e.g., optical systems,[12–15] semiconductor quantum dots,[16–18] and superconducting qubits.[19–22]

The standard LZ model is defined in an infinite time domain and the general solution, e.g., of the two-level case, is described by Weber's parabolic cylinder functions.[3,4] In the case of ideal driving, complete population transfer could be achieved through the adiabatic passage of the linear LZ sweep. As well as in other analogs of the LZ protocol, population transfer via avoided level crossings possesses the advantage of being insensitive to the pulse area in comparison with the usual resonant $\pi$-pulse scheme.[23] On the other hand, as the adiabatic evolution is often required in these protocols, it indicates a slow speed. This is an issue related to the generic quandary of quantum manipulations, i.e., how they could be implemented accurately and rapidly. Various strategies have been proposed to tackle this issue, such as the composite adiabatic passage technique[24,25] and transitionless quantum driving in terms of the counter-diabatic protocol[26–30] or the short-cut protocol.[31–33] In most of these cases, a complicated design of the driving field, e.g., an auxiliary counter-diabatic field of time-dependent form in the transition-less driving algorithm, is required to achieve the high-fidelity population transfer.

In this Letter, we propose a tangent-pulse-driven quantum model for nonadiabatic population transfer and demonstrate that, conditioned to a matching sweep frequency, the transition dynamics of the system, including the two-level case as well as its multi-level extension, is fully controllable in an analytical manner. Contrary to those protocols based on the transition-less driving algorithm, no auxiliary field is needed in the scheme and the nonadiabatic dynamics of the model itself that undergoes avoided level crossings could realize complete population transfer. Not only that, but for the imperfect pulsing process with truncation, we show that the high velocity of the nonadiabatic evolution of the proposed model can suppress the error caused by the cutoff of the driving field. As the protocol involves only a matching condition about the fixed sweep frequency, it stands for an ideal design for fast and accurate population control and may substitute the LZ model for potential applications.

The model we are considering is described by the following Hamiltonian

$$H(t) \equiv \Omega(t)\cdot \boldsymbol{J} = \eta_1 J_x + \eta_2 \tan(\gamma t) J_z, \qquad (1)$$

where $\boldsymbol{J}$ denotes the angular-momentum operator with the components satisfying $[J_i, J_j] = i\varepsilon_{ijk}J_k$. The $z$ component of the external field $\Omega_z(t)$ assumes a tangent-shape form (see Fig. 1(a)) with $t \in (-\frac{\pi}{2\gamma}, \frac{\pi}{2\gamma})$, and $\eta_{1,2}$ and the sweep frequency $\gamma$ are fixed constants which satisfy the matching condition (setting $\hbar = 1$)

$$\eta_1^2 = \eta_2^2 + \gamma^2. \qquad (2)$$

Given a general tangent-shape pulse, e.g., $\tilde{\Omega}(t) = (\tilde{\eta}_1, 0, \tilde{\eta}_2 \tan(\gamma t))$, the above condition for the frequency could be fulfilled either by tuning the $x$ component so that $\tilde{\eta}_1 \to \eta_1 = \sqrt{\tilde{\eta}_2^2 + \gamma^2}$, or by an overall modulation on the intensity of the field, $\Omega(t) \equiv \gamma\tilde{\Omega}(t)/\sqrt{\tilde{\eta}_1^2 - \tilde{\eta}_2^2}$, provided that $\tilde{\eta}_1 > \tilde{\eta}_2$.

Firstly, we show that the dynamics of the system governed by the Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H(t)|\psi(t)\rangle \qquad (3)$$

could be solved analytically. To this goal, we invoke a time-dependent transformation on the wavefunction $|\psi(t)\rangle = G(t)|\psi^{\rm g}(t)\rangle$, where $G(t) = e^{i\varphi J_z}e^{i(\gamma t+\pi/2)J_y}$ with $\varphi = -\arcsin\frac{\gamma}{\eta_1}$. In the rotating frame defined

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by $G(t)$, the state $|\psi^{\rm g}(t)\rangle$ satisfies a new Schrödinger equation, $i\partial_t|\psi^{\rm g}(t)\rangle = H^{\rm g}(t)|\psi^{\rm g}(t)\rangle$, in which the effective Hamiltonian $H^{\rm g}(t)$ is obtained as

$$\begin{aligned} H^{\rm g}(t) &= G^{\dagger}(t)H(t)G(t) - iG^{\dagger}(t)\partial_t G(t) \\ &= -\eta_2 \cos^{-1}(\gamma t) J_z. \end{aligned} \tag{4}$$

This simple form of $H^{\rm g}(t)$, i.e., containing only the Cartan generator $J_z$ but with vanishing other generators is what one usually expects in the algebraic approach to this kind of time-dependent quantum systems. The previous examples successfully managed by the algebraic method mostly involve driving fields with coordinate time-varying components.[34–36] The present system is distinctly different since there is only one component depending on time and the corresponding sweep generates a typical transition dynamics with avoided level crossings which is in close analogy to that of the LZ model.

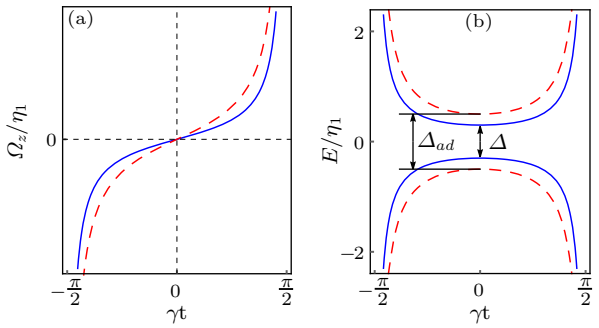


**Fig. 1.** Adiabatic versus nonadiabatic scanning processes of the tangent-pulse driven model (1) with a matching sweep frequency: (a) comparison of the time dependency of the $z$-component of the driving field between the adiabatic sweep with $\gamma/\eta_1 \to 0$ (dashed line) and the nonadiabatic sweep with $\gamma/\eta_1 = 0.8$ (solid line); (b) the corresponding adiabatic (dashed line) and diabatic (solid line) energy levels, $E^{\rm ad}_{\pm}(t)$ and $E_{\pm}(t)$ over the constant $\eta_1$ of the model with $j = \frac{1}{2}$, by which the associated gaps at the crossing point $t = 0$ are shown to be $\Delta_{\rm ad} = 1$ and $\Delta = \eta_2/\eta_1 = 0.6$, respectively.

To proceed, noticing that $H^{\rm g}(t)$ is Abelian along the time, the time evolution of the system in the rotating frame is described by the operator

$$U^{\rm g}(t,t_0) = \exp[-i\int_{t_0}^{t} H^{\rm g}(t')dt'] = \exp[i\Theta(t,t_0)J_z], \tag{5}$$

where $\Theta(t,t_0) = \eta_2 \int_{t_0}^{t} \cos^{-1}(\gamma t')dt'$. That is to say, the Schrödinger equation in this representation possesses a stationary-state solution $|\psi^{\rm g}_m(t)\rangle = e^{im\Theta(t,t_0)}|m\rangle$ in which $|m\rangle$ denotes the eigenstate of $J_z$ with magnetic quantum number $m$. Hence the time evolution operator of the original Schrödinger Eq. (3) is given by

$$U(t,t_0) = G(t)U^{\rm g}(t,t_0)G^{\dagger}(t_0), \tag{6}$$

and the basic solution to its wavefunction, the so-called diabatic base, is obtained as

$$\begin{aligned} |\psi_m(t)\rangle &= G(t)|\psi^{\rm g}_m(t)\rangle \\ &= e^{im\Theta(t,t_0)} \sum_{m'} \mathcal{D}^{j}_{m'm}\Big(\gamma t + \frac{\pi}{2}\Big) e^{im'\varphi}|m'\rangle, \end{aligned} \tag{7}$$

where $\mathcal{D}^{j}_{m'm}(\phi) \equiv \langle m'|e^{i\phi J_y}|m\rangle$, and the index $m'$ of the summation is taken over $-j, -j+1, \ldots, j$ with $j$ being the azimuthal quantum number. Subsequently, the diabatic energy levels of the system can be calculated according to $E_m(t) \equiv \langle\psi_m(t)|H(t)|\psi_m(t)\rangle$.

The result described above is applicable to the general $SU(2)$ system, i.e., to the angular momentum with arbitrary quantum number $j$. For the simplest two-level ($j = \frac{1}{2}$) system, there is

$$\mathcal{D}^{\frac{1}{2}}(\phi) = \begin{pmatrix} \cos\frac{\phi}{2} & \sin\frac{\phi}{2} \\ -\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{pmatrix}, \tag{8}$$

thus the diabatic bases are shown to be

$$|\psi_{\pm\frac{1}{2}}(t)\rangle = e^{\pm\frac{i}{2}\Theta(t,t_0)} \begin{pmatrix} \cos(\frac{\gamma t}{2} \pm \frac{\pi}{4}) e^{i\varphi/2} \\ -\sin(\frac{\gamma t}{2} \pm \frac{\pi}{4}) e^{-i\varphi/2} \end{pmatrix}. \tag{9}$$

The corresponding diabatic energy levels are obtained as $E_{\pm}(t) = \mp\frac{1}{2}\eta_2 \cos^{-1}(\gamma t)$, which together with the adiabatic $E^{\rm ad}_{\pm}(t) = \mp\frac{1}{2}\eta_1 \cos^{-1}(\gamma t)$ are depicted in Fig. 1(b). It is seen that at the beginning and the ending points of the sweep, the diabatic basis state $|\psi_{\frac{1}{2}}(t)\rangle$ tends to $|+\rangle = \binom{1}{0}$ as $t \to -\frac{\pi}{2\gamma}$ and to $|-\rangle = \binom{0}{1}$ as $t \to +\frac{\pi}{2\gamma}$ (up to a phase term); and the basis state $|\psi_{-\frac{1}{2}}(t)\rangle$ tends reciprocally to $|-\rangle$ or $|+\rangle$ as $t \to \mp\frac{\pi}{2\gamma}$. Therefore the complete population transfer $|+\rangle \leftrightarrow |-\rangle$ could be realized through the ideal tangent-pulse sweep, whether the driving process is adiabatic or nonadiabatic.

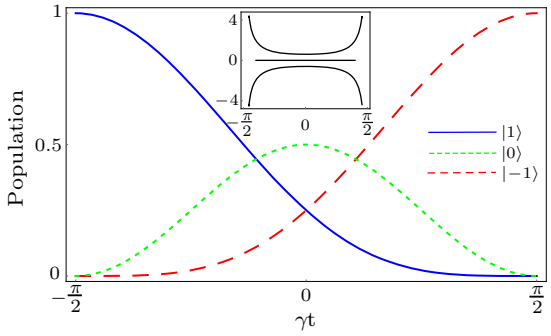


**Fig. 2.** Nonadiabatic population transfer along the time evolution in the tangent-pulse driven model (1) with $j = 1$. The initial state is in $|1\rangle$. The inset describes the diabatic levels $E_m(t)/\eta_1$ ($m = 1, 0, -1$) with $\gamma/\eta_1 = 0.8$ and they exhibit the avoided crossing at $t = 0$.

The Hamiltonian (1) with high quantum number $j$ accounts for a multi-level extension of the sweep protocol associated with avoided crossings. In particular, the exact solvability of the system makes it an ideal scenario to manifest the behavior of the wavefunction undergoing multichannel transitions. Firstly, it is direct to show that the driving process results in a transition from the state $|m\rangle$ to $|-m\rangle$ for all possible

$m = -j, -j+1, \ldots, j$. According to Eq. (6), the evolution operator generated by the overall sweep during $t \in (-\tau, \tau)$ with $\tau \to \frac{\pi}{2\gamma}$ takes the form of

$$U(\tau, -\tau) = e^{i\varphi J_z} e^{i\pi J_y} e^{i[\Theta(\tau)-\varphi]J_z}, \tag{10}$$

where $\Theta(\tau) = \eta_2 \int_{-\tau}^{\tau} \cos^{-1}(\gamma t) dt$. In view of the transformation $e^{i\pi J_y}|m\rangle \to |-m\rangle$, the transition probability between any two states $|m\rangle$ and $|m'\rangle$ is immediately yielded: $|\langle m'|U(\tau, -\tau)|m\rangle|^2 = \delta_{m',-m}$. To illustrate further the multichannel transition process of the wavefunction, we resort to the three-level case with $j = 1$. The corresponding matrix $\mathcal{D}^j_{m'm}(\phi)$ is shown as

$$\mathcal{D}^1(\phi) = \begin{pmatrix} \cos^2\frac{\phi}{2} & \frac{\sin\phi}{\sqrt{2}} & \sin^2\frac{\phi}{2} \\ -\frac{\sin\phi}{\sqrt{2}} & \cos\phi & \frac{\sin\phi}{\sqrt{2}} \\ \sin^2\frac{\phi}{2} & -\frac{\sin\phi}{\sqrt{2}} & \cos^2\frac{\phi}{2} \end{pmatrix}. \tag{11}$$

By taking $\phi = \gamma t + \frac{\pi}{2}$ and substituting the above expression into Eq. (7), one obtains the diabatic bases for the three-level system

$$\begin{aligned} |\psi_0(t)\rangle =& \frac{\sqrt{2}}{2}\cos(\gamma t)e^{i\varphi}|1\rangle - \sin(\gamma t)|0\rangle \\ & - \frac{\sqrt{2}}{2}\cos(\gamma t)e^{-i\varphi}|-1\rangle, \\ |\psi_{\pm 1}(t)\rangle =& e^{\pm i\Theta(t,t_0)}\Big\{\frac{1}{2}[1 \mp \sin(\gamma t)]e^{i\varphi}|1\rangle \\ & \mp \frac{\sqrt{2}}{2}\cos(\gamma t)|0\rangle \\ & + \frac{1}{2}[1 \pm \sin(\gamma t)]e^{-i\varphi}|-1\rangle\Big\}. \end{aligned} \tag{12}$$

Starting from an initial state $|\psi(-\tau)\rangle = |1\rangle$, the wavefunction will undergo transitions via the channels $|1\rangle \to |-1\rangle$ and $|1\rangle \to |0\rangle \to |-1\rangle$, and will evolve precisely to the ending state $|-1\rangle$ up to a phase factor. We depict in Fig. 2 the corresponding population transfer along the time evolution of the driving process. It is seen that the maximal population in the intermediate state $|0\rangle$ is $p = \frac{1}{2}$ which occurs at the point $t = 0$.

In the practical scanning process, the driving field has finite intensity and the cutoff of the sweep will cause some losses of the transition probability. Consider that the truncation of the field pulse is symmetric. For the evolution generated during the period $t \in [-\tau_c, \tau_c]$, the goal now is to find the matrix of transition probabilities $\hat{P}(\tau_c)$, with the matrix element defined as $P_{m'm}(\tau_c) \equiv |\langle m'|U(\tau_c, -\tau_c)|m\rangle|^2$. Here $\delta \equiv \frac{\pi}{2} - \gamma\tau_c$ denotes the deviation of the maximal phase angle from that of the ideal tangent pulse. Since the evolution operator $U(\tau_c, -\tau_c)$ is specified explicitly in Eq. (6), $P_{m'm}(\tau_c)$ could be calculated directly via

$$P_{m'm}(\tau_c) = |\langle m'|e^{i(\pi-\delta)J_y} e^{i\Theta(\tau_c)J_z} e^{-i\delta J_y}|m\rangle|^2, \tag{13}$$

where $\Theta(\tau_c) \equiv \Theta(\tau_c, -\tau_c)$. As the representatives $\mathcal{D}^j_{k'k}(-\delta) = \langle k'|e^{-i\delta J_y}|k\rangle$ for $j = \frac{1}{2}$ and $j = 1$ are already given in Eqs. (8) and (11), the corresponding matrices of the transition probabilities of these two cases are readily obtained. In the following we focus on the influence of the sweep frequency $\gamma$ on the transition probability and demonstrate that the high velocity of the scanning rate could suppress the error caused by the truncation of the driving pulse. We stress that this is a general result for the described driven model, although it will be illustrated in the following via the simplest case of $j = \frac{1}{2}$.

Explicitly, for the two-level system undergoing an imperfect driving process with the symmetric truncation, the probability of the transition $|+\rangle \leftrightarrow |-\rangle$ is given by

$$P_{-+} = P_{+-} = 1 - \cos^2\frac{\Theta(\tau_c)}{2}\sin^2\delta. \tag{14}$$

To reveal the influence of the sweep frequency $\gamma$ on the population transfer, let us denote by $\delta_0 = \arctan\frac{\Omega_x}{\Omega_z(\tau_c)}$ the deviation of the pulsed field vector $\Omega(t)$ from the ideal $z$ axis at the points $t = \pm\tau_c$. In view of $\Omega_x = \eta_1$, $\Omega_z(\tau_c) = \eta_2\tan(\gamma\tau_c)$, and the matching condition of the frequency $\eta_2^2 = \eta_1^2 - \gamma^2$, one has

$$\tan\delta = \sqrt{1 - (\gamma/\eta_1)^2}\tan\delta_0. \tag{15}$$

Equations (14) and (15) indicate that, for the fixed values of $\Omega_x$ and $\Omega_z(\tau_c)$, the cutoff error to the transition probability dominated by $\delta$ could be suppressed by increasing the sweep frequency $\gamma$. In particular, as long as the frequency is modulated within the matching condition and satisfies $\gamma/\eta_1 \to 1$, the high-fidelity population transfer could be achieved by the nonadiabatic evolution even when there exists dramatic truncation of the driving field, i.e., with a finite deviation $\delta_0$ of the field vector from the $z$ axis at $t = \pm\tau_c$ (see Fig. 3).

The above characterized dynamics of the tangent protocol, in which the population transfer could be enhanced by accelerating the scanning rate, implies a rare and intriguing character that has not ever been found in other existing quantum driven models. Firstly, in the linear LZ protocol the nonadiabaticity of the evolution is known to induce unwanted transitions to the population transfer. A second example appropriately serving as a reference is the counter-diabatic protocol based on the transition-less quantum driving. Given a Hamiltonian $H(t)$, the protocol cancels the nonadiabatic part of the evolution under $H(t)$ by introducing an auxiliary counter-diabatic field and ensures that the system evolving under the total Hamiltonian $H_{cd}(t) = H(t) + H'(t)$ always remains in the instantaneous eigenstate of $H(t)$. For the Hamiltonian specified by Eq. (1), there is $H_{cd}(t) = H(t) + \dot{\delta}_{cd}(t)J_y$ and the corresponding time evolution operator reads

$$U_{cd}(t, t_0) = e^{i\delta_{cd}(t)J_y} e^{i\Theta_{cd}(t,t_0)J_z} e^{-i\delta_{cd}(t_0)J_y}, \tag{16}$$

where $\delta_{cd}(t) = \pi - \arccos\frac{\Omega_z(t)}{\Omega(t)}$ and $\Theta_{cd}(t, t_0) = \int_{t_0}^{t}\Omega(\tau)d\tau$ with $\Omega(t) = \sqrt{\Omega_x^2 + \Omega_z^2(t)}$.

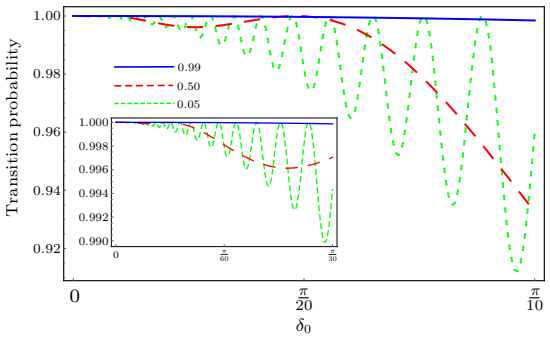


**Fig. 3.** Transition probabilities yielded by the imperfect pulsing processes with truncation. The error caused by the cutoff of the driving field is shown to be suppressed when the scanning rate of the protocol increases. An ultrahigh fidelity ($1 - P_{-+} \sim 10^{-4}$) is attainable through the nonadiabatic evolution ($\gamma/\eta_1 = 0.99$) even when there is dramatic truncation of the driving field ($\delta_0 \sim \frac{\pi}{30}$).

Consider a similar truncation of the scanning process of the counter-diabatic protocol, i.e., $t \in [-\tau_c, \tau_c]$ with $\gamma\tau_c = \frac{\pi}{2} - \delta$. The matrix of transition probabilities related to the protocol is specified accordingly as $P^{cd}_{m'm}(\tau_c) \equiv |\langle m'|U_{cd}(\tau_c, -\tau_c)|m\rangle|^2$, where $U_{cd}(\tau_c, -\tau_c)$ denotes the corresponding evolution operator described in Eq. (16). In view of $\delta_{cd}(-\tau_c) = \delta_0$ and $\delta_{cd}(\tau_c) = \pi - \delta_0$, it is readily recognized that for the two-level system the transition probability reads

$$\begin{aligned} P^{cd}_{-+}(\tau_c) &= |\langle -|e^{i(\pi-\delta_0)J_y} e^{i\Theta_{cd}(\tau_c)J_z} e^{-i\delta_0 J_y}|+\rangle|^2 \\ &= 1 - \cos^2\frac{\Theta_{cd}(\tau_c)}{2}\sin^2\delta_0, \end{aligned} \tag{17}$$

where $\Theta_{cd}(\tau_c) \equiv \Theta_{cd}(\tau_c, -\tau_c)$. Compared with Eq. (14), the critical difference is that the cutoff error in the counter-diabatic protocol is dominated by $\delta_0$ instead of $\delta$ that was shown in the previously described protocol. Indeed, since the transition-less driving algorithm pursues the evolution in such a way that the state remains in the adiabatic state of $H(t)$, it turns out to be a general result that the cutoff error in all its resulting protocols is independent of the sweep frequency.

In summary, we have proposed a design for the population control which unifies the high operation rate and robustness as its intrinsic character. Its simple form of the tangent-pulse sweep could rival the linear LZ protocol. We have shown that the generated dynamics associated with avoided level crossings, whatever adiabatic or nonadiabatic, could yield the desired population transfer. Compared with those existing nonadiabatic protocols based on the transition-less driving algorithm, the present scheme possesses distinct superiorities: (1) no auxiliary time-varying field but a simple matching condition for the sweep frequency is involved; and (2) for imperfect pulsing processes with truncation, the cutoff error could be suppressed by enhancing the scanning rate of the protocol. We emphasize that the matching condition of the fixed frequency in the design does not add technical complexity to the tangent-pulse driving and is readily achievable for experimental implementation, e.g., by the Bose–Einstein condensates in an accelerated optical lattice[37] or by the nitrogen-vacancy center in diamond.[28] In particular, for the electron spin of the nitrogen-vacancy center, the coherent time could be 500 μs. If the field strength $\eta_1$ is set as 0.1 MHz and $\gamma$ a bit less, the duration of the sweep is about $10\pi$ μs, which can be well implemented within the decoherence time. Finally, we expect that the finding of the exactly solvable tangent-pulse driven quantum model is helpful to advance further the study of the issue of the solvability for more general time-dependent quantum systems.

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