Quasinormal Modes of a Noncommutative-Geometry-Inspired Schwarzschild Black Hole *

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(Received 18 August 2017)

The quasinormal modes (QNMs) of massless scalar field perturbation in a noncommutative-geometry-inspired Schwarzschild black hole spacetime are studied using the third-order Wentzel–Kramers–Brillouin approximative approach. The result shows that the noncommutative parameter plays an important role for the quasinormal (QNM) frequencies.

PACS: 04.70.−s, 04.30.−w DOI: 10.1088/0256-307X/35/1/010401

In recent years, noncommutative-geometry-inspired black holes aroused the interest of numerous researchers because coordinate noncommutativity is supposed to remove the so-called paradox of black hole information loss.[1] Nicolini *et al.* first found a noncommutative-geometry-inspired Schwarzschild black hole solution in four dimensions.[2] In their study, the effect of noncommutativity is incorporated in the mass term of the gravitational source taking the mass density to be a Gaussian mass distribution instead of the conventional Dirac delta function. Subsequently, the model was extended to include the electric charge,[3] rotation[4] and extra-spatial dimensions.[5,6] Since then, many works on noncommutative-geometry-inspired black holes have been carried out[7−23] (for a comprehensive review of noncommutative-geometry-inspired black holes, see Ref. [24] and references therein).

On the other hand, the quasinormal modes (QNMs) in the black hole spacetime have been an attractive subject for a long time since QNMs depend only on the macroscopic properties of the black hole, and are thus widely regarded as a direct way to identify the existence of the black hole through measuring QNM frequencies using gravitational wave detectors in the near future. In addition, it is believed that QNMs have close connection with AdS/CFT correspondence and loop quantum gravity.

QNMs were pointed out by Vishveshwara in Ref. [25], while QNM frequencies were coined by Press.[26] So far, various numerical, analytical and semianalytical methods to compute QNM frequencies have been developed, such as the Mashhood method, the Wentzel–Kramers–Brillouin (WKB) approximative approach, integration of the wavelike equations, phase integral approaches, method of Laplace transforms, the Frobenius method, method of continued fractions, monodromy method, etc.[27−35] (for a detailed understanding of QNMs of black holes, see Refs. [36–38] and references therein). Among these methods, the WKB approximative approach is a semi-analytical approach used in the calculation of the QNM frequencies of black holes frequently[39−64] (this method was applied for the first time by Schutz and Will,[28] and was then extended to the third and sixth orders by Iyer and Will[29] and Konoplya[31] to increase accuracy). However, to the best of my knowledge, the WKB method has not been applied to noncommutative-geometry-inspired black holes.

In this Letter, we study QNMs of the noncommutative-geometry-inspired Schwarzschild black hole using the WKB approximation. The purpose is to explore the effect of the noncommutative parameter on QNMs. The result shows that the noncommutative parameter has an important effect on QNMs of the noncommutative-geometry-inspired Schwarzschild black hole.

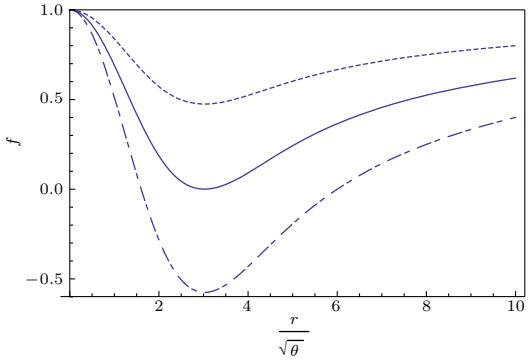

Fig. 1. The metric function f versus r, for various values of $\frac{M}{\sqrt{\theta}}$. The dashed, solid and dot-dashed lines correspond to $\frac{M}{\sqrt{\theta}} < 1.90412$, $\frac{M}{\sqrt{\theta}} = 1.90412$ and $\frac{M}{\sqrt{\theta}} > 1.90412$, respectively.

The noncommutative-geometry-inspired Schwarzschild black hole is described by the metric[2]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2), \quad (1)$$

where the metric function is

$$f(r) = 1 - \frac{4M}{r\sqrt{\pi}}\gamma\Big(\frac{3}{2}, \frac{r^2}{4\theta}\Big). \quad (2)$$

Here M is the total mass of the black hole, θ is a positive parameter with dimension of length squared, $\sqrt{\theta} \sim l_{\rm P}$ with $l_{\rm P}$ being the Planck length, and the

*Supported by the Natural Science Foundation of Education Department of Shannxi Province under Grant No 15JK1077, and the Doctorial Scientific Research Starting Fund of Shannxi University of Science and Technology under Grant No BJ12-02.

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lower incomplete Gamma function in the above equation is defined to be

$$\gamma\Big(\frac{3}{2},\frac{r^2}{4\theta}\Big)=\int_0^{\frac{r^2}{4\theta}} t^{\frac{1}{2}}e^{-t}dt. \tag{3}$$

In the limit $\frac{r}{\sqrt{\theta}}\to\infty$, Eq. (1) reduces the Schwarzschild metric. For the different values of $\frac{M}{\sqrt{\theta}}$, the noncommutative-geometry-inspired Schwarzschild black hole has different horizons: (i) for $\frac{M}{\sqrt{\theta}}>1.90412$, there are two horizons. (ii) For $\frac{M}{\sqrt{\theta}}=1.90412$, there is one degenerate horizon (extremal black hole). (iii) For $\frac{M}{\sqrt{\theta}}<1.90412$, there is no horizon (see Fig. 1). Therefore, for a noncommutative-geometry-inspired Schwarzschild black hole, the range of the noncommutative parameter θ is

$$0<\theta\leq\Big(\frac{M}{1.90412}\Big)^2. \tag{4}$$

Table 1. Quasinormal frequencies of massless scalar perturbations in the noncommutative-geometry-inspired Schwarzschild black hole spacetime for various values of the noncommutative parameter θ.

(l,n)	θ	ω	(l,n)	θ	ω
$l=1,\ n=0$	0.01	$0.29114-0.0980014i$	$l=1,\ n=0$	0.18	$0.290551-0.0999151i$
	0.05	$0.29114-0.0980014i$		0.2	$0.287728-0.096964i$
	0.1	$0.29112-0.0980015i$		0.22	$0.283388-0.0915783i$
	0.12	$0.291125-0.0982266i$		0.24	$0.278354-0.0850454i$
	0.14	$0.291525-0.099114i$		0.26	$0.273416-0.0787232i$
	0.16	$0.29161-0.100213i$		0.2758	$0.26988-0.0744508i$
$l=2,\ n=0$	0.01	$0.483211-0.0968049i$	$l=2,\ n=0$	0.18	$0.482649-0.0971593i$
	0.05	$0.483211-0.0968049i$		0.2	$0.481729-0.0962942i$
	0.1	$0.483212-0.0968079i$		0.22	$0.480364-0.0945725i$
	0.12	$0.48322-0.0968592i$		0.24	$0.478712-0.0921738i$
	0.14	$0.48322-0.0970471i$		0.26	$0.476963-0.0893755i$
	0.16	$0.483098-0.0972771i$		0.2758	$0.475597-0.0870308i$
$l=2,\ n=1$	0.01	$0.463192-0.29581i$	$l=2,\ n=1$	0.18	$0.461173-0.297498i$
	0.05	$0.463192-0.29581i$		0.2	$0.455213-0.293767i$
	0.1	$0.463203-0.295826i$		0.22	$0.445895-0.286786i$
	0.12	$0.463354-0.296081i$		0.24	$0.434555-0.277761i$
	0.14	$0.463743-0.296986i$		0.26	$0.422695-0.268236i$
	0.16	$0.463589-0.298049i$		0.2758	$0.413607-0.261227i$
$l=3,\ n=0$	0.01	$0.675206-0.0965121i$	$l=3,\ n=0$	0.18	$0.674742-0.0965267i$
	0.05	$0.675206-0.0965121i$		0.2	$0.674181-0.0959655i$
	0.1	$0.675206-0.0965145i$		0.22	$0.673408-0.0949062i$
	0.12	$0.675205-0.0965395i$		0.24	$0.672516-0.0933756i$
	0.14	$0.675179-0.0966152i$		0.26	$0.671609-0.091437i$
	0.16	$0.675057-0.0966761i$		0.2758	$0.670928-0.0896446i$
$l=3,\ n=1$	0.01	$0.660414-0.292344i$	$l=3,\ n=1$	0.18	$0.658335-0.29263i$
	0.05	$0.660414-0.292344i$		0.2	$0.655025-0.290175i$
	0.1	$0.660419-0.292358i$		0.22	$0.650244-0.285811i$
	0.12	$0.660457-0.292498i$		0.24	$0.644568-0.280004i$
	0.14	$0.660466-0.292917i$		0.26	$0.638551-0.273344i$
	0.16	$0.660001-0.293274i$		0.2758	$0.633718-0.267802i$
$l=3,\ n=2$	0.01	$0.634839-0.494118i$	$l=3,\ n=2$	0.18	$0.629835-0.495267i$
	0.05	$0.634839-0.494118i$		0.2	$0.619496-0.48972i$
	0.1	$0.634866-0.494155i$		0.22	$0.604183-0.480198i$
	0.12	$0.635092-0.494532i$		0.24	$0.586009-0.468323i$
	0.14	$0.635493-0.495653i$		0.26	$0.567136-0.456014i$
	0.16	$0.634615-0.49666i$		0.2758	$0.552608-0.447159i$

Before discussing QNMs of the noncommutative-geometry-inspired Schwarzschild black hole, let us check the regularity of the spacetime described by metric (1). To demonstrate this, it is necessary to calculate the Ricci scalar R, the Ricci square $R_{\mu\nu}R^{\mu\nu}$ and the Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. Direct calculation reveals that they are expressed as

$$R=\frac{4M}{\sqrt{\pi}}\Big(\frac{1}{\theta^{\frac{3}{2}}}-\frac{r^2}{8\theta^{\frac{5}{2}}}\Big)e^{-\frac{r^2}{4\theta}}, \tag{5}$$

$$\lim_{r\to0}R=\frac{4M}{\sqrt{\pi}\theta^{\frac{3}{2}}}, \tag{6}$$

$$R_{\mu\nu}R^{\mu\nu}=\frac{8M^2}{\pi}\Big(\frac{1}{2\theta^3}-\frac{r^2}{8\theta^4}+\frac{r^4}{64\theta^5}\Big)e^{-\frac{r^2}{2\theta}}, \tag{7}$$

$$\lim_{r\to0}R_{\mu\nu}R^{\mu\nu}=\frac{4M^2}{\pi\theta^3}, \tag{8}$$

$$\begin{aligned}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}=&\frac{16M^2}{\pi}\Big[\frac{12}{r^6}\gamma^2\Big(\frac{3}{2},\frac{r^2}{4\theta}\Big)\\&-\Big(\frac{2}{r^3\theta^{\frac{3}{2}}}+\frac{1}{2r\theta^{\frac{5}{2}}}\Big)\gamma\Big(\frac{3}{2},\frac{r^2}{4\theta}\Big)e^{-\frac{r^2}{4\theta}}\\&+\Big(\frac{1}{4\theta^3}+\frac{r^4}{64\theta^5}\Big)e^{-\frac{r^2}{2\theta}}\Big],\end{aligned} \tag{9}$$

$$\lim_{r\to0}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}=\frac{8M^2}{3\pi\theta^3}. \tag{10}$$

Equations (5)–(10) show that the noncommutative-geometry-inspired Schwarzschild black hole spacetime is regular everywhere, including the origin. In this

study, we focus on the massless scalar field perturbation. The massless scalar field is governed by the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi)=0, \tag{11}$$

where $-g$ is the absolute value of the determinant of the noncommutative-geometry-inspired Schwarzschild black hole spacetime metric, and Φ is the scalar field.

Introducing

$$\Phi=e^{-i\omega t}Y_{l,m}(\vartheta,\varphi)\frac{\Psi(r)}{r}, \tag{12}$$

where $Y_{l,m}(\vartheta,\varphi)$ is spherical harmonics with l and m being the angular quantum number and magnetic quantum, respectively, and substituting Eqs. (1) and (6) into Eq. (5), we obtain the radial component of the Klein–Gordon equation in tortoise coordinate $x=\int\frac{dr}{f(r)}$,[31,32]

$$\frac{d^2\Psi}{dx^2}+[\omega^2-V(r)]\Psi(x)=0, \tag{13}$$

where ω is the complex QNM frequency and the effective potential $V(r)$ is of the form

$$V(r)=f(r)\Big[\frac{l(l+1)}{r^2}+\frac{1}{r}\frac{d}{dr}f(r)\Big]. \tag{14}$$

For convenience, we take $M=1$ henceforth. Thus $f(r)$ reduces to

$$f(r)=1-\frac{4}{r\sqrt{\pi}}\gamma\Big(\frac{3}{2},\frac{r^2}{4\theta}\Big). \tag{15}$$

According to Eq. (4), the black hole has horizons only when the noncommutative parameter θ satisfies the condition $\theta\leq 0.2758$.

The boundary conditions defining QNMs can be written as

$$\Psi(x)\sim\pm e^{\pm i\omega x}\ \text{as}\ x\longrightarrow\mp\infty. \tag{16}$$

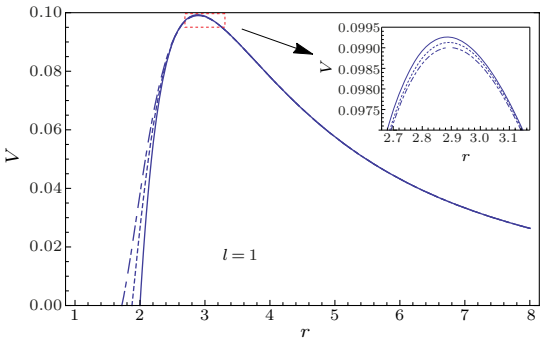

Fig. 2. The effective potential V versus r for the massless scalar field perturbation with $l=1$ in the noncommutative-geometry-inspired Schwarzschild black hole spacetime. The dashed and dot-dashed lines correspond to $\theta=0.24$ and 0.27, respectively. The corresponding one in the Schwarzschild black hole spacetime is also plotted (solid line) for comparison.

In Fig. 2, we show the dependence of effective potential V on radial coordinate r for the massless scalar field in the noncommutative-geometry-inspired Schwarzschild black hole spacetime for $l=1$ and $\theta=0.24$ and 0.27. The corresponding effective potential in the Schwarzschild black hole spacetime is also plotted for comparison.

From this figure, one can find that the height of the potential barrier becomes lower compared with that in the Schwarzschild spacetime, and the height of the potential barrier decreases as the noncommutative parameter θ increases.

The real parts and the absolute values of the imaginary parts of the QNM frequencies as a function of θ are plotted in Fig. 3. For comparison, the corresponding results in the Schwarzschild spacetime are also shown in Fig. 3 and listed in Table 2.

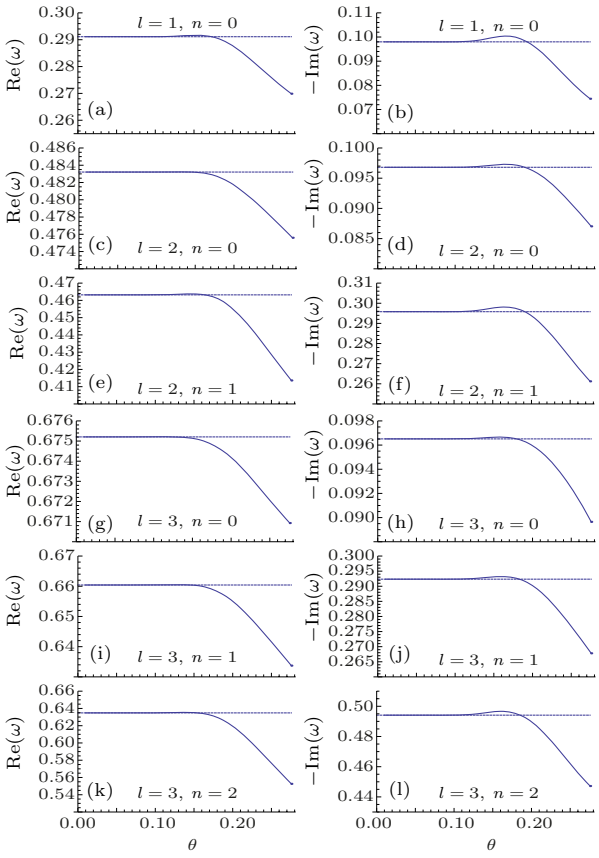

Fig. 3. Real parts and imaginary parts of QNM frequencies of the massless scalar perturbations in the noncommutative-geometry-inspired Schwarzschild black hole spacetime are plotted as a function of θ. Circle points at the end of lines correspond to cases of extremal noncommutative-geometry-inspired Schwarzschild black holes. Dashed lines denote values of the real or imaginary parts of QNM frequencies of the corresponding Schwarzschild black holes.

As the effective potential V has a peak and has the form of a potential barrier, one can compute the QNM frequencies by adopting the WKB approximate approach. In this work, we evaluate QNM frequencies using the third-order WKB approximation. The formula for the QNM frequencies can be found in Ref. [29]. It is known that the accuracy of the third-

order WKB approximation is excellent for any $n < l$, where n is the overtune number. Therefore, we only list the results for $n < l$ in Table 1.

Table 2. Quasinormal frequencies of massless scalar perturbations in the Schwarzschild spacetime.

(l, n)	ω
$l = 1, n = 0$	$0.291114 - 0.0980014i$
$l = 2, n = 0$	$0.483211 - 0.0968049i$
$l = 2, n = 1$	$0.463192 - 0.29581i$
$l = 3, n = 0$	$0.675206 - 0.0965121i$
$l = 3, n = 1$	$0.660414 - 0.292344i$
$l = 3, n = 2$	$0.634839 - 0.494118i$

Table 1 and Fig. 3 tell us that: (i) for all ω computed, $I_m(\omega)$ was negative, thus the black hole is stable under scalar perturbations. (ii) Explicitly, the noncommutative parameter plays an important role for QNM frequencies. (iii) For fixed l and n and varying θ, the real part and the absolute value of the imaginary part of the QNM frequency in the noncommutative-geometry-inspired two-horizon Schwarzschild black hole spacetime are smaller or larger compared with the corresponding ones in the Schwarzschild black hole spacetime. This means that the oscillations of the massless scalar field dampen more slowly or quickly in the noncommutative-geometry-inspired two-horizon Schwarzschild black hole spacetime. However, the real part and the absolute value of the imaginary part of the QNM frequency in the extremal noncommutative geometry inspired Schwarzschild black hole spacetime are always smaller compared with the corresponding ones in the Schwarzschild black hole spacetime. Thus the oscillations of the massless scalar field dampen more slowly in the extremal noncommutative-geometry-inspired Schwarzschild black hole spacetime.

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