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# Controlling of the Polarization States of Electromagnetic Waves Using Epsilon-near-Zero Metamaterials *

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*We demonstrate theoretically that the epsilon-near-zero materials can be utilized to control effectively the polarization conversion of an electromagnetic wave through reflection. The significant feature differing from all other means based on whatever natural materials or metamaterials is that for TM incident wave, the reflected phase is a constant, while for TE wave, the reflected phase is a linear function of the incident angle. The phase difference between them covers the range from $-180^\circ$ to $0^\circ$, and the polarization conversions from linear states to elliptical or circular states can be obtained by only adjusting the incident angle. Because no complex structures are employed, our proposal promises a simple approach for manipulating polarization conversion at both terahertz and optical frequencies.*

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The manipulation of electromagnetic (EM) wave polarization is one of the essential issues in optical communities. In particular, it may be of interest to have a simple approach to effectively control the polarization conversions from linear state to circular one, and vice versa, due to a variety of optical applications,[1–4] such as the spin–orbit interactions of light.[5] A conventional way for this aim utilizes the phase retardation effect between two orthogonally polarized wave components when they pass through a birefringent material.[6] However, all natural birefringent materials have a small ratio between refractive indices, giving rise to relatively thick retardation plates. With the advent of artificial metamaterials that trigger a number of fascinating phenomena, such as negative refraction[7,8] and invisibility cloaking,[9,10] they provide another technique for manipulating polarization conversion.[11–17] It was shown that through total reflection, an anisotropic metamaterial plate designed by H-shaped structures can be used to effectively manipulate EM wave polarization.[11] However, when the desired working frequencies go into high region, such as terahertz or even optical frequencies, the complicated structures bring challenges on nano-fabricated technology. In addition, the energy loss resulting from structure resonances may decrease the conversion efficiency. The planar metamaterials, called metasurfaces, may overcome the loss problem moderately,[16,17] because of ultrathin thickness compared with working wavelength, but the complex structures are still required.

Recently, epsilon-near-zero (ENZ) materials have attracted much attention, due to their intriguing properties.[18–26] It was shown that for both transverse electric (TE) and transverse magnetic (TM) polarized waves, when they strike obliquely from air to ENZ material, the internal total reflections occur. In such a process, however, both polarized waves suffer extremely different physics, leading to distinct Goos–Hänchen (G-H) effect: for TE polarization, the G-H shift is a constant for any incident angle excepting zero; for TM polarization, there is no G-H effect.[26] For fabrications in practice, the ENZ materials in terahertz region can be obtained by doped semiconductors or 2D graphenes;[27] in the optical region, the metals at plasma frequency behave as ENZ materials,[27] or they can be fabricated by layered structures.[28,29] This research progress in ENZ materials provides an opportunity to control EM wave polarization in the terahertz or optical region.

In this work, we show theoretically that the ENZ metamaterials can also be used to manipulate the polarization state of EM wave through total reflection. The significant feature differing from all other means based on any natural materials or metamaterials is that for TM incident wave, the reflected phase is a constant, while for TE wave, the reflected phase is a linear function of the incident angle. The phase difference between them covers the range from $-180^\circ$ to $0^\circ$, and the polarization conversions from linear states to elliptical or circular states can be obtained by only adjusting the incident angle. Compared to the previous work[11] in which one state conversion is at one working frequency, while another state conversion should be shifted to another frequency, our proposal is more convenient for controlling the polarization conversion, particularly at high frequencies.

To demonstrate our idea, we start from studying a perfect ENZ material which is a lossless ENZ with exactly zero critical angle. Figure 1 shows the considered configuration, in which region 1 is air and region 2 is the ENZ material with parameters $\varepsilon_2 \approx 0$ and $\mu_2 = 1$. The model system is infinitely expanded in $x$ and $z$ directions. Suppose a linearly polarized

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wave with an incident angle $\theta_{\rm in}$ striking obliquely the ENZ material from air, which is given in term of the electric field vector $\boldsymbol{E}^{\rm in} = \boldsymbol{E}_0 \exp(i\boldsymbol{k}_{\rm in}\boldsymbol{r} + i\omega t)$, where $E_0$ is the amplitude of the electric field vector, $|\boldsymbol{k}_{\rm in}| = k_0$ is the wave vector, and $\omega$ is the angular frequency. The angle $\phi$ indicates the direction of the electric field vector with respect to the $z$ axis. As is well known, for an arbitrary linearly polarized wave, it can be superposed by two TE and TM polarizations without phase differences between them. In this way, the incident wave can be expressed further by $\boldsymbol{E}_0 = E_{\parallel}\hat{p} + E_{\perp}\hat{s}$, where $E_{\parallel} = E_0 \sin\phi$, $E_{\perp} = E_0 \cos\phi$, and $\hat{p}$ ($\hat{s}$) is the unit vector in the TM (TE) component. After reflection by the ENZ material, the reflected wave is $\boldsymbol{E}^{\rho} = (\rho_{\rm p} E_{\parallel}\hat{p} + \rho_{\rm s} E_{\perp}\hat{s}) \exp(i\boldsymbol{k}_{\rho}\boldsymbol{r} + i\omega t)$, where $\rho_{\rm p} = |\rho_{\rm p}| \exp(i\varphi_{\rm p})$ and $\rho_{\rm s} = |\rho_{\rm s}| \exp(i\varphi_{\rm s})$ are the reflection coefficients, with $\varphi_{\rm p}$ and $\varphi_{\rm s}$ denoting corresponding reflection phases. Conventionally, the polarization state of reflected wave is defined by the time variation of the tip of the electric field vector at a fixed point in space. If the locus of the tip is a circle, the reflected wave is circularly polarized. For a mathematical discussion of polarizations, we employ two parameters, that is, $A = |\rho_{\rm p} E_{\parallel}|/|\rho_{\rm s} E_{\perp}|$ and $\Delta = \varphi_{\rm p} - \varphi_{\rm s}$ (the phase differences between TE and TM components). All possible polarization states are illustrated by two difference values. For instance, when $A = 1$ and $\Delta = \pm 90°$, the EM wave is circularly polarized.

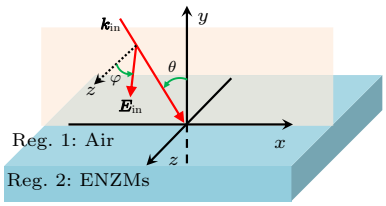


**Fig. 1.** (Color online) The schematic diagram of manipulating the polarization state of EM wave by using ENZ materials. A linearly polarized EM wave (in the $x$–$y$ plane) is obliquely incident from air (region 1) into ENZ material (region 2) with parameters $\varepsilon_2 \approx 0$ and $\mu_2 = 1$. The intersection angle between the electric field vector and the $z$ axis is defined as $\phi$.

To examine the polarization state of the reflected wave, now we analyze the reflection coefficients of both TE and TM components. By applying the continuous boundary conditions of EM fields at plane $y = 0$ for TE wave, the reflection coefficient is calculated as follows:

$$\rho_{\rm s} = \frac{\cos\theta_{\rm in} - i\sqrt{\sin^2\theta_{\rm in} - \varepsilon_2}}{\cos\theta_{\rm in} + i\sqrt{\sin^2\theta_{\rm in} - \varepsilon_2}} = |\rho_{\rm s}| \exp(i\varphi_{\rm s}), \quad (1a)$$

$$\varphi_{\rm s} = -2\arctan(\sqrt{\sin^2\theta_{\rm in} - \varepsilon_2}/\cos\theta_{\rm in}). \quad (1b)$$

For TM wave, the reflection coefficient is expressed as

$$\rho_{\rm p} = \frac{\varepsilon_2\cos\theta_{\rm in} - i\sqrt{\sin^2\theta_{\rm in} - \varepsilon_2}}{\varepsilon_2\cos\theta_{\rm in} + i\sqrt{\sin^2\theta_{\rm in} - \varepsilon_2}} = |\rho_{\rm p}| \exp(i\varphi_{\rm p}), \quad (2a)$$

$$\varphi_{\rm p} = -2\arctan[\sqrt{\sin^2\theta_{\rm in} - \varepsilon_2}/(\varepsilon_2\cos\theta_{\rm in})]. \quad (2b)$$

Based on these formulas, when $\varepsilon_2 = 0$ and $\theta_{\rm in} \neq 0$, then $|\rho_{\rm s}| = |\rho_{\rm p}| = 1$, $\varphi_{\rm s} = -2\theta_{\rm in}$ and $\varphi_{\rm p} = -\pi$. In this way, the phase difference is $\Delta = -\pi + 2\theta_{\rm in}$, which is only dependent on the incident angle. To visualize the above theoretical analysis, Fig. 2(a) shows the relationship between the reflectance $R = |\rho|^2$ (transmittance $T = 1 - R$) and the incident angle for TE waves, while Fig. 2(a) shows that for TM waves. Obviously, the internal total refection of both polarizations results in a unity reflectance for oblique incidence. Figure 2(c) displays the reflection phase, in which the red line is for TE polarization and the red dashed line is for TM polarization. For TE, the reflection phase is a linear function of incident angle; for TM, the reflection phase is a constant value (i.e., $-180^{\circ}$), verifying the fact that the ENZ material for TM polarization serves as a perfect magnetic conductor. The phase differences $\Delta$ for all incident angles are illustrated in Fig. 2(d), which are located in the range of $(-180°, 0°)$.

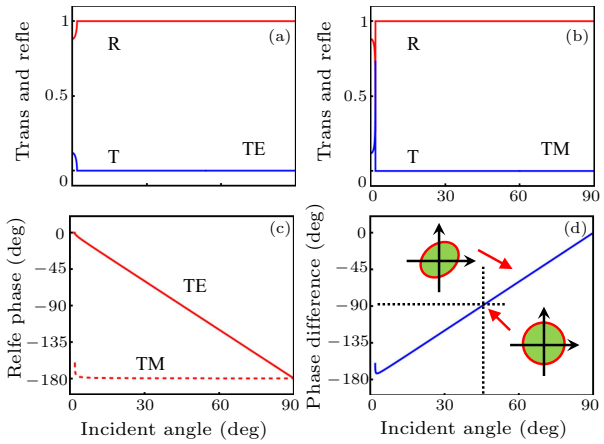


**Fig. 2.** (Color online) Lossless ENZ materials. (a) The reflection and transmission of TE incident wave. (b) The reflection and transmission of TM incident wave. (c) The reflection phases of TE and TM waves. (d) The phase differences between TE and TM components in reflected wave. In particular, at $\phi = 45°$ and $\theta_{\rm in} = 45°$, the phase difference is $\Delta = 90°$, leading a circular polarized state of reflected wave. In calculations, the parameters of the ENZ material are set as $\varepsilon_2 = 10^{-3}$ and $\mu = 1$. Otherwise, the polarization state of reflected waves are elliptical.

Based on the above theoretical results about reflection coefficients, now let us check possible polarization states of the reflected wave. Based on the abovementioned parameters signing polarization states, we know $A = \tan\phi$ and $\Delta = -\pi + 2\theta_{\rm in}$ for the current case, which are controlled by two angles $\phi$ and $\theta_{\rm in}$. The parameter $A$ belongs to the range $(0, \infty)$ when the angle $\phi$ varies within $(0°, 90°)$. Apparently, when $\phi = 45°$ and $\theta_{\rm in} = 45°$, then $A = 1$ and $\Delta = -90°$, which means that the reflected wave is circularly polarized. Otherwise, the reflected wave is elliptically polarized. To quantify the polarization conversion, the conversion efficiency can be defined by $\eta = (|\rho_{\rm p} E_{\parallel}|^2 + |\rho_{\rm s} E_{\perp}|^2)/(|E_{\parallel}|^2 + |E_{\perp}|^2)$. For the lossless ENZ material, there is no energy dissipation

for both components in the process of total reflection, leading to a perfect conversion, with efficiency 100%.

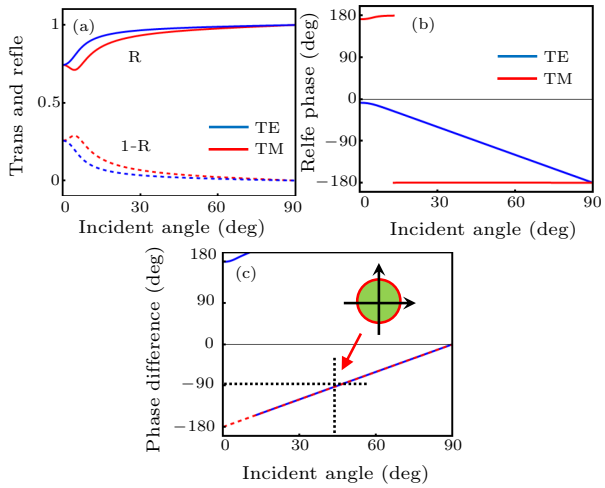


**Fig. 3.** (Color online) Low-loss ENZ materials. (a) The reflection and transmission of both TE and TM incident waves. (b) The reflection phases of TE and TM waves. (c) The blue line is the phase differences between TE and TM components in reflected wave, and for comparison, the phase difference of the perfect case, that is, $\Delta = -\pi + 2\theta_{\rm in}$, is shown by the red dashed line. In all the calculations, the parameters of the ENZ material are $\varepsilon_2 = 10^{-3} + 10^{-2}i$ and $\mu_2 = 1$.

In real situations, however, the ENZ materials have loss, such as doped semiconductors at terahertz.[27] In the following we investigate the influence of material loss on controlling the polarization state of reflected wave using the ENZ material. For simplicity, we consider a complex permittivity $\varepsilon_2 = \varepsilon_2' + i\varepsilon_2''$, with imaginary part $\varepsilon_2''$ indicating the loss. When a linearly polarized EM wave is incident from air onto the loss ENZ material, the reflection coefficients can be theoretically obtained by Eq. (1) for TE wave and Eq. (2) for TM wave. Firstly, we discuss a low-loss case. As a demonstration, an ENZ material with $\varepsilon_2 = 10^{-3} + 0.01i$ is taken under consideration. Figure 3(a) illustrates the reflectance of both TE and TM waves for full incident angles, and their corresponding reflection phases are plotted by blue and red lines in Fig. 3(b). Due to the material loss, the reflectance of both polarizations cannot reach unity, particularly at small incident angles. In Fig. 3(c), the blue line illustrates the phase differences between two components, which perfectly coincide with that of the lossless case (shown by the red dashed line). On the other hand, we can obtain $A(\phi, \theta_{\rm in}) = \tan\phi|\rho_{\rm p}(\theta_{\rm in})|/|\rho_{\rm s}(\theta_{\rm in})|$, which still varies from 0 to infinity as the angle $\phi$ changes from $0^\circ$ to $90^\circ$. Based on these results, it is evident that the polarization state of reflected wave still can be controlled effectively by two angles $\phi$ and $\theta_{\rm in}$. Now let us examine how to realize the polarization conversion from linear state to circular state, which is of most interest to us. Similarly the incident angle is at $\theta_{\rm in} = 45^\circ$, giving rise to $\Delta = -90^\circ$. Meanwhile, at this incident angle, it has $|\rho_{\rm s}| = 0.99$ (calculated by Eq. (1(a)) and $|\rho_{\rm p}| = 0.98$ (calculated by the Eq. (2(a)). To meet another condition $A(\phi, \theta_{\rm in}) = 1$, then $\tan\phi = |\rho_{\rm s}(\theta_{\rm in})|/|\rho_{\rm p}(\theta_{\rm in})| = 1.01$, which is satisfied when $\phi = 45.3^\circ$. Consequently, for the linearly incident EM wave, when $\theta_{\rm in} = 45^\circ$ and $\phi = 45.3^\circ$, the polarization of the reflected wave is the circular state. After calculation, the conversion efficiency reaches approximately $\eta = 97\%$, revealing an excellent performance of manipulating the polarization state by the loss ENZ material.

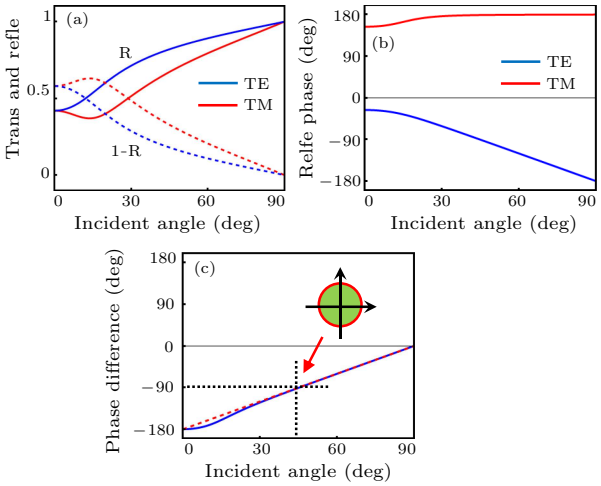


**Fig. 4.** (Color online) The high-loss ENZ material. (a) The reflection and transmission of both TE and TM incident waves. (b) The reflection phases of TE and TM waves. (c) The blue line is the phase difference $\Delta = \varphi_{\rm p} - \varphi_{\rm s} - 360^\circ$, while the red dashed line is $\Delta = -\pi + 2\theta_{\rm in}$ for comparison. In all the calculations, we set $\varepsilon_2 = 10^{-3} + 10^{-1}i$ and $\mu_2 = 1$.

Alternately another high-loss case is concerned, that is, $\varepsilon_2 = 10^{-3} + 0.1i$. It is noted that such a loss level is large enough in research field of ENZ materials, which can make many fascinating phenomena disappear.[24,26] Similarly, based on Eqs. (1) and (2), the reflectance and the corresponding reflection phases of both polarizations can be figured out, as shown in Figs. 4(a) and 4(b), respectively. Obviously, as the loss becomes larger, the ENZ material absorbs more EM energy in the process of reflection, resulting in a larger decline in reflectance. Nevertheless, the influence of loss on the phase differences between two components is not so serious. As shown by the blue line in Fig. 4(c), the phase difference is nearly a linear function of incident angle, especially at large angles. For obtaining a reflected wave of circular polarization, the required phase difference $\Delta = -90^\circ$ can be met when $\theta_{\rm in} = 45^\circ$, and at this incident angle, we know $|\rho_{\rm s}| = 0.91$ and $|\rho_{\rm p}| = 0.82$. On the other hand, the condition $A(\phi, \theta_{\rm in}) = 1$ requires $\phi = 48.0^\circ$. After performing some computations, the conversion efficiency is about $\eta = 74\%$, which is moderated. In the light of the above discussions on ENZ materials with two loss levels, we can know that when the material loss is introduced, our proposed approach still works well for manipulating the polarization state of reflected wave.

In particular, the polarization conversion from linear state to circular one still can be realized via adjusting two angles $\phi$ and $\theta_{\rm in}$. The main influence of loss reflects in conversion efficiency. For low loss, the conversion efficiency is quite high; while for high loss, the conversion efficiency is not desired.

In conclusion, we have shown that the ENZ materials can be employed to manipulate the polarization conversions from a linear to elliptic and circular states, by adjusting the incident angle. The efficiencies are 100% for lossless ENZ materials. We also analyze the influences of ENZ material losses on the polarization conversion. When the loss is not very high, the linear-to-circular polarization conversion is still feasible with high efficiencies, which reveals a robust feature of proposed approach. We expect that our suggestion can be confirmed experimentally at terahertz or optical frequencies, considering the recent progress of ENZ materials.[28,29] Some works suggested the tunable zero index materials through external magnetic field or temperature,[30,31] which might service as building blocks for tuning the working frequencies of the ENZ-based polarization conversions. Although the studies of this work are focused on the plane wave, our proposal can also work for a beam with small angular width, for example, a Gaussian beam with a large waist.

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